

PHYSICS 101 - Conceptual Physics

Units (SI):

Length: m = meter

Time: s = second

Mass: kg = kilogram

Velocity: m/s

Acceleration: m/s²

Momentum: kg m/s

Force: N = Newton = kg m/s²

Energy: J = Joule = Nm = kg m²/s² = Ws ; eV = electron-Volt = $1.602 \cdot 10^{-19}$ J

Power: W = Watt = J/s

Charge: C = Coulomb

Currents: A = Ampere = C/s

Electric Field: N/C = V/m

Electric Potential: V = Volt = J/C

Electric Resistance: Ω = Ohm = V/A

Magnetic Field: T = Tesla = Vs/m² = 10,000 Gauss

Prefixes:

kilo = k = 10^3 = 1000 = Thousand, Mega = M = 10^6 = 1,000,000 = Million,

Giga = G = 10^9 = 1,000,000,000 = Billion, Tera = T = 10^{12} = Trillion,

Peta = P = 10^{15} = Quadrillion

centi = c = 10^{-2} = 0.01 = 1-hundreth, milli = m = 10^{-3} = 0.001 = 1-thousandth,

micro = μ = 10^{-6} = 0.000,001 = 1-millionth, nano = n = 10^{-9} , pico = p = 10^{-12} , femto = f = 10^{-15}

Useful Constants:

Gravitational constant: $G = 6.67 \cdot 10^{-11}$ Nm²/kg²

Gravitational acceleration at surface of Earth: $g = 9.81$ m/s²

Mass of Sun: $1.99 \cdot 10^{30}$ kg

Distance from Earth to Sun: $1.50 \cdot 10^{11}$ m

Mass of Moon: $7.35 \cdot 10^{22}$ kg

Distance from Earth to Moon: $3.84 \cdot 10^8$ m

Mass of Earth: $5.97 \cdot 10^{24}$ kg

Radius of Earth: $6.38 \cdot 10^6$ m

Earth's magnetic field: 0.5 Gauss = 0.0005 T (Magnetic north pole near Australia)

Elementary charge: $e = 1.602 \cdot 10^{-19}$ C

Permittivity constant: $\epsilon_0 = 8.854 \cdot 10^{-12}$ F/m

Permeability constant: $\mu_0 = 4\pi \cdot 10^{-7}$ H/m

k (electrostatic force constant) = $1/4\pi\epsilon_0 = 8.99 \cdot 10^9$ Nm²/C²

Speed of Light $c = 2.998 \cdot 10^8$ m/s = $1/\sqrt{\epsilon_0 \mu_0}$

Natural Science - The Scientific Method

- 1) Collect observations; categorize and develop detailed descriptions
- 2) Identify crucial parameters and conditions; develop systematic (quantitative, if possible) descriptions of these as well as measured outcomes (use precise definitions for all observables); ignore “irrelevant” or “extraneous” perturbations and/or control for those. Avoid the “correlation = causation” fallacy – there could be “hidden variables” that influence both correlated observables
- 3) If possible, do systematic experiments varying **one** of the relevant parameters at a time; look for systematic changes in outcome depending on each [in-class analog: Do labs with changing parameters].
- 4) Develop a hypothesis (a – quantitative, if possible – relationship between parameters and outcomes); collect more data / do more experiments to test hypothesis
- 5) If your hypothesis describes your data approximately, call it a “Model”; if it agrees with all observations, call it a “Law”
- 6) A coherent set of “Laws” is called a “Theory”; all parts of a Theory must be logically (and mathematically, if applicable) consistent with each other as well as with previously established theories (unless you can prove those wrong)
- 7) Apply “Occam’s razor”: Your theory should have (only) the minimum number of ingredients required to describe **all** relevant observations; do not invoke “extra-natural causes”
- 8) Derive testable predictions from your theory or law; in particular, explore all new and previously unforeseen consequences [in-class analog: Do problems]
- 9) Test your predictions against observations and experiments; discard or modify theory (e.g., limit range of applicability) if test results disagree with predictions.
- 10) All theories are “tentative” at some level – there could always be contradictory observations in the future – but don’t discard well-tested theories needlessly!

