Work and Energy

- Newton showed momentum **p** = m**v** is an important concept
  - Relevant for collisions - momentum is **conserved**
  - Can tell you about the change in motional state during collision
  - Change in momentum = Impulse = F **x** Δt
  - MOMENTUM = HISTORY
- Leipnitz proposed $(^1/_2)$ mv^2 as relevant quantity
  - Compare distance needed to stop moving object - 2x the velocity => 4x the distance! (average v **X** time to get v to zero)
  - Nowadays called "kinetic energy"
  - Many other forms of energy - the sum of all of them is conserved
  - Work = Energy **transfer** from one system to another
- Momentum tells you "oomph", energy tells you "ouch"

Work and Energy

- Work: The total **effect** a force **F** has **on** an object (mass point, closed system,…), based on the change in its position (configuration in space)
  - Given by Force times Displacement $\Delta W = F\Delta s$
  - Changes motional (kinetic) state or internal state, mechanical state,…
  - Changes total **energy** of the object/system
  - Can be positive or negative (get or give, add or extract)
  - Can be transferred from one object to another, but no object can "have" work
  - Analogy: Transfer of cash
- Energy: The **ability** to **do** work
  - Property **of** the object (mass point, closed system,…)
  - Changed by work done **on (or by)** object/system
  - Can be exchanged between objects/systems, but **can not** be created or destroyed (energy conservation)
  - Analogy: Net Worth

Work and Kinetic Energy I

- Prototype example - free fall starting at rest:
  - Force: mg ; Acceleration g ; Velocity $v(t) = g \cdot t$;
    Distance fallen $\Delta s = \frac{1}{2} gt^2$
  - Work done after time t:
    $\Delta W = F\Delta s \, / = mg \cdot (\frac{1}{2} gt^2) = \frac{1}{2}m \cdot (gt)^2 = \frac{1}{2}m \cdot (v(t))^2$
 - $= \text{K.E.} \, (t)$

- In general: CHANGE of kinetic energy of some object:
  $\Delta K.E. = \Delta \frac{1}{2}mv^2 = m \, v_{\text{aver}} \, \Delta v = m \, v_{\text{aver}} \, a \, \Delta t = F \, v_{\text{aver}} \, \Delta t = F\Delta s$
- If F includes ALL forces acting on an object, $\Delta K.E. = F\Delta s$ is equal to the work ΔW done on the object
  - Note: negative work - energy diminishes! Still requires a real force! Example: braking, stopping a bullet,…

Work and Kinetic Energy II

- Dimension: Displacement times Force
  Unit: Nm = J (Joule) (pronounce like "cool")

1) Specify a force F acting **on** an object/system

2) Multiply displacement in the direction of the force with that force:
   $\Delta W = F \, \Delta s \, \cos\phi$

   - If **F** is in the direction of Δs, then positive work is done on object
   - If **F** is in opposite direction of Δs, then negative work is done on object
   - If **F** is perpendicular to direction of Δs, then no work is done at all.

3) => ΔW is work down by **this particular force F**.

4) Sum work done due to all (external) forces acting on object:
   $\Delta W = \sum \mathbf{F}_i \cdot \Delta \mathbf{s} = \sum \Delta W_i$

4) Set equal to **change** in total kinetic energy of object:
   $\Delta K.E. = \frac{m}{2} v_f^2 - \frac{m}{2} v_i^2 = \Delta W$

- Examples: Pushing car up incline, car rolling down (demo); Catching a ball; ball bouncing off wall, ball getting stuck in wall

Work and Kinetic Energy - Example

- Car rolling down ramp: Gravity does work, normal force doesn't => only displacement in vertical direction counts

