Summary

Units (SI):
Length: m = meter
Time: s = second
Mass: kg = kilogram; atomic mass unit \( u = 1.661 \times 10^{-27} \text{ kg} = m(^{12}\text{C})/12 \)
Velocity: m/s
Acceleration: m/s²
Momentum: kg m/s
Force: N = Newton = kg m/s²
Energy: J = Joule = Nm = kg m²/s² = Js; eV = electron-Volt = 1.602 × 10⁻¹⁹ J
Power: W = Watt = J/s
Charge: C = Coulomb
Currents: A = Ampere = C/s
Electric Field: N/C = V/m
Electric Potential: V = Volt = J/C
Electric Resistance: \( \Omega = \text{Ohm} = V/A \)
Magnetic Field: T = Tesla = Vs/m² = 10,000 Gauss
Amount: mol (1 mol = \( N_A \) molecules = A gram, where \( A \) is atomic mass)
Density \( \rho \): kg/m³
Pressure: Pa = Pascal = N/m²; 100,000 Pa ≈ 1 atmosphere
Temperature: K = Kelvin (C = Celsius, F = Fahrenheit);
\[ 32^\circ F = 0^\circ C = 273.15 K; 212^\circ F = 100^\circ C = 373.15K \]
Heat: J or 1 calorie = 4.187 J; 1 food calorie = 1000 calories = 4187 J
Frequency: Hz = 1/s

Prefixes:
kilo = k = 10³ = 1000 = Thousand
Mega = M = 10⁶ = 1,000,000 = Million
Giga = G = 10⁹ = 1,000,000,000 = Billion
Tera = T = 10¹² = Trillion, Peta = P = 10¹⁵ = Quadrillion
centi = c = 10⁻² = 0.01 = 1-hundreth
milli = m = 10⁻³ = 0.001 = 1-thousandth
micro = \( \mu \) = 10⁻⁶ = 0.000,001 = 1-millionth
nano = n = 10⁻⁹, pico = p = 10⁻¹², femto = f = 10⁻¹⁵
**Useful Constants:**

Gravitational constant: $G = 6.67 \times 10^{-11} \text{Nm}^2/\text{kg}^2$

Gravitational acceleration at surface of Earth: $g = 9.81 \text{ m/s}^2$

Mass of Sun: $1.99 \times 10^{30} \text{ kg}$

Distance from Earth to Sun: $1.50 \times 10^{11} \text{ m}$

Mass of Moon: $7.35 \times 10^{22} \text{ kg}$

Distance from Earth to Moon: $3.84 \times 10^8 \text{ m}$

Mass of Earth: $5.97 \times 10^{24} \text{ kg}$

Radius of Earth: $6.38 \times 10^6 \text{ m}$

Earth’s magnetic field: about 0.5 Gauss = 0.0005 T ($5 \times 10^{-5} \text{ T}$)

(Magnetic north pole near Australia)

Elementary charge: $e = 1.602 \times 10^{-19} \text{ C}$

Permittivity constant: $\varepsilon_o = 8.854 \times 10^{-12} \text{ F/m}$

Permeability constant: $\mu_o = 4\pi \times 10^{-7} \text{ H/m}$

$k$ (electrostatic force constant) = $1/4\pi\varepsilon_o = 8.99 \times 10^9 \text{ Nm}^2/\text{C}^2$

Speed of Light: $c = 2.998 \times 10^8 \text{ m/s} = 1/\sqrt{\varepsilon_o\mu_o}$

Avogadro’s number:

$N_A = 6.022 \times 10^{23} \text{ molecules/mol} = \text{number of } ^{12}\text{C atoms in 12 g of carbon}$

Universal Gas Constant: $R = 8.32 \text{ J/mol/K}$

Density of water: $1000 \text{ kg/m}^3$, air (sea level): $1.25 \text{ kg/m}^3$, of iron: $7874 \text{ kg/m}^3$

Atmospheric pressure at sea level: $101,300 \text{ Pa}$

Typical speed of sound: $330 \text{ m/s} – 340 \text{ m/s in air}$

Frequency of “middle A” musical note: $440 \text{ Hz}$

Electron mass: $m_e = 9.109 \times 10^{-31} \text{ kg}; E = mc^2 = 510,999 \text{ eV} = 511 \text{ keV}$

Proton mass: $m_p = 1.673 \times 10^{-27} \text{ kg}; E = mc^2 = 938,272,030 \text{ eV} = 938.3 \text{ MeV}$

Neutron mass: $m_n = 1.675 \times 10^{-27} \text{ kg}; E = mc^2 = 939,565,360 \text{ eV} = 939.6 \text{ MeV}$

Planck’s constant:

$h = 6.63 \times 10^{-34} \text{ Js}; 6.63 \times 10^{-25} \text{ kg m/s x nm} = 4.17 \times 10^{-15} \text{ eV / Hz}$

$\Rightarrow 1/h = 1.51 \times 10^{-33} \text{ Hz/J} = 2.4 \times 10^{14} \text{ Hz/eV};$

$2 \text{ eV corresponds to } \lambda = 620 \text{ nm and } f = 4.84 \times 10^{14} \text{ Hz (yellow light)}$
**Natural Science - The Scientific Method**

1) Collect observations; categorize and develop detailed descriptions

2) Identify crucial parameters and conditions; develop systematic (quantitative, if possible) descriptions of these as well as measured outcomes (use precise definitions for all observables); abstract from “irrelevant” or “extraneous” perturbations and/or control for those. Avoid the “correlation = causation” fallacy – there could be “hidden variables” that influence both correlated observables

3) If possible, do systematic experiments varying **one** of the relevant parameters at a time; look for systematic changes in outcome depending on each [in-class analog: Do labs with changing parameters].