Non-science

- “Theories” leading to untestable predictions (predictions that are too vague to be “falsifiable” or that refer to unmeasurable phenomena)
- Ad-hoc hypotheses (a new one for every new observation), anecdotal evidence, testimonials
- value statements, matters of taste or opinion, exclusive reliance on authority or precedent, normative statements (“you should...!”)

AND NOW TO PHYSICS ->

Motion in 1 dimension

Average velocity for time interval $\Delta t = t_1 \dots t_2$

$$v_{av} = \frac{\text{change in position during the time } \Delta t}{\text{elapsed time } \Delta t} = \frac{x(t_2) - x(t_1)}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$$

Instantaneous velocity at time t

= limit of the average velocity for a very short time interval around t ; dx/dt

Relative velocity addition

$$\vec{v}(x \text{ relative to } B) = \vec{v}(x \text{ relative to } A) + \vec{v}(A \text{ relative to } B)$$

(Average) Acceleration for time interval $\Delta t = t_1 \dots t_2$

$$\mathbf{a} = \frac{\text{change in velocity during time } \Delta t}{\text{elapsed time interval } \Delta t} = \frac{\Delta \mathbf{v}}{\Delta t}$$

Motion in 1 dimension with constant acceleration a , initial velocity v_0 and displacement x_0 :

$$v(t) = v(0) + a t = v_0 + a t$$

$$v_{av}, (0 \dots t_1) = 1/2 [v_0 + v(t_1)]$$

$$x(t) = x_0 + v_0 t + 1/2 a t^2$$

Forces – some Examples

Weight (Gravity): $\mathbf{F}_{\text{Weight}} = -mg$ (in negative vertical direction, up = +);

$g = 9.81 \text{ m/s}^2$ near Earth's surface (approximately 10 m/s^2)

Force exerted by spring (Hooke's Law): $\mathbf{F}_{\text{el}} = -kx$ (opposite to the direction of the displacement x from relaxed state of spring; with k = spring constant)

Tension T : Force along direction of string, at each end it is the same as the force exerted by the string on the attachment point (towards the string).

Normal force $\mathbf{F}_n$: Equal and opposite to sum of perpendicular (to a surface) components of all other forces (makes net perpendicular force zero)

Static Friction $\mathbf{f}_{\text{stat}}$ (object at rest): equal and opposite to sum of parallel components of all other forces, canceling them if the sum is not too big:

$$|\mathbf{f}_{\text{stat}}| \leq \mu_s |\mathbf{F}_n|.$$

Kinetic friction $\mathbf{f}_{\text{kin}}$ (moving object): Force opposite to direction of motion along surface, $|\mathbf{f}_{\text{kin}}| = \mu_k |\mathbf{F}_n|$.

Newton's First Law

When all forces applied to an object balance out to zero (cancel each other, net force = 0), then this object will not accelerate relative to an inertial system; it will be in equilibrium: If at rest, it will stay at rest, if in motion, it will continue to move in the same direction, with constant velocity $\Rightarrow \vec{a} = 0$.

Newton's Second Law

$\vec{a} = \vec{F}/m$; acceleration equals net force divided by mass. More detailed:

$$\vec{F}_{\text{resultant}} = \sum_{\text{all forces } i=1 \dots N} \vec{F}_i = m \vec{a}$$

(Sum over all forces, including direction!)

Note: Forces add like **vectors** (parallelogram rule or add each of the 3 cartesian components individually!)

Newton's Third Law

Forces always come in pairs (interaction between two objects A and B):

$$\vec{F}_{\text{Action}} (\text{A on B}) = - \vec{F}_{\text{Reaction}} (\text{B on A})$$

Momentum

For a single object: $\vec{p} = m \vec{v}$

Change in momentum (impulse): $\vec{J} = \Delta \vec{p} = \sum \vec{F} \cdot \Delta t$

Because of Newton's 3rd Law, impulse transferred from object A to object B is equal in magnitude but opposite in sign (direction) to simultaneously transferred impulse from B to A.

System of particle with no external force:

Total momentum $\sum \vec{p}_i = \vec{p}_1 + \vec{p}_2 + \dots$ is conserved

(same before as after any collision between any particles in the system)

Momentum conservation in two-particle collision:

$$m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f} \quad (i = \text{initial}, f = \text{final})$$

If sum of all kinetic energies is also conserved: ELASTIC collision.

