Summary

Units (SI):
Length: m = meter
Time: s = second
Mass: kg = kilogram; atomic mass unit $u = 1.661 \times 10^{-27}$ $kg = m^{(12)C}/12$
Velocity: m/s
Acceleration: m/s$^2$
Momentum: kg m/s
Force: N = Newton = kg m/s$^2$
Energy: J = Joule = Nm = kg m$^2$/s$^2 = Ws$; eV = electron-Volt = $1.602 \times 10^{-19}$ J
Power: W = Watt = J/s
Charge: C = Coulomb
Currents: A = Ampere = C/s
Electric Field: N/C = V/m
Electric Potential: V = Volt = J/C
Electric Resistance: Ω = Ohm = V/A
Magnetic Field: T = Tesla = Vs/m$^2 = 10,000$ Gauss
Amount: mol (1 mol = $N_A$ molecules = A gram, where $A$ is atomic mass)
Density $\rho$: kg/m$^3$
Pressure: Pa = Pascal = N/m$^2$; 100,000 Pa $\approx 1$ atmosphere
Temperature: K = Kelvin (C = Celsius, F = Fahrenheit);
$32^\circ$F $= 0^\circ$C $= 273.15$ K; $212^\circ$F $= 100^\circ$C $= 373.15$K
Heat: J or 1 calorie $= 4.187$ J; 1 food calorie $= 1000$ calories $= 4187$ J
Frequency: Hz $= 1$/s

Prefixes:
kilo $= k = 10^3 = 1000 = $Thousand
Mega $= M = 10^6 = 1,000,000 = $Million
Giga $= G = 10^9 = 1,000,000,000 = $Billion
Tera $= T = 10^{12} = $Trillion, Peta $= P = 10^{15} = $Quadrillion
centi $= c = 10^{-2} = 0.01 = 1$-hundreth
milli $= m = 10^{-3} = 0.001 = 1$-thousandth
micro $= \mu = 10^{-6} = 0.000,001 = 1$-millionth
nano $= n = 10^{-9}$, pico $= p = 10^{-12}$, femto $= f = 10^{-15}$
Useful Constants:
Gravitational constant: \( G = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2 \)
Gravitational acceleration at surface of Earth: \( g = 9.81 \text{ m/s}^2 \)
Mass of Sun: \( 1.99 \times 10^{30} \text{ kg} \)
Distance from Earth to Sun: \( 1.50 \times 10^{11} \text{ m} \)
Mass of Moon: \( 7.35 \times 10^{22} \text{ kg} \)
Distance from Earth to Moon: \( 3.84 \times 10^8 \text{ m} \)
Mass of Earth: \( 5.97 \times 10^{24} \text{ kg} \)
Radius of Earth: \( 6.38 \times 10^6 \text{ m} \)
Earth’s magnetic field: about 0.5 Gauss = 0.0005 T (5 \times 10^{-5} \text{ T})
  (Magnetic north pole near Australia)
Elementary charge: \( e = 1.602 \times 10^{-19} \text{ C} \)
Permittivity constant: \( \varepsilon_o = 8.854 \times 10^{-12} \text{ F/m} \)
Permeability constant: \( \mu_o = 4\pi \times 10^{-7} \text{ H/m} \)
\( k \) (electrostatic force constant) = \( 1/4\pi\varepsilon_o = 8.99 \times 10^9 \text{ Nm}^2/\text{C}^2 \)
Speed of Light: \( c = 2.998 \times 10^8 \text{ m/s} = 1/\sqrt{\varepsilon_o \mu_o} \)
Avogadro's number:
\( N_A = 6.022 \times 10^{23} \) molecules/mol = number of \(^{12}\text{C} \) atoms in 12 g of carbon
Universal Gas Constant: \( R = 8.32 \text{ J/mol/K} \)
Density of water: 1000 kg/m\(^3\), air (sea level): 1.25 kg/m\(^3\), of iron: 7874 kg/m\(^3\)
Atmospheric pressure at sea level: 101,300 Pa
Typical speed of sound: 330 m/s – 340 m/s in air
Frequency of “middle A” musical note: 440 Hz
Electron mass: \( m_e = 9.109 \times 10^{-31} \text{ kg} \); \( E = mc^2 = 510,999 \text{ eV} = 511 \text{ keV} \)
Proton mass: \( m_p = 1.673 \times 10^{-27} \text{ kg} \); \( E = mc^2 = 938,272,030 \text{ eV} = 938.3 \text{ MeV} \)
Neutron mass: \( m_n = 1.675 \times 10^{-27} \text{ kg} \); \( E = mc^2 = 939,565,360 \text{ eV} = 939.6 \text{ MeV} \)
Planck’s constant:
\( h = 6.63 \times 10^{-34} \text{ Js} = 6.63 \times 10^{-25} \text{ kg m/s x nm} = 4.17 \times 10^{-15} \text{ eV / Hz} \)
=> \( 1/h = 1.51 \times 10^{33} \text{ Hz J} = 2.4 \times 10^{14} \text{ Hz/eV} \)
2 eV corresponds to \( \lambda = 620 \text{ nm} \) and \( f = 4.84 \times 10^{14} \text{ Hz} \) (yellow light)
Natural Science - The Scientific Method