4) Develop a hypothesis (a – quantitative, if possible – relationship between parameters and outcomes); collect more data / do more experiments to test hypothesis

5) If your hypothesis describes your data approximately, call it a “Model”; if it agrees with all observations, call it a “Law”

6) A coherent set of “Laws” is called a “Theory”; all parts of a Theory must be logically (and mathematically, if applicable) consistent with each other as well as with previously established theories (unless you can prove those wrong)

7) Apply “Occam’s razor”: Your theory should have (only) the minimum number of ingredients required to describe **all** relevant observations; do not invoke “extra-natural causes”

8) Derive testable predictions from your theory or law; in particular, explore all new and previously unforeseen consequences [in-class analog: Do problems]

9) Test your predictions against observations and experiments; discard or modify theory (e.g., limit range of applicability) if test results disagree with predictions.

10) All theories are “tentative” at some level – there could always be contradictory observations in the future – but don’t discard well-tested theories needlessly!

**Non-science**

- “Theories” leading to untestable predictions (predictions that are too vague to be “falsifiable” or that refer to unmeasurable phenomena)
- Ad-hoc hypotheses (a new one for every new observation), anecdotal evidence, testimonials
- value statements, matters of taste or opinion, exclusive reliance on authority or precedent, normative statements (“you should…!”)

…and now to PHYSICS…
Motion in 1 dimension
Average velocity for time interval $\Delta t = t_1 \ldots t_2$
$$v_{av} = \frac{\text{change in position during the time } \Delta t}{\text{elapsed time } \Delta t} = \frac{x(t_2) - x(t_1)}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$$

Instantaneous velocity at time $t$
= average velocity for a very short time interval around $t$; $dx/dt$

Relative velocity addition
$$\vec{v}(x \text{ relative to } B) = \vec{v}(x \text{ relative to } A) + \vec{v}(A \text{ relative to } B)$$

Acceleration for time interval $\Delta t = t_1 \ldots t_2$
$$a = \frac{\text{change in velocity during time } \Delta t}{\text{elapsed time interval } \Delta t} = \frac{\Delta v}{\Delta t}$$

Motion in 1 dimension with constant acceleration $a$:
$$v(t) = v(0) + a \ t = v_0 + a \ t$$
$$v_{av, (0 \ldots t_1)} = \frac{1}{2} [v_0 + v(t_1)]$$
$$x(t) = x_0 + v_0 t + \frac{1}{2} a t^2$$

Forces – some Examples
Weight (Gravity): \( \vec{F}_{\text{Weight}} = -mg \) (in negative vertical direction, up = +); \( g = 9.81 \text{ m/s}^2 \) near Earth’s surface
Force exerted by spring (Hooke's Law): \( \vec{F}_{\text{el}} = -kx \) (opposite to the direction of the displacement $x$ from relaxed state of spring; with $k$ = spring constant)
Normal force $\vec{F}_n$: Equal and opposite to sum of perpendicular (to a surface) components of all other forces (makes net perpendicular force zero)
Static Friction $f_{\text{stat}}$ (object at rest): equal and opposite to sum of parallel components of all other forces, canceling them if the sum is not too big: \( |f_{\text{stat}}| \leq \mu_s |\vec{F}_n| \).
Kinetic friction $f_{\text{kin}}$ (moving object): Force opposite to direction of motion along surface, \( |f_{\text{kin}}| = \mu_k |\vec{F}_n| \).
Tension $T$: Force along direction of string, at the end it is the same as the force exerted by the string on the attachment point (towards the string).
**Newton's First Law**
When all forces applied to an object balance out to zero (cancel each other), then this object will not accelerate relative to an inertial system; it will be in equilibrium: If at rest, it will stay at rest, if in motion, it will continue to move in the same direction, with constant velocity $\Rightarrow \ddot{a} = 0$.

**Newton's Second Law**
$a = F/m$; acceleration equals net force divided by mass. More detailed:
\[
\vec{F}_{\text{resultant}} = \sum_{i=1}^{N} \vec{F}_i = m \ddot{a}
\]
(Sum over all forces, including direction!)

**Note:** Forces add like vectors (parallelogram rule!)

**Newton's Third Law**
Forces always come in pairs (interaction between two objects A and B):
$F_{\text{Action}} \text{ (A on B)} = - F_{\text{Reaction}} \text{ (B on A)}$

**Momentum**
For a single object: $\vec{p} = m \vec{v}$
Change in momentum (impulse): $\vec{J} = \Delta \vec{p} = \sum \vec{F} \Delta t$

Because of Newton’s 3rd Law, impulse transferred from object A to object B is equal in magnitude but opposite in sign (direction) to simultaneously transferred impulse from B to A.

System of particle with no external force: Total momentum $\sum \vec{p}_i = \vec{p}_1 + \vec{p}_2 + \ldots$ is conserved (same before as after any collision between any particles in the system)

Momentum conservation in two-particle collision:
$m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f} \ (i = \text{initial}, f = \text{final})$

If sum of all kinetic energies is also conserved (no work done on system as a whole, no energy converted to other forms; see below): ELASTIC collision. Otherwise (friction, conversion to heat/deformation,\ldots): INELASTIC collision. If 2 objects stick together after collision: Completely inelastic
If collision is completely inelastic: $m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = (m_1 + m_2) \vec{v}_f$

**Work and Energy**

Work $\Delta W$ on some object (particle or system) due to a force $F$ if that object is displaced by $\Delta x$ in the direction of the force: $\Delta W = F \Delta x = F \Delta s \cos \phi$

(only the part of the total displacement $\Delta s$ that is in the direction of the force counts; no displacement = no work). Work done on object can be positive (transferred TO object) or negative (transferred FROM object).

**Work-Energy Theorem:** Net work done on an object changes the ENERGY of that object: $\Delta E$ (particle or system) = $\Delta W$ (done on that particle or system)

**Conservation of total Energy**

The total amount of energy within a system can only be changed by exchanging energy with another system. IF the energy of one object (particle, system,...) goes UP, then the energy of another object (particle, system,...) must go DOWN by the same amount, and vice versa!