Otherwise (friction, conversion to heat...): INELASTIC collision.

If 2 objects stick together after collision: Completely inelastic collision,

$$m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = (m_1 + m_2) \vec{v}_f$$

Work and Energy

Work ΔW **on** some object (particle or system) **due** to a force F if that object is displaced by Δs in the direction of that force: $\Delta W = F \Delta s = F \Delta s \cos \phi$ (only the part of the total displacement Δs that is in the direction of the force counts; no displacement = no work). Work done on object (= transferred TO object) can be positive (displacement in direction of force), zero (displacement perpendicular to force) or negative (displacement opposite to direction of force (work is transferred FROM object)).

Work-Energy Theorem: **Net** work done on an object changes the **ENERGY** of that object: ΔE (particle or system) = ΔW (done on that particle or system)

Power: Rate of work done on system per unit time: $P = \Delta W / \Delta t = \mathbf{F} \cdot \mathbf{v}$

Conservation of total Energy

The total amount of energy within a system can only be changed by exchanging energy (work) with another system. IF the energy of one object (particle, system...) goes UP, then the energy of another object (particle, system...) must go DOWN by the same amount, and vice versa!

Types of Energy

Kinetic energy (E_{kin} or **KE**) of a moving particle: $\text{K.E.} = \frac{m}{2} v^2$ (depends only on speed; is never negative; has no direction; depends on coordinate system used for $\mathbf{v}$)

If the only (relevant, changing) energy of a system is kinetic energy, then the work-energy theorem states: $\Delta \text{K.E.} = \Delta W$

Potential Energy (symbol: PE, E_{pot} or U)

Some forces can do work that depends only on initial and final position of the particle(s) that this work is being done on. Example: Gravity – only the difference in height between initial and final state matters.

These forces are called “conservative” because they can “store” work and return it without losses. Any **negative** work done by a **conservative** force increases stored “potential” for future **positive** work by the **same** force.

Work done by conservative force stored in form of potential energy U .

At reference point $U(\mathbf{r}_{\text{ref}}) = 0$ ($\mathbf{r}_{\text{ref}}$ can be chosen for convenience).

Example: height above some reference surface h , $h = 0 \rightarrow U_{\text{grav}} = 0$.

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Examples:

- Approximate Gravitational Potential Energy (near Earth surface; general case see later): $U_{\text{grav}} = mgh$ (h is height above reference surface)
- Particle attached to a spring, stretching the spring by moving a distance x from the spring's relaxed state: $U_{\text{el}} = (1/2) k x^2$ (k = spring constant; $x = 0 \Rightarrow U_{\text{el}} = 0$ for relaxed state of spring)

Total mechanical energy

Sum of kinetic and potential energy of a particle/system: $E_{\text{mech}} = \text{K.E.} + U$

IF no external (non-conservative) forces present: $E_{\text{mech}} = \text{const.}$

($\Delta E = 0$) $\Rightarrow$ sum of kinetic and potential energy is conserved.

Otherwise (in the presence of non-conservative forces doing work):

$$\Delta E_{\text{mech}} = \frac{m}{2} v_f^2 - \frac{m}{2} v_i^2 + \Delta U = \Delta W_{\text{nonconservative}}$$

Examples:

Car rolling UP the ramp DEcreases kinetic energy and INcreases potential energy; going back DOWN INcreases kinetic energy (car speeds up) and DEcreases potential energy (mgh becomes less). Sum stays constant, so once you have $E_{\text{mech}} = \frac{m}{2} v^2 + mgh$ at **one** position, you can find velocity at any other position by solving $\frac{m}{2} v^2 = E_{\text{mech}} - mgh$

Other forms of energy: (see later during PHYS101-102)

Energy stored in objects (distortion, chemical energy, ..., mass! ¹)

Kinetic energy due to random motion of particles in a system = internal energy (most disordered and least “useful” form of energy; “heat”)

General form of gravitational energy; electromagnetic energy

Energy stored in fields (e.g. electromagnetic fields)

Energy can be transmitted by fields and waves (sound, light, radiation,...)