1) Collect observations; categorize and develop detailed descriptions
2) Identify crucial parameters and conditions; develop systematic (quantitative, if possible) descriptions of these as well as measured outcomes (use precise definitions for all observables); abstract from “irrelevant” or “extraneous” perturbations and/or control for those. Avoid the “correlation = causation” fallacy – there could be “hidden variables” that influence both correlated observables
3) If possible, do systematic experiments varying one of the relevant parameters at a time; look for systematic changes in outcome depending on each [in-class analog: Do labs with changing parameters].
4) Develop a hypothesis (a – quantitative, if possible – relationship between parameters and outcomes); collect more data / do more experiments to test hypothesis
5) If your hypothesis describes your data approximately, call it a “Model”; if it agrees with all observations, call it a “Law”
6) A coherent set of “Laws” is called a “Theory”; all parts of a Theory must be logically (and mathematically, if applicable) consistent with each other as well as with previously established theories (unless you can prove those wrong)
7) Apply “Occam’s razor”: Your theory should have (only) the minimum number of ingredients required to describe all relevant observations; do not invoke “extra-natural causes”
8) Derive testable predictions from your theory or law; in particular, explore all new and previously unforeseen consequences [in-class analog: Do problems]
9) Test your predictions against observations and experiments; discard or modify theory (e.g., limit range of applicability) if test results disagree with predictions.
10) All theories are “tentative” at some level – there could always be contradictory observations in the future – but don’t discard well-tested theories needlessly!

Non-science

- “Theories” leading to untestable predictions (predictions that are too vague to be “falsifiable” or that refer to unmeasurable phenomena)
- Ad-hoc hypotheses (a new one for every new observation), anecdotal evidence, testimonials
- value statements, matters of taste or opinion, exclusive reliance on authority or precedent, normative statements (“you should…!”)

...and now to PHYSICS...
**Motion in 1 dimension**

Average velocity for time interval $\Delta t = t_1 \ldots t_2$

$$v_{av} = \frac{\text{change in position during the time } \Delta t}{\text{elapsed time } \Delta t} = \frac{x(t_2) - x(t_1)}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$$

**Instantaneous velocity at time t**

$= \text{average velocity for a very short time interval around } t ; \frac{dx}{dt}$

**Relative velocity addition**

$\vec{v}(x \text{ relative to } B) = \vec{v}(x \text{ relative to } A) + \vec{v}(A \text{ relative to } B)$

**Acceleration for time interval $\Delta t = t_1 \ldots t_2$**

$$a = \frac{\text{change in velocity during time } \Delta t}{\text{elapsed time interval } \Delta t} = \frac{\Delta v}{\Delta t}$$