Power: Rate of work done on system per unit time: $P = \Delta W / \Delta t = F \cdot v$

**Types of Energy**

**Kinetic energy** of a moving particle: $\text{K.E.} = \frac{m}{2} \vec{v}^2$ (depends only on speed; is never negative; has no direction; depends on coordinate system used for $v$)

If the only (relevant, changing) energy of a system is kinetic energy, then the work-energy theorem states: $\Delta \text{K.E.} = \Delta W$

**Potential Energy**

Some forces can do work that depends only on initial and final position of the particle(s) that this work is being done on. Example: Gravity – only the difference in height between initial and final state matters: $\Delta W = mg \Delta h$

These forces are called “conservative” because they can “store” work and return it without losses. Any work done against a conservative force increases stored “potential” for future *positive* work by the same force.

Work done by conservative force stored in form of potential energy:

$U (\vec{r}) = \Delta W_{\text{ext}} (\vec{r}_{\text{ref}} \rightarrow \vec{r}) = -F_{\text{conservative}} \Delta s \cos \phi$

At reference point $U (\vec{r}_{\text{ref}}) = 0$ ($\vec{r}_{\text{ref}}$ can be chosen for convenience).

Examples:

- Particle attached to a spring, stretching the spring by moving a distance $x$ from the spring’s relaxed state: $U_{el} = (k / 2) x^2$
(k = spring constant; x = 0 $\Rightarrow$ $U_{el} = 0$ for relaxed state of spring); 
more general: totally elastic distortion of objects

- Approximate Gravitational Potential Energy (near Earth surface):
  $U_{grav} = mgh$ ($h$ is height above reference point)

- More general Gravitational Potential Energy: $U_{grav} = -GmM/r$
  (see later in semester; here reference point is $\infty$ far away)

- Electrostatic Potential Energy (charged particle in static electric field)
  $\Rightarrow$ see even later

**Total mechanical energy**

Sum of kinetic and potential energy of a particle: $E_{mech} = K.E. + U$

IF no external (dissipative) forces present: $E_{particle} = E_{mech} = \text{const.}$ ($\Delta E = 0$)

$\Rightarrow$ sum of kinetic and potential energy stays constant; for example:
go ing UP the ramp DEcreases kinetic energy (negative work done by gravity) and INcreases potential energy; going back DOWN INcreases kinetic energy (car speeds up) and DEcreases potential energy ($mgh$ becomes less).

Sum stays constant, so once you have $E_{mech} = \frac{m}{2} v^2 + mgh$ at one position, you can find velocity at any other position by solving $\frac{m}{2} v_f^2 = E_{mech} - mgh$

If EXTERNAL forces DO work: $\Delta E_{mech} = \frac{m}{2} v_f^2 - \frac{m}{2} v_i^2 + \Delta U = \Delta W_{ext}$

**Other forms of energy:** (see later during PHYS101-102)

Energy stored in objects (distortion, chemical energy, ..., mass!) 1

Kinetic energy due to random motion of particles in a system = internal energy (most disordered and least “useful” form of energy; “heat”)

Energy stored in fields (e.g. electromagnetic fields)

Energy can be transmitted by fields and waves (sound, light, electromagnetic radiation,...)

Energy can be positive (e.g., kinetic energy always is) or negative (e.g., general gravitational energy, binding energy of molecules in solid or liquid, binding energy of electrons in atoms, binding energy of nucleons in nuclei etc.) or both (depending on reference point)

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1 *Note:* According to Einstein, mass is just one form of energy ($E = mc^2$) and therefore only the sum of all energy types including mass energy is conserved!
The total sum of ALL these forms of energy in a CLOSED system is always conserved; energy of an interacting system can only change if it is exchanged with other system (in form of work or heat).

**Motion in a circle**

Position described by angle $\theta$ [in radians], radius $R$

Distance around the perimeter of a circle [in m]: $D = \theta [\text{in radians}] \cdot R [\text{in m}]$

Period of 1 revolution: $T$; rotational speed in “rounds per second” $\text{rps} = 1/T$

Angular velocity: $\omega = \Delta \theta [\text{in radians}] / \Delta t [\text{in s}] = 2\pi / T = 2\pi \text{rps} = |\vec{v}| / R$;

linear (tangential) speed $v = R\omega = 2\pi R \text{rps}$

Centripetal acceleration: $a_{\text{centr}} = \vec{v}^2 / R = \omega^2 R$

$\Rightarrow$ required centripetal force $F = m a_{\text{centr}}$

**Moment of Inertia $I$**:

- Single Particle of mass $m$ at distance $R$ from axis: $I = mR^2$
- Extended Objects: $I = \sum_p (m_p r_p^2)$; increases with total mass and with the square of the average distance $r_p$ from axis of that mass
- Hollow cylindrical shell (total mass $M$, radius $R$): $I = MR^2$
- Solid cylinder: $I = MR^2/2$
- Solid sphere: $I = 2/5 MR^2$
- Thin rod of length $l$ (axis through center): $1/12 \, Ml^2$
- Thin rod (axis through edge): $1/3 \, Ml^2$

Kinetic energy for rotational motion: $\text{K.E.} = \frac{1}{2} I \omega^2$

**Angular momentum**:

- Single Particle at distance $R$ from axis: $L = mRv = mR^2 \omega = I\omega$
- Extended object: $L = I\omega$ (conserved if no external torque is present)

Points along axis of rotation; positive if rotation is counter-clockwise, negative if rotation is clockwise (right hand rule: curling fingers = sense of rotation, thumb points in direction of $L$).

Total angular momentum of closed system is conserved if no external force present $\Rightarrow$ if moment of inertia increases, angular velocity must decrease!