- Final velocity same as for free fall! $\frac{1}{2}\,mv^2 = mg\Delta h$

- Car moving up ramp: Weight does negative work $\Delta W = -mg\Delta h$

- Slows down car or I have to supply additional work

- Car on flat surface:
  Friction slows car down -
  Negative work done $\Delta W = -\frac{1}{2}mv^2$
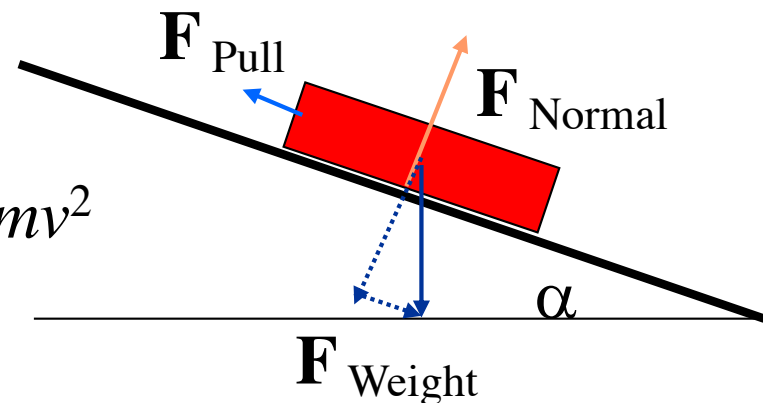
Important Notes

- Even if $\Sigma\mathbf{F}_i \neq 0$, Work done can be zero:
  - No displacement: Holding a book, pushing against a wall
  - Direction of displacement perpendicular to Force: Moving sideways (constant height) in gravity field, circular motion (constant speed)
  - Normal forces and static friction **never** do work, but tension can (only if string moves, and only in equal and opposite amount on each end) and kinetic friction can, too (always negative).
- Kinetic energy (like all types of energy) is a scalar quantity (unlike momentum!): It has no direction!
  K.E. = $^m/_2 \, v^2 = W$ (0 -> v), always positive
  - Independent of direction (circular motion at constant speed doesn't change kinetic energy)
  - **Depends on system of reference**
- **Everything** I said **only** valid in Inertial System of Reference

Power

- Work done per unit time:
  $P = \Delta W / \Delta t$ (average)

- Unit: Watt = Joule/s, kW (kiloWatt) ->
  new unit for energy&work: Ws, kWh…

- $\Delta W = \mathbf{F} \cdot \Delta \mathbf{s} \Rightarrow P = \mathbf{F} \cdot \Delta \mathbf{s} / \Delta t = \mathbf{F} \cdot \mathbf{v}$

- Example:
  1000 kg Car accelerating from 0 to 20m/s in 5 s $\Rightarrow F = 4000$ N,
  $v_{ave} = 10$ m/s $\Rightarrow P_{ave} = 40$ kW = 53 hp .

- Different way to calculate:
 K.E. (final) = 200,000 J = ΔW in 5 s

Do NOT confuse!

- Position (displacement), velocity, speed, acceleration
- Force, momentum, impulse
- Energy, work, power, heat

Other Types of Energy - Potential Mechanical Energy

- So far: Considered only changes in kinetic energy due to work done.
    - Analogy: Pure Cash Economy
- But: Some systems are able to "store" the work for you (when you put work in) and "give back" the **same** amount (when they do positive work on something else).
    - Analogy: Bank Account. You pay money in (ending up with less cash) - the money is stored for you - you can withdraw it again (get cash back)
- Such systems are called "conservative" (they conserve your work/money for you)
- **IF** energy stored depends on position/configuration of object/system: Call it "Potential Mechanical Energy" U or U_{pot}

Potential Energy - Example

- Car moving up ramp: You must push against gravity and thereby transfer work to system $\Delta W = mg\Delta h$

- Amount of work depends only on initial and final position

- Can be retrieved as positive work on the way back down

- Increases the total energy of the car but not its kinetic energy

- Therefore, ascribe to a new type of energy: Mechanical potential energy of the system "car" + gravitation: $U_{grav} = mgh$