Energy can be positive (e.g., kinetic energy always is) or negative (e.g., general gravitational energy, binding energy of molecules in solid or liquid, binding energy of electrons in atoms, binding energy of nucleons in nuclei etc.) or both (depending on reference point)

The total sum of ALL these forms of energy in a CLOSED system is always conserved; energy of an interacting system can only change if it is exchanged with other system (in form of work or heat).

¹ Note: According to Einstein, mass is just one form of energy ($E = mc^2$) and therefore only the sum of all energy types including mass energy is conserved!

Motion in a circle

Position described by angle θ (measured in radians), circle radius R

Period of revolution = time once around: T

Speed of rotation in “rounds per second” $rps = 1/T$

Angular velocity: $\omega = 2\pi / T = 2\pi rps = v_{\parallel} / R$; $v_{\parallel}$ tangential speed (along perimeter of circle): $v_{\parallel} = R\omega = 2\pi R rps$

Centripetal acceleration: $a_{\text{centr}} = v_{\parallel}^2 / R = \omega^2 R$

=> centripetal force necessary for circular motion: $F = m a_{\text{centr}}$

Angular momentum:

- Single Particle at distance R from axis: $L = mRv = mR^2\omega = I\omega$

- Extended object: $L = M\langle R^2 \rangle \omega = I\omega$; $\langle R^2 \rangle$ = average squared distance from the axis, and I = moment of inertia (see below).

Total angular momentum of any system is conserved if there is no external torque (see below) -> if moment of inertia (radius!) increases, angular velocity must decrease and vice versa!

Moment of Inertia I :

- Single Particle at distance R from axis: $I = mR^2$

- Extended Objects: $I = \sum_p (m_p r_p^2)$; increases with total mass and average squared distance from axis of rotation

- Cylindrical Shell, bicycle wheel (radius R , mass M): $I = MR^2$

- Solid cylinder or wheel: $I = MR^2/2$ (some mass is closer to axis than R !)

- Solid sphere: $I = 2/5 MR^2$

- Thin rod of length L (axis through center): $1/12 ML^2$

Force along perimeter at distance $\mathbf{r}$ from axis creates Torque $\tau = r \perp \mathbf{F}_{\parallel}$

A torque changes angular momentum: $\tau = \Delta L / \Delta t$

Torque = lever arm $\times$ tangential force; only the part $\mathbf{F}$ of perpendicular to $\mathbf{r}$, i.e., along the circumference = $\mathbf{F}_{\parallel}$ is counted)

No external forces or only radial forces = no external torque

Requirement for equilibrium: Sum of all forces AND sum of all torques must both be zero! -> L is conserved

Object at rest NOT toppling over: Center of gravity of object must be above supported area (“footprint”) of object.

Newton's GENERAL Law of Gravitation

Gravitational Force of mass M on mass m at distance d from center to center:

$F = -GMm/d^2$ pointing from mass m back to M

Gravitational acceleration of m towards M : $a = GM/r^2$

Tidal force difference across a distance D : $\Delta F = 2 GmM D/r^3$

Gravitational Potential Energy between two masses at distance d (center to center): $U_{\text{grav}} = - GmM /d$ (if reference point is at infinite separation)

Escape speed from planet with mass M and radius R : $v_{\text{esc}} = \sqrt{2GM / R}$

Kepler's Laws

- 1) Orbits of satellites (moons, planets,...) are elliptical
- 2) A line from the gravitating body (sun, planet,...) to the satellite sweeps out equal areas in equal times (conservation of angular momentum) $\Rightarrow$ speed is larger if distance is smaller
- 3) The square of the period T of an orbit around a mass M is proportional to the cube of a : $T^2 \propto a^3$ (a = major half axis of ellipse = average of shortest plus longest distance from M ; R for circular orbit) $\Rightarrow a \propto T^{2/3} \Rightarrow \omega \propto a^{-3/2}$
Complete formula $T = 2\pi(a^3/GM)^{1/2}$
 $\Rightarrow$ on a circular orbit of radius R the velocity is $v_{\text{orbit}} = \sqrt{GM / R}$

Projectile motion near Earth's surface (neglect air resistance)