A change in angular momentum (rotational status) requires external torque:

tangential force $F$ at distance $l$ relative to axis: $\tau = l \cdot F \cdot \sin\theta = \Delta L / \Delta t$

(lever arm $l$ times force; only the part of $F$ perpendicular to $l$ counts)
If net external torque (sum of all torques) = 0 -> $\vec{L}$ is conserved

Requirement for equilibrium: Sum of all forces AND sum of all torques must both be zero! Center of gravity must be above support area.
Gravitation
Gravitational Force due to mass \( M \) on mass \( m \) at distance \( r \): \( F = -\frac{GMm}{r^2} \)
\( r \) is distance from center of mass \( M \) to center of mass \( m \)
Gravitational Force \( F \) points back to its source (the center of the “attractor”)
Gravitational acceleration of \( m \) towards \( M \): \( a = \frac{GM}{r^2} \) (independent of \( m \))
Tidal acceleration difference across an object of size \( D \) due to a mass \( M \) at distance \( r \): \( \Delta a = 2 \frac{GM}{r^3} D \)
Gravitational Potential Energy between two masses at distance \( r \) (center to center): \( U_{grav} = -\frac{GmM}{r} \) (reference point assumed at infinite separation)
Escape speed from planet with mass \( M \) and radius \( R \): \( v_{esc} = \sqrt{\frac{2GM}{R}} \)

Kepler’s Laws
1) Orbits of satellites (moons, planets,...) are elliptical
2) A line from the gravitating body (sun, planet,...) to the satellite
   sweeps out equal areas in equal times (conservation of angular momentum) \( \Rightarrow \) speed is larger if distance is smaller
3) The square of the period \( T \) of an orbit around a mass \( M \) is proportional to the cube of \( a \): \( T^2 \propto a^3 \) \( (a = \text{major half axis of ellipse}) \) \( \Rightarrow T \propto \sqrt{a^3} \).
   Complete formula \( T = 2\pi \sqrt{\left(\frac{a^3}{GM}\right)} \)
   \( \Rightarrow \) on a circular orbit of radius \( R \) the velocity is \( v_{orbit} = \sqrt{\frac{GM}{R}} \)

Projectile motion near Earth’s surface (neglect air resistance)
Horizontal (\( x \)) and vertical (\( y \)) motion are independent of each other
Horizontal components: \( v_x = v_{x0} = \text{const.} ; \quad x(t) = x_0 + v_{x0}t \)
Vertical components: \( v_y(t) = v_{y0} - gt ; \quad y(t) = y_0 + v_{y0}t - \frac{1}{2} gt^2 \)
Total motion is simply combination of horizontal and vertical one. Vertical motion usually determines total time for path (until impact). Initial launch at angle \( \theta \), above horizontal: \( v_{x0} = v_0 \cos(\theta_0) ; v_{y0} = v_0 \sin(\theta_0) \)
Total speed = \( v = \sqrt{v_x^2 + v_y^2} \) ; final angle \( \theta = \arctan\left(\frac{v_y}{v_x}\right) \).
Maximum height of trajectory is \( y_{max} = y_0 + \frac{1}{2} v_{y0}^2 / g \) (assuming \( v_{y0} > 0 \));
Total time elapsed for trajectory from ground back to ground: \( \Delta t = \frac{2 v_{y0}}{g} \)
Maximum distance traveled (from ground to ground) is \( \Delta x_{max} = \frac{2 v_{x0} v_{y0}}{g} \)
Electrostatics: Charge
Positive or negative, measured in Coulomb [C], conserved, quantized (in units of e). Equal sign charges repel, opposite sign charges attract.
Electron: Charge \( q = -e \). Proton: Charge \( q = +e \). Most atoms have \( q = 0 \).

Force between charges:
Coulomb Force \( \vec{F}_{on \ q \ due \ to \ Q \ at \ distance \ r} = k \frac{Qq}{r^2} \), \( k = 8.99 \times 10^9 \) \text{Nm}^2/\text{C}^2.

Alternative Formulation:
Charge \( Q \) (or any distribution of such charges) create an electric field \( \vec{E} \).
Charge \( q \) experiences a force \( \vec{F} = q\vec{E} \) in this field (include sign of \( q \) ).
Field lines begin at positive charges and end at negative ones or go on forever; density of field lines indicates strength of field.
Examples:
- Electric field at distance \( r \) from a single spherical charge: \( \vec{E} = \frac{kQ}{r^2} \)
- Electric field between two large, conducting parallel plates: \( \vec{E} = \text{const.} \)

Conductors
- Contain huge amounts of “free” charges
- In presence of electric field, charges will flow to counteract field
- Unless new charges are constantly supplied (current), field will be canceled \( \Rightarrow \) no field inside conductors
- All charges sit on the outside surface of the conductor
- External field will rearrange free charges such that conductor experiences net force towards external charge even in absence of net charge (“induction”)
- Conductor connected to ground will have net charge if one type of its charges is pushed away by nearby other charges of same sign

Insulators
- Contain no free charges
- Can pick up a few “extra charges” – a little net charge
- External field can polarize individual molecules such that material experiences net force towards external charge
- Polarization can weaken but never cancel external field
**Potential**
Moving charge \( q \) along electric field \( \mathbf{E} \) does work: \( W = Fd = qEd \)
Electrostatic forces are conservative \(-\) work stored in electrostatic potential energy \( U_{\text{pot}} \). Electrostatic potential energy \( U_{\text{pot}} \) of a charged object divided by its charge \( q \) equals potential \( V \) [measured in Volt = V]. A charge \( q \) changes its energy by \( q\Delta V \) when moving from a point with potential \( V_1 \) to a point with potential \( V_2 = V_1 + \Delta V \).
1 V potential difference between 2 points means there is a potential to do 1 J of work per Coulomb moved from the higher potential point to the other.

**Current**
Net flow of charge in a given direction per unit time, measured in Ampere [A = C/s]
Count positive charges +, negative charges – and reverse sign for charges moving in opposite direction. Net current = sum of all currents
Current in conductor with resistance \( R \) requires electric field \(-\) potential difference \( \Delta V = -RI \). (Ohm’s Law).
Resistance measured in Ohm [Ω], is proportional to length of conductor, is inversely proportional to cross section, and is usually greater if conductor is hot.
Currents require complete (closed circuit) path and non-electrostatic “pump” (EMF = ElectroMotive Force = energy per charge) to keep running. Current is identically the same everywhere around a simple closed loop circuit.
Currents heat up conductors; power (energy transfer per unit time) is \( P \) [Watt] = \( \Delta V \) [Volt] x \( I \) [Ampere] = \( RI^2 = \Delta V^2/R \)
Typical speed of electrons in wire: \( 10^6 \) m/s random (internal energy), but only \( 0.1 \) mm/s on average in direction of electric force. Electric fields (that “tell the electrons to move”) travel with (nearly) speed of light, \( c \). Power can be delivered both by currents going in one direction only (DC, ex.: battery driven) and by currents going back and forth (AC, currents driven by generators, household and HV circuits).