**Motion in 1 dimension with constant acceleration $a$:**

$$v(t) = v(0) + a \ t = v_0 + a \ t$$

$$v_{av, \ (0 \ \ldots \ t_1)} = \frac{1}{2} [v_0 + v(t_1)]$$

$$x(t) = x_0 + v_0 \ t + \frac{1}{2} a \ t^2$$

**Forces – some Examples**

Weight (Gravity): $\vec{F}_{\text{weight}} = -mg$ (in negative vertical direction, up = +); $g = 9.81$ m/s$^2$ near Earth’s surface

Force exerted by spring (Hooke’s Law): $\vec{F}_{el} = -kx$ (opposite to the direction of the displacement $x$ from relaxed state of spring; with $k =$ spring constant)

Normal force $F_n$: Equal and opposite to sum of perpendicular (to a surface) components of all other forces (makes net perpendicular force zero)

Static Friction $f_{\text{stat}}$ (object at rest): equal and opposite to sum of parallel components of all other forces, canceling them if the sum is not too big: $|f_{\text{stat}}| \leq \mu_s |F_n|$.

Kinetic friction $f_{\text{kin}}$ (moving object): Force opposite to direction of motion along surface, $|f_{\text{kin}}| = \mu_k |F_n|$.

Tension $T$: Force along direction of string, at the end it is the same as the force exerted by the string on the attachment point (towards the string).
Newton's First Law
When all forces applied to an object balance out to zero (cancel each other), then this object will not accelerate relative to an inertial system; it will be in equilibrium: If at rest, it will stay at rest, if in motion, it will continue to move in the same direction, with constant velocity $\Rightarrow \mathbf{a} = 0$.

Newton’s Second Law
$a = \mathbf{F}/m$; acceleration equals net force divided by mass. More detailed:

$$
\mathbf{F}_{\text{resultant}} = \sum_{i=1}^{N} \mathbf{F}_i = m \mathbf{a}
$$

(Sum over all forces, including direction!)

Note: Forces add like vectors (parallelogram rule!)

Newton’s Third Law
Forces always come in pairs (interaction between two objects A and B):

$$
\mathbf{F}_{\text{Action}} (A \text{ on } B) = - \mathbf{F}_{\text{Reaction}} (B \text{ on } A)
$$

Momentum
For a single object: $\mathbf{p} = m \mathbf{v}$

Change in momentum (impulse): $\mathbf{J} = \Delta \mathbf{p} = \sum \mathbf{F} \Delta t$

Because of Newton’s 3$^{rd}$ Law, impulse transferred from object A to object B is equal in magnitude but opposite in sign (direction) to simultaneously transferred impulse from B to A.

System of particle with no external force: Total momentum $\sum \mathbf{p}_i = \mathbf{p}_1 + \mathbf{p}_2 + \ldots$ is conserved (same before as after any collision between any particles in the system)

Momentum conservation in two-particle collision:

$$
m_1 \mathbf{v}_{1i} + m_2 \mathbf{v}_{2i} = m_1 \mathbf{v}_{1f} + m_2 \mathbf{v}_{2f} \quad (i = \text{initial, } f = \text{final})
$$

If sum of all kinetic energies is also conserved (no work done on system as a whole, no energy converted to other forms; see below): ELASTIC collision. Otherwise (friction, conversion to heat/deformation,..): INELASTIC collision. If 2 objects stick together after collision: Completely inelastic

If collision is completely inelastic: $m_1 \mathbf{v}_{1i} + m_2 \mathbf{v}_{2i} = (m_1 + m_2) \mathbf{v}_{f}$
Work and Energy

Work $\Delta W$ on some object (particle or system) due to a force $F$ if that object is displaced by $\Delta x$ in the direction of the force: $\Delta W = F \Delta x = F \Delta s \cos \phi$ (only the part of the total displacement $\Delta s$ that is in the direction of the force counts; no displacement = no work). Work done on object can be positive (transferred TO object) or negative (transferred FROM object).

Work-Energy Theorem: Net work done on an object changes the ENERGY of that object: $\Delta E$ (particle or system) = $\Delta W$ (done on that particle or system)

Conservation of total Energy

The total amount of energy within a system can only be changed by exchanging energy with another system. IF the energy of one object (particle, system,...) goes UP, then the energy of another object (particle, system,...) must go DOWN by the same amount, and vice versa!