$\mathbf{F}_{Pull}$

$\mathbf{F}_{Normal}$

$\mathbf{F}_{Weight}$

α

y

x
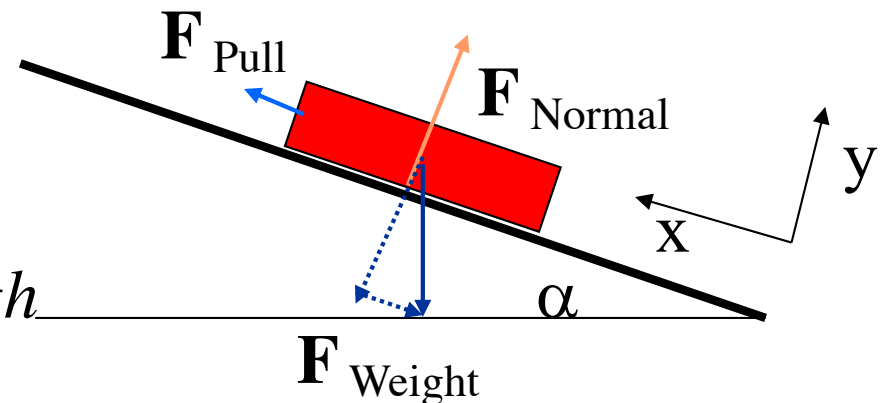
Total Mechanical Energy

- Dimension: Same as Work
  Unit: Nm = J (Joule)   Symbol: $E = K.E. + U$

1) Specify all **non-conservative** [*)] forces acting **on** a system

2) Multiply displacement in the direction of the net non-conservative force with that force:
  $$\Delta W_{nc} = F\, \Delta s\, \cos\phi$$

3) Set equal to change in total energy:
  $$\Delta E = {}^m/_2 v_f^2 - {}^m/_2 v_i^2 + \Delta U = \Delta W_{nc}$$

4) If no non-conservative forces are present: TME = conserved!!!
  $$E = K.E. + U = \text{const.} \Rightarrow \Delta K.E. = -\Delta U$$

[*)] We include all non-conservative forces like friction and push/pull, plus all forces that we don't want to include in the system, so the separation between WORK and TOTAL MECHANICAL ENERGY depends on our definition of the system.

Example: Gravitational Potential Energy I
*)

- Motion in vertical (y-) direction only:

$$\Delta U = -W_{grav} = mg\Delta y$$

- External force: Lift mass m from height y_i to height y_f (without increasing velocity) => Work gets stored as gravitational potential energy $\Delta U_{grav} = mg\,(y_f - y_i) = mg\,\Delta y$
- Free fall (no external force): Total energy conserved, change in kinetic energy compensated by change in potential energy $\Delta K.E. = {}^m/_2\, v^2 = -mg\,\Delta y$
- Example: Throw baseball upwards with 20 m/s (accelerate over 0.5m). Maximum height? Force needed?

*) Here the system consists of Earth plus object!

Gravitational Potential Energy II

- Important point: Potential energy has **no** absolute zero (like kinetic energy does - in a given reference frame!).
- Depends on choice of **reference point**: **You decide** where you want U to be 0.
- Call that point $h = 0$.
- Define potential energy as $U_{grav} = mgh$.
- => Total energy $E = {}^m/_2\, v^2 + mgh$.
- Choice arbitrary - other choice means constant offset in definition of U and E.
- No **observable** depends on that choice! All that counts are differences ΔU. BUT: you must specify reference point when quoting E and U!

Work on a pendulum - an Example

- Slowly pushing a pendulum bob sideways:
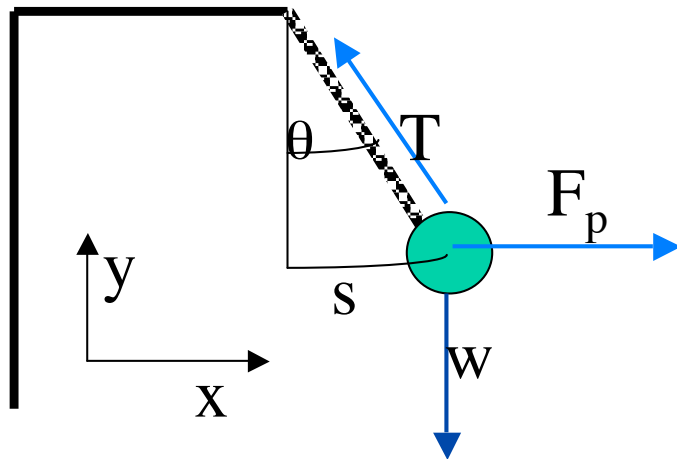
1) Tension does no work (perpendicular to motion)
2) Pushing force does work - equal to potential energy stored by gravitational force
3) After letting go, gravitational potential energy gets converted to kinetic energy at the bottom