**Series Circuit**
Same current \( I \) has to go through all elements in series (doesn’t matter which one comes “first”). Voltage supplied by battery etc. gets “divided up” among
various elements. Total resistance is sum of individual resistances. The largest voltage drop occurs across the largest resistor. Current (and therefore power) in each element smaller than if connected directly (without the others) to battery. Any break in circuit and all current ceases.

**Parallel Circuit**
Current that must be supplied by battery is the sum of the currents to all of the various branches. Each branch sees the full battery voltage, independent from the other branches. Smallest resistor draws the largest current and therefore the largest power. Any one element (branch) can have a break without affecting the others.

**Magnetic fields**
Due to charges in motion (electric currents, spinning electrons,...) Field lines can form closed loops, but never end or begin. Example: Permanent dipole, solenoid (or short coil): Field lines can be described as if emanating from “North pole” and converging towards “South pole” (but connect through the interior of the dipole!) Cannot separate “north pole” from “south pole” – chopping in half will simply yield 2 new dipoles.
Earth has a magnetic dipole field with magnetic north pole close to geographic south pole (Australia). Field of straight wire: circles around it, falls off like $1/r$.

**Magnetic materials**
Most materials react only slightly to magnetic fields. All materials have a slight diamagnetic response (see below) to strong magnetic fields. In some special materials, there are additional responses which can totally overpower the usual diamagnetic response.
Examples:
*Ferromagnets*: Iron, Cobalt, Nickel and some Rare Earth compounds
Magnetic properties due to electron spins (aligned within domains).
Normally random orientation of domains – no net magnetism
External field can orient or grow/shrink domains – strong magnetic response (magnetization) – leading to attraction towards other magnets
For some alloys, domains can keep orientation indefinitely –→ permanent magnets. Permanent dipoles can attract (unlike poles) or repel (like poles); net force on dipole is zero in homogeneous magnetic field but torque tends to align dipole with field direction

*Paramagnets*: A weak magnetization in the direction of external field; attraction to strong magnets. Some materials like (liquid) oxygen, gadolinium,…

*Diamagnets*: A (usually extremely weak) magnetization opposite to the direction of external field; always leads to (usually feeble) repulsion from strong magnets. Property of nearly all materials (including water; –→ floating frog) usually too weak to detect and/or masked by ferro- or paramagnetism. Exception: Superconductors are perfect diamagnets (strong expulsion of all external magnetic fields –→ floating permanent magnet above cold superconducting disk).

**Magnetic forces**
Acts only on moving charges (including permanent or induced magnets: equal poles repel, opposite poles attract)  
Magnitude proportional to $q$, $v$ and $B_{\text{perp}}$ (the part of $B$ perpendicular to $v$):  
$$ F = qv |B_{\text{perp}}| $$
Acts perpendicular to both $v$ and $B$ (right hand rule). In vector notation:
$$ F = qv \times B $$
Typical motion in homogeneous field: Circle of radius $R = \frac{mv}{qB}$ with angular velocity $\omega = \frac{qB}{m}$, neither speeding up nor slowing down (no change in kinetic energy –→ magnetic forces don’t do work on moving charges).  
Force on wire: $ILB$ ($L$=Length), sideways (perpendicular to field and wire). Two parallel wires with current in the same direction attract:
$$ F = 2 \cdot 10^{-7} \text{ N per 1 m of length if wires are 1 m apart and both carry } I = 1 \text{ A.}$$
Second Semester – PHYS102

Atomic structure of matter
Size of typical atoms: about 1 Ångström (10⁻¹⁰ m)
Atomic Mass \( A = 12 \times \text{Mass of Atom} / (\text{Mass of } ^{12}\text{C atom}) \)
  approximately = \( Z + N \) (number of protons and neutrons in nucleus);
\( Z \) determines element, \( N \) which isotope of that element
  H: \( A = 1 \) (or 2 or 3); He: 4 (or 3), C: 12 (13,14…), O: 16, …
Molecular Mass = Sum of atomic masses of all atoms in the molecule
  (H₂: 2, O₂: 32, H₂O: 18)
1 mol of a compound = as many grams as molecular mass
  (1 mol \(^{12}\text{C} = 12 \text{ g}, 1 \text{ mol } ^{13}\text{C} = 13 \text{ g})
1 mol of anything contains \( N_A = 6.022 \times 10^{23} \) molecules or atoms

Deformation of solids
Elongation or compression: \( \Delta L/L \propto F \) (tension)
  inversely proportional to cross section and “strength” (Young’s modulus)
Hooke’s law: \( F = -kx \)
Bending beam: Outside layer elongated, inside layer compressed

Density
\[ \rho = \frac{\text{mass}}{\text{Volume}} \ [\text{kg/m}^3] \]
  • Air: \approx 1.25 \text{ kg/m}^3 \text{ at sea level – decreases continuously with height (50\% at 5.6 km)}
  • Water: 1000 \text{ kg/m}^3 \text{ (ice a little less)}
  • Iron: 7,874 \text{ kg/m}^3
  • Iridium: 22,650 \text{ kg/m}^3

Pressure
\[ P = \frac{\text{force}}{\text{surface area}} \ [\text{N/m}^2 = \text{Pa}] \]
Hydrostatic pressure in fluid w/ density \( \rho \) changes with height: \( \Delta p = -\rho g \Delta h \)
Air pressure at sea level: 101,300 \text{ Pa} \text{ (weight of } \approx 1 \text{ kg or } \approx 10 \text{ N on } 1 \text{ cm}^2)\)
Buoyant force: Equal to weight of displaced fluid (upwards) = \( g V_{\text{obj}} \rho_{\text{fluid}} \)
Pascal’s Principle: Change pressure somewhere in liquid – gets transmitted to all other parts (modulo hydrostatic pressure difference)
Boyle’s Law: Pressure and density are proportional in (ideal) gases; $PV = \text{const.}$ (for constant amount of gas and constant temperature)

Bernoulli’s principle: Pressure is lower in faster flowing fluid

**Internal Energy (Part of Total Energy of a System)**
All types of energy of particles making up an object that are random and undirected (no overall motion of the object); can be changed through work on the object or through heat transferred from one object to another through direct contact (conduction), flow (convection) or (electromagnetic) radiation. Measured in J or calories ($1 \text{ calorie} = 4.2 \text{ J}$; warms 1 gram of water by $1^\circ \text{C}$).