Power: Rate of work done on system per unit time: $P = \frac{\Delta W}{\Delta t} = F \cdot v$

Types of Energy

Kinetic energy of a moving particle: K.E. = $\frac{m}{2} \vec{v}^2$ (depends only on speed; is never negative; has no direction; depends on coordinate system used for $\vec{v}$)
If the only (relevant, changing) energy of a system is kinetic energy, then the work-energy theorem states: $\Delta$ K.E. = $\Delta W$

Potential Energy

Some forces can do work that depends only on initial and final position of the particle(s) that this work is being done on. Example: Gravity – only the difference in height between initial and final state matters: $\Delta W = mg \Delta h$
These forces are called “conservative” because they can “store” work and return it without losses. Any work done against a conservative force increases stored “potential” for future positive work by the same force.
Work done by conservative force stored in form of potential energy:
$U (\vec{r}) = \Delta W_{ext} (\vec{r}_{ref} \rightarrow \vec{r}) = -F_{\text{conservative}} \Delta s \cos \phi$
At reference point $U (\vec{r}_{ref}) = 0$ ($\vec{r}_{ref}$ can be chosen for convenience).

Examples:
- Particle attached to a spring, stretching the spring by moving a distance $x$ from the spring’s relaxed state: $U_{el} = (k / 2) x^2$
  ($k$ = spring constant; $x$ = 0 $\Rightarrow$ $U_{el} = 0$ for relaxed state of spring); more general: totally elastic distortion of objects
• Approximate Gravitational Potential Energy (near Earth surface): 
  \[ U_{\text{grav}} = mgh \quad (h \text{ is height above reference point}) \]
• More general Gravitational Potential Energy: 
  \[ U_{\text{grav}} = -\frac{GmM}{r} \]
  (see later in semester; here reference point is \( \infty \) far away)
• Electrostatic Potential Energy (charged particle in static electric field)
  -> see even later

**Total mechanical energy**
Sum of kinetic and potential energy of a particle: 
\[ E_{\text{mech}} = \text{K.E.} + U \]
IF no external (dissipative) forces present: 
\[ E_{\text{particle}} = E_{\text{mech}} = \text{const}. \quad (\Delta E = 0) \]
=> sum of kinetic and potential energy stays constant; for example:
go to UP the ramp DEcreases kinetic energy (negative work done by gravity) and INcreases potential energy; going back DOWN INcreases kinetic energy (car speeds up) and DEcreases potential energy \((mgh)\) becomes less.
Sum stays constant, so once you have \( E_{\text{mech}} = \frac{m}{2} v^2 + mgh \) at one position,
you can find velocity at any other position by solving \( \frac{m}{2} v^2 = E_{\text{mech}} - mgh \)
If EXTERNAL forces DO work: 
\[ \Delta E_{\text{mech}} = \frac{m}{2} v_f^2 - \frac{m}{2} v_i^2 + \Delta U = \Delta W_{\text{ext}} \]

**Other forms of energy:** (see later during PHYS101-102)
Energy stored in objects (distortion, chemical energy, \ldots, mass! \(^1\))
Kinetic energy due to random motion of particles in a system = internal energy (most disordered and least “useful” form of energy; “heat”)
Energy stored in fields (e.g. electromagnetic fields)
Energy can be transmitted by fields and waves (sound, light, electromagnetic radiation,\ldots)
Energy can be positive (e.g., kinetic energy always is) or negative (e.g., general gravitational energy, binding energy of molecules in solid or liquid, binding energy of electrons in atoms, binding energy of nucleons in nuclei etc.) or both (depending on reference point)
The total sum of ALL these forms of energy in a CLOSED system is always conserved; energy of an interacting system can only change if it is exchanged with other system (in form of work or heat).