- On the way up: Net work done by external force (pushing) increases total energy $(K.E. + U_{grav})$ stored in the system
- On the way down: No other (non-grav.) force => Total Energy conserved $(\Delta E = 0)$
  $\Delta E = \Delta K.E. + \Delta U_{grav} = 0 \Rightarrow \Delta K.E. = {}^m/_2 v^2 = -\Delta U_{grav} = -mg\Delta h$

Elastic Potential Energy

- So far: Considered system object - gravitational field;
  Potential energy = mgh
- Now: Consider system -> mass attached to spring;
  Potential energy = work that can be done by spring
  - Call x = 0 unstretched position of spring
  - Force exerted by spring $F_x = -kx$
  - Work done by spring $\Delta W = -k/2 \, (x_f^2 - x_i^2)$
  - Potential energy stored in spring-cart system:
    $U_{elas} = -\Delta W = k/2 \, (x_f^2 - x_0^2)$
    where x_0 is the point where we declare U_{elas} to be = 0.
  - $x_0 = 0 \Rightarrow U_{elas} = k/2 \, x^2$ (convenient, **not** unique)
  - Note: $U_{elas} > 0$ stretched **and** compressed

Elastic Potential Energy cont'd.

- No other (non-elastic) force =>
  Total Energy conserved ($\Delta E = 0$)
  $\Delta E = \Delta \text{K.E.} + \Delta U = 0 \implies \Delta^m/_2 \, v^2 = -\Delta U$
  Example: Oscillation

- Non-elastic force present =>
  $\Delta E = \Delta \text{K.E.} + \Delta U = \Delta W \implies$
  $\Delta^m/_2 \, v^2 = -\Delta U + \Delta W$

- Elastic and gravitational force present =>
  $E = \text{K.E.} + U_{elas} + U_{grav} =$
  $^m/_2 \, v^2 + k/2 \, x^2 + mgh$.
  (several bank accounts)

- Note: Elastic forces are conservative because work done only depends on initial and final position.

Total Mechanical Energy
- Final Version

1) Specify all forces acting on an object

2) Separate out all **conservative** forces (Work done depends only on initial and final position). Incorporate them into the system of the object as being able to store potential energy U

3) Add all **non-conservative** forces acting **on** the system (= all other forces), call the result "net non-conservative force".

4) Multiply displacement in the direction of the net force with that force:
$$\Delta W_{ext} = \mathbf{F} \cdot \Delta \mathbf{s} = F \, \Delta s \, \cos\phi$$

5) Set equal to change in total energy:
$$\Delta E = {}^m/_2 \, v_f^2 - {}^m/_2 \, v_i^2 + \Delta U = \Delta W_{ext} \; ; \; \Delta \, {}^m/_2 \, v^2 = -\Delta U + \Delta W_{ext}$$

Examples: Pumpkin falling on spring-loaded platform (without and with air resistance); bungee jump

Other types of Energy

1) Electromagnetic energy (see later in the semester).
   Examples: Charged capacitors (electrostatic energy), current-carrying coils (magnetic energy), …

2) Chemical energy (really a special kind of electromagnetic energy).
   Examples: Batteries, fuel, …

3) Sound, light, nuclear,…

4) Internal energy -see next semester (PHYS102)
   Note: Forces that create heat are NOT conservative – internal energy can only partially be converted back to other forms of energy or to work. Examples: Dragging a cart with friction over the floor; object impacting on floor, marble "eating its way" through ice block, braking, …

The sum of all types of energy is ALWAYS conserved!!!

Important Points

- Energy concept is useful:
  - Calculate change in velocity without knowing Force as F(t)
  - Understand levers, hydraulic systems, mechanical advantage, power plants…
- Energy concept is fundamental: Energy is conserved!
  - Kinetic energy (always positive - "cash")
  - Gravitational energy (can be + or -; "checking account with overdraft")
  - Elastic potential energy (always positive - "savings")
  - Chemical, electrical, sound, light, …. Energy
  - internal energy (less useful - "cash hidden in a mattress")
- Efficiency = fraction of useful work/total energy transferred