**Temperature**
Measure of (proportional to) average internal energy per particle (really: per “degree of freedom”); measured in Kelvin, Celsius or Fahrenheit (thermometer); tends to even out between different objects or systems over time (by transfer of heat from warmer to colder object).

Heat transfer: Conduction (direct contact), Convection (movement of fluid with different temperature) and Radiation (only heat transfer in vacuum)

Heat capacity $C$ – amount of internal energy change needed to increase temperature of an object by $1^\circ \text{C}$.

Specific heat capacity $c = C/\text{mass}$ – amount of energy needed to increase temperature of 1 kg mass of a substance by $1^\circ \text{C}$.

E.g. water has specific heat capacity $c = 4200 \text{ J/kg/}^\circ \text{C} = 1 \text{ cal/g/}^\circ \text{C}$

Newton’s Law of cooling/heating: Heat transfer rate proportional to temperature difference $\Rightarrow$ will slow as temperatures approach each other.

**Phase Change**
Change in internal energy without change in temperature (e.g., transition solid -> liquid -> gas -> plasma ->…).

Example: Water

“Latent heat of melting”: 80 cal (335 J) melts 1 g of ice at $0^\circ \text{C}$

“Latent heat of evaporating”: 540 cal (2255 J) convert 1 g of water to vapor (at $100^\circ \text{C}$ and standard pressure)
Evaporation can occur at any temperature (if vapor pressure at that temperature exceeds partial pressure of surrounding water vapor; else condensation). **Boiling**: vapor pressure equals or exceeds air pressure -> gas can form *inside* the liquid, not only on surface

Vapor pressure increases with temperature and is 1 atm \((10^5 \text{ Pa})\) at 100 \(^\circ\)C

**Triple point**: All 3 phases exist together (0.01 \(^\circ\)C, 600 Pa)

**First Law of Thermodynamics:**
\[
\Delta E_{\text{internal}} = (\text{Work done on system}) + (\text{Heat added}) - (\text{Work done by system}) - (\text{Heat removed})
\]
[assuming no other form of energy changes in system]

\[\Rightarrow \Delta E_{\text{internal}} = P(-\Delta V) + Q (Q = \text{heat}; \text{positive if added, negative if removed})\]

Energy conservation! ("You can not win")

For **ideal** gases:
\[
E_{\text{internal}} = \frac{3}{2} nRT
\]

\[PV = nRT \quad [P = \text{pressure in Pascal}, V = \text{volume in } m^3, n = \text{number of mols}, R = \text{gas constant } = 8.3 \text{ J/mol/K}, T = \text{temperature in K}]\]

\[\Rightarrow \text{Volume of 1 mol of gas at } 0 \, ^\circ\text{C and normal atmospheric pressure } = 22.4 \text{ liters}\]

**Ideal Heat Engine (Carnot Machine):**
Remove Heat \(Q_1\) from reservoir at higher temperature \(T_{\text{hot}}\)
Exhaust less heat \(Q_2 = T_{\text{cool}} \times T_{\text{hot}}\) at lower temperature \(T_{\text{cool}}\)
Get work \(\Delta W = (1 - T_{\text{cool}}/T_{\text{hot}}) \times Q_1\) out – efficiency \(e = 1 - T_{\text{cool}}/T_{\text{hot}}\)

Refrigerator or heat pump \(\Rightarrow\) works in reverse: Remove Heat \(Q_1\) from cold reservoir at \(T_{\text{cool}},\) put in mechanical work \(\Delta W = (T_{\text{hot}} / T_{\text{cool}} - 1) \times Q_1\) to exhaust more heat \(Q_2 = T_{\text{hot}} / T_{\text{cool}} \times Q_1\) at higher temperature \(T_{\text{hot}}\).

**Second Law of Thermodynamics:**
- No machine can simply convert heat into work without exhausting some of the heat into a colder reservoir
- No machine can beat the Carnot machine’s efficiency
- Heat can never flow spontaneously from cold to warm without external input of work
- Entropy (disorder) can never decrease; it always tends to increase over time (e.g. when heat flows from warm to cold)
**Oscillations:**
Period: $T$ [sec]; Frequency: $f = 1/T$, unit Hertz (1 Hz = 1 oscillation/sec)
Harmonic Oscillator: $x(t) = A \sin(2\pi ft)$; $f$ independent of amplitude
[$x =$ excursion, $A =$ amplitude=maximum excursion from equilibrium/rest]

Pendulum of length $l$: $f = \frac{1}{2\pi} \sqrt{\frac{g}{l}}$

Mass $m$ on spring (spring constant $k$): $f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$

Oscillator will react most strongly to external periodic disturbance near its own intrinsic frequency (resonance) -> amplitude can build up over time.

**Waves:**
Wave velocity $v_{\text{wave}}$, wave length $\lambda$ (distance between crests), frequency $f$:
$v_{\text{wave}} = \frac{\lambda}{T} = \lambda f$; $\Rightarrow \lambda = v_{\text{wave}} / f$

Excursions perpendicular to propagation: Transverse wave (2 polarization directions); Excursions in direction of propagation: Longitudinal wave

Doppler effect: Emitter moving through medium – $\lambda$ shortened and $f$ increased in direction of motion and vice versa in opposite direction;

Receiver moving through medium – apparent $f$ reduced if moving in direction of wave and increased if moving against.