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\(^1\) *Note:* According to Einstein, mass is just one form of energy \((E = mc^2)\) and therefore only the sum of all energy types including mass energy is conserved!
**Motion in a circle**

Position described by angle $\theta$ [in radians], radius $R$

Distance around the perimeter of a circle [in m]: $D = \theta [\text{in radians}] \cdot R [\text{in m}]$

Period of 1 revolution: $T$; rotational speed in “rounds per second” $\text{rps} = 1/T$

Angular velocity: $\omega = \Delta \theta [\text{in radians}] / \Delta t [\text{in s}] = 2\pi / T = 2\pi \text{rps} = |\vec{\omega}| / R$

Linear (tangential) speed $v = R\omega = 2\pi R \text{rps}$

Centripetal acceleration: $a_{\text{centr}} = \vec{v}^2 / R = \omega^2 R$

=>$\text{required centripetal force } F = m a_{\text{centr}}$

**Moment of Inertia } I :**

- Single Particle of mass $m$ at distance $R$ from axis: $I = mR^2$
- Extended Objects: $I = \sum_p (m_p r_p^2)$; increases with total mass and with the square of the average distance $r_p$ from axis of that mass
- Hollow cylindrical shell (total mass $M$, radius $R$): $I = MR^2$
- Solid cylinder: $I = MR^2/2$
- Solid sphere: $I = 2/5 MR^2$
- Thin rod of length $l$ (axis through center): $1/12 \cdot Ml^2$
- Thin rod (axis through edge): $1/3 \cdot Ml^2$

Kinetic energy for rotational motion: $\text{K.E.} = \frac{1}{2} I \omega^2$

**Angular momentum:**

- Single Particle at distance $R$ from axis: $
 L = mRv = mR^2 \omega = I \omega$
- Extended object: $L = I \omega$ (conserved if no external torque is present)

Points along axis of rotation; positive if rotation is counter-clockwise, negative if rotation is clockwise (right hand rule: curling fingers = sense of rotation, thumb points in direction of $L$).

Total angular momentum of closed system is conserved if no external force present -> if moment of inertia increases, angular velocity must decrease!

A change in angular momentum (rotational status) requires external torque: tangential force $F$ at distance $l$ relative to axis: $\tau = l \cdot |F| \sin \theta = \Delta L / \Delta t$

(lever arm $l$ times force; only the part of $F$ perpendicular to $l$ counts)

If net external torque (sum of all torques) = 0 -> $\vec{L}$ is conserved

Requirement for equilibrium: Sum of all forces AND sum of all torques must both be zero! Center of gravity must be above support area.
**Gravitation**

Gravitational Force due to mass \( M \) on mass \( m \) at distance \( r \): \( F = -\frac{GMm}{r^2} \)

- \( r \) is distance from center of mass \( M \) to center of mass \( m \)
- Gravitational Force \( F \) points back to its source (the center of the “attractor”)
- Gravitational acceleration of \( m \) towards \( M \): \( a = \frac{GM}{r^2} \) (independent of \( m \))
- Tidal acceleration difference across an object of size \( D \) due to a mass \( M \) at distance \( r \): \( \Delta a = 2 \frac{GM}{r^3} \)
- Gravitational Potential Energy between two masses at distance \( r \) (center to center): \( U_{\text{grav}} = -\frac{GmM}{r} \) (reference point assumed at infinite separation)
- Escape speed from planet with mass \( M \) and radius \( R \): \( v_{\text{esc}} = \sqrt{\frac{2GM}{R}} \)

**Kepler’s Laws**

1) Orbits of satellites (moons, planets,…) are elliptical
2) A line from the gravitating body (sun, planet,…) to the satellite sweeps out equal areas in equal times (conservation of angular momentum) => speed is larger if distance is smaller
3) The square of the period \( T \) of an orbit around a mass \( M \) is proportional to the cube of \( a \): \( T^2 \propto a^3 \) (\( a \) = major half axis of ellipse) => \( T \propto \sqrt{a^3} \).