Horizontal and vertical motion are independent of each other

Horizontal components: $v_x = v_{x0} = \text{const.}$; $x(t) = x_0 + v_{x0}t$

Vertical components: $v_y(t) = v_{y0} - gt$; $y(t) = y_0 + v_{y0}t - \frac{1}{2}gt^2$

Total motion is simply combination of horizontal and vertical one. Vertical motion usually determines total time for path (until impact). Initial launch at angle θ_0 above horizontal with total speed v_0 : $v_{x0} = v_0 \cos(\theta_0)$; $v_{y0} = v_0 \sin(\theta_0)$

Electrostatics

Charge

Positive or negative, measured in Coulomb [C], conserved, quantized (multiples of elementary charge e). Equal sign charges repel, opposite sign charges attract. Neutral atoms: $Z \cdot e$ positively charged nucleus, Z electrons with charge $-e$.

Force between charges:

Coulomb Force $\vec{F}_{\text{on } q \text{ due to } Q \text{ at distance } r} = k \frac{Qq}{r^2}$, $k = 1/4\pi\epsilon_0 = 8.99 \cdot 10^9 \text{ Nm}^2/\text{C}^2$.

Electric Field:

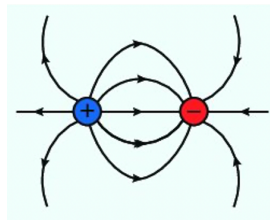
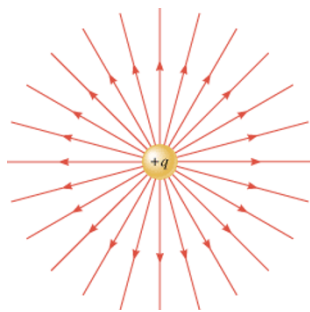
Charge Q (or any distribution of such charges) create(s) an electric field $\mathbf{E}$.

Any “test” charge q experiences a force $\mathbf{F} = q\mathbf{E}$ in this field ($\mathbf{E} = \mathbf{F}/q$).

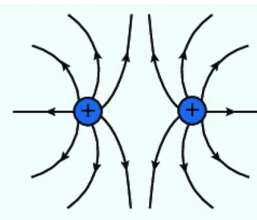
Field lines begin at positive charges and end at negative ones or go on forever; density of field lines indicates strength of field.

Examples:

- Electric field at distance d from a single spherical charge: $E = kQ/d^2$
- Electric field between two large, conducting parallel plates (capacitor):
 $\mathbf{E} = \text{const.}$ (pointing from positively charged to negatively charged plate)



Two opposite Charges



Two identical charges



Conductors

- Contain huge amounts of “free” charges
- In presence of electric field, charges will flow to counteract field
- Unless new charges are constantly supplied (current), field will be canceled -> no field inside conductors
- All charges sit on the outside
- External field will rearrange free charges such that conductor experiences net force towards external charge even in absence of net charge

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- Conductors connected to ground will have net charge if nearby charged object pushes its free charges.

Insulators

- Contain no free charges
- Can pick up a few “extra charges” – a little net charge
- External field can polarize individual molecules such that material experiences net force towards external charge
- Polarization can weaken but never cancel external field

Electrostatic Potential

Moving charge q along electric field $\mathbf{E}$ does work: $W = F \cdot d = qEd$

Electrostatic forces are conservative $\rightarrow$ work stored in electrostatic potential energy $U_{\text{el.st.}}$. Example: Electrostatic potential energy between two charges q_1, q_2 separated by distance d : $U_{\text{el.st.}} = k \cdot q_1 q_2 / d$

Electrostatic potential energy U_{pot} divided by charge q equals potential (voltage) V [measured in Volt = V]. A charge q changes its energy by

$\Delta U_{\text{el.st.}} = q(V_2 - V_1)$ when moving from a point with potential V_1 to a point with potential V_2 . 100 V means potential to do 100 J of work per Coulomb.

Current

Net flow of charge per unit time, measured in Ampere [$A = C/s$]

Current in conductor with resistance R requires electric field $\rightarrow$ potential difference $\Delta V = -RI$. (Ohm's Law).