Interference: Excursions of 2 overlapping waves add up; can lead to destructive interference (out of phase; less net excursion) or to constructive interference (in phase; more net excursion). Extreme case: Complete extinction

Standing wave: Wave interfering with its reflection in medium of length $L$; wave length must be integer fraction of a fundamental (either $2L$ or $4L$):

Nodes at both ends (string, ...): $\lambda_n = 2L/n, n = 1,2,3,...$ (harmonics)

Node at one end only (air column, ...): $\lambda_n = 4L/n, n = 1,2,3,...$

n-1 nodes between n maxima; resonant frequencies at $f_n = v_{\text{wave}} / \lambda_n$

Refraction: Waves change direction when wave velocity changes (e.g., transition from one medium to another); bend toward normal in “slower” medium

Diffraction: Each point along a wave is source of new (spherical/circular) wave (Huygen’s principle); waves can spread out “behind” corners and obstructions.
Sound and Music:
Sound: longitudinal wave, oscillations in density (pressure in fluids or position of molecules in solids). Requires medium – solid, liquid, gas.
Wave speed in air: 330 m/s (0°C) – 340 m/s (20°C).
Intensity (audible range): $10^{-12}$ W/m² (0 decibel) – 1 W/m² (120 decibel – pain threshold!); Intensity = Amplitude squared
10x more intensity: add 10 decibel; 10x more amplitude: add 20 decibel
Beat frequency: fluctuation of sound with frequency $f_1 - f_2$ if 2 sound waves with nearby frequencies $f_1, f_2$ interfere.
Concert A: $f = 440$ Hz; 1 octave = factor 2 in $f$

Induction
Wire moving in magnetic field -> force $qvB$ on each electron inside the wire
-> pushing a current along wire.
-> Working principle of electric generator (wire loop rotating in magnetic field)
Magnet moving relative to a “fixed” wire -> same force, but cause is electric field $E$ induced by changing magnetic field.
Changing magnetic field $B$ due to any cause: (non-static) electric field $E$
Induced field $E$ will circle “flux” of magnetic field, perpendicular to $B$
Energy gain per loop and per charge is proportional to area filled with magnetic field times the rate of change of the magnetic field.
Coil with several loops in changing magnetic field: Induced voltage is proportional to number of loops.
Transformers: Output voltage/Input voltage = # of secondary coil loops/# of primary coil loops.
Lenz’ law: Effects of a magnetic field change oppose that change.
=> Self-inductance: Induced EMF opposing change in $B$ (spark, back-EMF)
=> Jumping ring: Force pushes conductor out of region of changing $B$; Induction brake:
Induced EMF creates eddy currents that slow down moving conductor
Electromagnetic Waves
Maxwell’s Law: Changing electric field creates magnetic field circling it (perpendicular to $\mathbf{B}$).
Chain reaction: Changing $\mathbf{E}$ -> $\mathbf{B}$ (Maxwell); Changing $\mathbf{B}$ -> $\mathbf{E}$ (Faraday);
Changing $\mathbf{E}$… => Electromagnetic waves! Wave speed in vacuum = $c$
$\mathbf{E}$ perpendicular to $\mathbf{B}$, $\mathbf{E}$ and $\mathbf{B}$ perpendicular to propagation, $B = E/c$
Due to accelerated charges (antenna, oscillating molecules/atoms/nuclei…)

Typical wave lengths/frequencies
- Radio waves: $\lambda = 1\text{m} - 1000\text{ m} \mid f = 300 \text{kHz} - 300 \text{MHz}$
- Microwave: $\lambda = 1\text{mm} - 1\text{ m} \mid f = 300 \text{MHz} - 300 \text{GHz}$ (GHz = $10^9$ Hz)
- Infrared: $\lambda = 0.75\text{µm} - 1\text{ mm} \mid f = 300 \text{GHz} - 400 \text{THz}$ (THz = $10^{12}$ Hz)
- Visible Light: $\lambda = 375 \text{ nm} - 750 \text{ nm} \mid f = 400 - 800 \text{THz}$
- Ultraviolet: $\lambda = 10 \text{ nm} - 375 \text{ nm} \mid f = 0.8 \text{ PHz} - 30 \text{ PHz}$ (PHz = $10^{15}$ Hz)
- $\mathbf{X}$-ray: $\lambda = 10 \text{ pm} - 10 \text{ nm} \mid f = 30 \text{ PHz} - 30,000 \text{ PHz}$
- $\gamma$-ray: $\lambda < 10 \text{ pm} \mid f > 30,000 \text{ PHz}$

All objects/materials emit electromagnetic radiation over the full spectrum; maximum intensity centered on frequency which is proportional to temperature (ice-cold universe at 2.7 K: microwaves; room temperature: Infrared; sun’s temperature: visible light, etc.)

Color of Light:
Determined by mix of wave lengths/frequencies in light waves
“Pure” colors: 400 nm = violet, 500nm = blue-green, 600 nm = yellow, 700 nm = red
Sun maximum (6000K) around 550 nm = greenish-yellow
Color perception: Primary colors = red (600-700 nm), green (500-600), blue (400-500)
Additive colors: combination of relative intensities in these 3 ranges – e.g., yellow = red + green, cyan = green + blue, magenta = blue + red; white = equal mix of all 3
Subtractive (complementary) colors (cyan, magenta, yellow) remove intensity from reflected light – e.g. cyan + magenta remove all but blue; equal mix of all 3 = black
Color of light can get altered by scattering (wave-length dependent transmission vs. sideways scattering), absorption (removal of certain wave lengths in transmission or reflection) and diffraction (splitting up light into individual wave lengths); emission spectrum determines “initial color”
Geometrical Optics (follows from Fermat’s Principle of least time\(^2\)):

**Propagation:** constant medium => wave propagation follows straight lines = “rays” => Shadows (umbra/penumbra), pin hole camera

**Reflection:** Incoming angle = outgoing angle (on opposite side of normal)

**Absorption:** Wave energy dissipates, “ray” is stopped (often color-selective).

**Refraction:** Change in wave velocity \(v_{\text{wave}} = \frac{c}{n} (n = \text{index of refraction})\) =>

- going from small \(n\) to large \(n\): refracted part of wave bending towards normal \((n_1 \sin \theta_1 = n_2 \sin \theta_2)\); reflected portion has “phase jump” of \(\lambda/2\)
- going from large \(n\) towards small \(n\): bending away from normal, beyond maximum incoming angle: total internal reflection; no phase jump for reflected portion

**Dispersion:** \(n\) increases with shorter \(\lambda\) – blue more deflected than red

**Scattering:** Waves are absorbed by small particles and reemitted in all directions => decrease of intensity in the direction of the ray, diffuse background light in all other directions. Shorter-wavelength waves usually are scattered more (if scattering centers are tiny)

**Image:** Where light rays reaching your eye seem to be coming from

- **Real image:** Formed by actual light rays converging at its position; can be captured by screen or photosensor or cones+rods in your retina
- **Virtual image:** Cannot be captured; appears to be at a point where no actual light rays converge.