Complete formula \( T = 2\pi \sqrt{\frac{a^3}{GM}} \)

- \( => \) on a circular orbit of radius \( R \) the velocity is \( v_{\text{orbit}} = \sqrt{\frac{GM}{R}} \)

**Projectile motion near Earth’s surface (neglect air resistance)**

Horizontal (\( x \)) and vertical (\( y \)) motion are independent of each other

- Horizontal components: \( v_x = v_{x0} = \text{const.} \); \( x(t) = x_0 + v_{x0}t \)
- Vertical components: \( v_y(t) = v_{y0} - gt \); \( y(t) = y_0 + v_{y0}t - \frac{1}{2} gt^2 \)
- Total motion is simply combination of horizontal and vertical one. Vertical motion usually determines total time for path (until impact). Initial launch at angle \( \theta \) above horizontal: \( v_{x0} = v_0 \cos(\theta_0) \); \( v_{y0} = v_0 \sin(\theta_0) \)

- Total speed = \( v = \sqrt{v_x^2 + v_y^2} \); final angle \( \theta = \arctan\left(\frac{v_y}{v_x}\right) \).

- Maximum height of trajectory is \( y_{\text{max}} = y_0 + \frac{1}{2} v_{y0}^2/g \) (assuming \( v_{y0} > 0 \));
- Total time elapsed for trajectory from ground back to ground: \( \Delta t = 2 \frac{v_{y0}}{g} \)
- Maximum distance traveled (from ground to ground) is \( \Delta x_{\text{max}} = 2 v_{x0} v_{y0}/g \)
**Electrostatics: Charge**

Positive or negative, measured in Coulomb [C], conserved, quantized (in units of $e$). Equal sign charges repel, opposite sign charges attract.

Electron: Charge $q = -e$. Proton: Charge $q = +e$. Most atoms have $q = 0$.

**Force between charges:**

Coulomb Force $\vec{F}_{\text{on } q \text{ due to } Q}$ at distance $r = k \frac{Qq}{r^2}$, $k = 8.99 \times 10^9 \text{ Nm}^2/\text{C}^2$.

**Alternative Formulation:**

Charge $Q$ (or any distribution of such charges) create an electric field $\vec{E}$. Charge $q$ experiences a force $\vec{F} = q\vec{E}$ in this field (include sign of $q$!).

Field lines begin at positive charges and end at negative ones or go on forever; density of field lines indicates strength of field.

Examples:

- Electric field at distance $r$ from a single spherical charge: $\vec{E} = kQ/r^2$
- Electric field between two large, conducting parallel plates: $\vec{E} = \text{const.}$

**Conductors**

- Contain huge amounts of “free” charges
- In presence of electric field, charges will flow to counteract field
- Unless new charges are constantly supplied (current), field will be canceled -> no field inside conductors
- All charges sit on the outside surface of the conductor
- External field will rearrange free charges such that conductor experiences net force towards external charge even in absence of net charge (“induction”)
- Conductor connected to ground will have net charge if one type of its charges is pushed away by nearby other charges of same sign

**Insulators**

- Contain no free charges
- Can pick up a few “extra charges” – a little net charge
- External field can polarize individual molecules such that material experiences net force towards external charge
- Polarization can weaken but never cancel external field
Potential
Moving charge $q$ along electric field $\mathbf{E}$ does work: $W = Fd = qEd$
Electrostatic forces are conservative $\rightarrow$ work stored in electrostatic potential energy $U_{\text{pot}}$. Electrostatic potential energy $U_{\text{pot}}$ of a charged object divided by its charge $q$ equals potential $V$ [measured in Volt = V]. A charge $q$ changes its energy by $q\Delta V$ when moving from a point with potential $V_1$ to a point with potential $V_2 = V_1 + \Delta V$.
1 V potential difference between 2 points means there is a potential to do 1 J of work per Coulomb moved from the higher potential point to the other.