Resistance measured in Ohm [Ω], proportional to length of conductor, inversely proportional to cross section, greater if conductor is hot.

Currents require complete (closed circuit) path and non-electrostatic “pump” (EMF = ElectroMotive Force = energy per charge) to keep running.

Currents heat up conductors; power (energy transfer per unit time) is

$$P [\text{Watt}] = \Delta V [\text{Volt}] \times I [\text{Ampere}] = RI^2 = \Delta V^2 / R$$

Typical speed of electrons in wire: 10^6 m/s random (internal energy), but only 0.1 mm/s on average in direction of electric force. Electric fields (that “tell the electrons to move”) travel with (nearly) speed of light, c .

AC current: electrons just slosh back and forth – none makes the whole journey.

Series Circuit

The same current has to go through all elements (doesn't matter which one comes "first"). Voltage supplied by battery etc. gets "divided up" among various elements. Total resistance is sum of individual resistances.

The largest voltage drop occurs across the largest resistor.

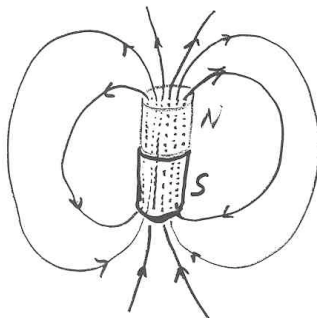
Parallel Circuit

Current that must be supplied by battery is the sum of the currents to all of the various branches. Each branch sees the full battery voltage, independent from the other branches. Smallest resistor draws the largest current.

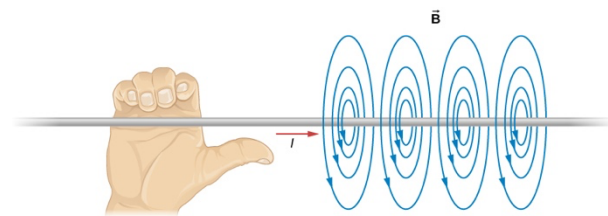
Magnetic fields

Due to charges in motion (electric currents, spinning electrons, ...)

Field lines can form closed loops, but never end or begin.



Example: Permanent dipole, solenoid (or short coil):
Field lines can be described *as if* emanating from "North pole" and converging towards "South pole" (but connect through the interior of the dipole!) Earth has a magnetic dipole field with magnetic north pole close to geographic south pole.



Field of straight wire: circles around it, falls off like $1/r$.

Magnetic materials

Most materials are non-magnetic or react only slightly to magnetic fields

Exceptions:

Ferromagnets: Iron, Cobalt, Nickel and some Rare Earth compounds

Magnetic properties due to electron spins (aligned within domains).

Normally random orientation of domains – no net magnetism

External field can shrink or expand domains – strong magnetic response (magnetization) – leading to attraction towards other magnets

For some alloys, domains can keep orientation indefinitely → permanent magnets. Permanent dipoles can attract (opposite poles) or repel (equal poles); net force on dipole is zero in homogeneous magnetic field but torque tends to align dipole with field direction (→ compass needle)

Paramagnets: Weak magnetization in the direction of external field; attraction to strong magnets. Some materials like (liquid) oxygen, gadolinium,...

Diamagnets: A (usually extremely weak) magnetization opposite to the direction of external field; always leads to (usually feeble) repulsion from strong magnets. Property of nearly all materials (including biological tissue; floating frog) usually too weak to detect and/or masked by ferro- or paramagnetism. Exception: Superconductors are perfect diamagnets (strong expulsion of all external magnetic fields).

Magnetic forces

Acts only on moving charges (including permanent or induced magnets: equal poles repel, opposite poles attract)

Proportional to q , $\mathbf{v}$ and $\mathbf{B}_{\text{perp}}$ (the part of $\mathbf{B}$ perpendicular to $\mathbf{v}$)

Acts perpendicular to both $\mathbf{v}$ and $\mathbf{B}$ (right hand rule)

Typical motion in homogeneous field: Circle of radius $R = mv/qB$ with angular velocity $\omega = qB/m$, neither speeding up nor slowing down (no change in kinetic energy → magnetic forces don't do work on moving charges).

Force on wire: ILB (L =Length), sideways (perpendicular to field and wire).