Current
Net flow of charge in a given direction per unit time, measured in Ampere [A = C/s]
Count positive charges +, negative charges – and reverse sign for charges moving in opposite direction. Net current = sum of all currents
Current in conductor with resistance $R$ requires electric field $\rightarrow$ potential difference $\Delta V = -RI$. (Ohm’s Law).
Resistance measured in Ohm [Ω], is proportional to length of conductor, is inversely proportional to cross section, and is usually greater if conductor is hot.
Currents require complete (closed circuit) path and non-electrostatic “pump” (EMF = ElectroMotive Force = energy per charge) to keep running. Current is identically the same everywhere around a simple closed loop circuit.
Currents heat up conductors; power (energy transfer per unit time) is $P$ [Watt] = $\Delta V$ [Volt] x $I$ [Ampere] = $RI^2 = \Delta V^2/R$
Typical speed of electrons in wire: $10^6$ m/s random (internal energy), but only 0.1 mm/s on average in direction of electric force. Electric fields (that “tell the electrons to move”) travel with (nearly) speed of light, $c$.
Power can be delivered both by currents going in one direction only (DC, ex.: battery driven) and by currents going back and forth (AC, currents driven by generators, household and HV circuits).

Series Circuit
Same current $I$ has to go through all elements in series (doesn’t matter which one comes “first”). Voltage supplied by battery etc. gets “divided up” among
various elements. Total resistance is sum of individual resistances.
The largest voltage drop occurs across the largest resistor. Current (and therefore power) in each element smaller than if connected directly (without the others) to battery. Any break in circuit and all current ceases.

**Parallel Circuit**
Current that must be supplied by battery is the sum of the currents to all of the various branches. Each branch sees the full battery voltage, independent from the other branches. Smallest resistor draws the largest current and therefore the largest power. Any one element (branch) can have a break without affecting the others.

**Magnetic fields**
Due to charges in motion (electric currents, spinning electrons,...)
Field lines can form closed loops, but never end or begin.
Example: Permanent dipole, solenoid (or short coil) : Field lines can be described as if emanating from “North pole” and converging towards “South pole” (but connect through the interior of the dipole!) Cannot separate “north pole” from “south pole” – chopping in half will simply yield 2 new dipoles.
Earth has a magnetic dipole field with magnetic north pole close to geographic south pole (Australia).
Field of straight wire: circles around it, falls off like $1/r$.

**Magnetic materials**
Most materials react only slightly to magnetic fields. All materials have a slight diamagnetic response (see below) to strong magnetic fields. In some special materials, there are additional responses which can totally overpower the usual diamagnetic response.
Examples:
*Ferromagnets*: Iron, Cobalt, Nickel and some Rare Earth compounds
Magnetic properties due to electron spins (aligned within domains).
Normally random orientation of domains – no net magnetism
External field can orient or grow/shrink domains – strong magnetic response (magnetization) – leading to attraction towards other magnets
For some alloys, domains can keep orientation indefinitely –> permanent magnets. Permanent dipoles can attract (unlike poles) or repel (like poles); net force on dipole is zero in homogeneous magnetic field but torque tends to align dipole with field direction

*Paramagnets:* A weak magnetization in the direction of external field; attraction to strong magnets. Some materials like (liquid) oxygen, gadolinium,…

*Diamagnets:* A (usually extremely weak) magnetization opposite to the direction of external field; always leads to (usually feeble) repulsion from strong magnets. Property of nearly all materials (including water; -> floating frog) usually too weak to detect and/or masked by ferro- or paramagnetism. Exception: Superconductors are perfect diamagnets (strong expulsion of all external magnetic fields -> floating permanent magnet above cold superconducting disk).

**Magnetic forces**
Acts only on moving charges (including permanent or induced magnets: equal poles repel, opposite poles attract)
Magnitude proportional to $q$, $v$ and $B_{\text{perp}}$ (the part of $B$ perpendicular to $v$):
$$F = qv |B_{\text{perp}}|$$
Acts perpendicular to both $v$ and $B$ (right hand rule). In vector notation:
$$F = qv \times B$$
Typical motion in homogeneous field: Circle of radius $R = \frac{mv}{qB}$ with angular velocity $\omega = \frac{qB}{m}$, neither speeding up nor slowing down (no change in kinetic energy -> magnetic forces don’t do work on moving charges).
Force on wire: $ILB$ ($L$=Length), sideways (perpendicular to field and wire).
Two parallel wires with current in the same direction attract:
$$F = 2 \cdot 10^{-7} \text{ N per 1 m of length if wires are 1 m apart and both carry } I = 1 \text{ A.}$$