**Flat Mirrors:** Upright, equal-sized virtual image equally far from mirror (on other side) as object; interchanged front and back

- **Convex curved mirror:** Smaller, upright virtual image on other side of mirror
- **Concave curved mirror:** Object close => magnified, upright virtual image on other side of mirror; Object further away => inverted real image in front

**Object in denser medium:** Appears closer to surface than it actually is

- Light going through parallel-surface plate at an angle: Sideways offset
- Light going through continuously changing medium (atmosphere at different height or temperature): Bending (mirages, delayed sunsets…)

**Lenses:** Described by focal length \(f\) (point where parallel rays are focused)

- **Concavely curved (dispersing) lens:** Smaller, upright virtual image on object side of lens; used to correct near-sightedness (defocuses); virtual focal point.

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\(^2\) Light “rays” travel from A to B along that path which requires the least time
Convexly curved (focusing) lens: If object closer than \( f \) => Virtual, upright, magnified image on same side as object; if object farther than \( f \) => real, inverted image on opposite side of lens than object (magnified if distance less than twice \( f \)); real focal point. Used for far-sightedness, projection, cameras; focusing lens in eye makes real image on retina for visual perception

*Lens equations: \( 1/f = 1/\text{objectdistance} + 1/\text{imagedistance} \); magnification = - imagedistance / objectdistance (\( f < 0 \) if virtual focus / imagedistance < 0 if virtual image))

Diffraction + Interference

Huygen’s principle: Each point along a wave front becomes source of a new spherical wave -> “bending” around corners, diffraction, wave propagation

Thin film interference: reflected waves from both sides of film interfere => pattern of bright light (constructive interference; path length difference plus “phase jump” = zero or \( n\lambda \)) and dark (destructive interference; path length plus “phase jump” = \( \lambda/2 \) or \( n\lambda + \lambda/2 \)); depends on wave length => different colors are removed at different thickness (bands or rings of varying color)

Larger distance (cavities bounded by 2 mirrors) -> standing waves: interferometers (Fabry-Perot, Michelson); lasers

Double slit interference: Dark and bright stripes of equal thickness; spacing proportional to \( \lambda/s \) (\( s = \) slit separation) and to distance from slits to screen

Single slit: Broad central maximum, dim fringes; spacing of first minimum (diffraction limit) proportional to \( \lambda/w \) (\( w = \) slit width)

Diffraction grating: Many slits, narrow maxima separated by wide dark bands (spacing proportional to \( \lambda \) divided by slit separation); spectroscopy

Polarization: \( E \) field oscillates in one single direction (x or y only)

Polarizer/analyzer: transmits light oscillating only along its axis; if incoming light is polarized, amount (intensity) transmitted is \( I/I_0 = \cos^2(\phi) \)

(\( \phi \) = angle between direction of polarization and axis of polarizer)

Reflected light is typically polarized parallel to the reflecting plane

Emission and Absorption

Classical Physics: Electromagnetic radiation due to accelerated charges [e.g., antennae, “bremsstrahlung” (= braking radiation due to charges slowing down), synchrotron radiation (charges moving in circles), hot plasmas]
Atomic Physics: Electromagnetic radiation due to transition between different electron orbits of different (fixed) energy; energy difference carried away by a “photon” of frequency $f = \Delta E/h$ ($h =$ Planck’s constant)

Free atoms (dilute gas): Sharp, widely separated frequencies (emission after exciting atoms into higher orbits) with unique color (wave length) pattern for each type of atom/molecule (“spectroscopic fingerprint”); identical absorption lines (removing photons of same frequency from passing light beam)

2-step processes: Fluorescence (immediate re-emission at lower frequency) and Phosphorescence (re-emission of light after a while)

Stimulated emission (photon doubling) -> LASER

Incandescence: Broad, overlapping spectral lines -> continuous spectrum (due to interactions and Doppler broadening in dense materials); typical thermal radiation with peak frequency $f_{\text{max}} \propto$ Temperature.

Example: The peak frequency of the light emitted by the sun (surface temperature $T \approx 6000$K) corresponds to green-yellow light ($\lambda = 500$-$550$ nm)

Particle-Wave Duality

Microscopic particles travel like waves (=> interference, diffraction, standing waves, etc.) with wavelength $\lambda = h/p$ (momentum) and frequency $f = E/h$; waves interact with microscopic objects like particles carrying energy $E = hf$ and momentum $p = h/\lambda$. Examples: light travels as electromagnetic wave and interacts as point-like photons; particles like electrons travel as “matter waves” and interact like point-like objects, etc. Interaction is always “all or nothing”.

“Matter waves” are oscillations of an abstract “matter field” $\psi$; its intensity $= \text{field strength squared } |\psi|^2$ at some point gives probably of finding electron at that point. Stable orbits correspond to standing “matter waves”.

Photo-electric effect: energy of emitted electrons depends on frequency $f$ of absorbed light: $E = hf - W$ ($W =$ work required to emit electron; no emission if $hf < W$); number of emitted electrons proportional to light intensity

Heisenberg’s uncertainty principle: $\Delta p \Delta x \geq h/2\pi$ (you cannot measure position in some direction and momentum in the same direction simultaneously with infinite precision)
Quantum Mechanics

1) All objects (with momentum $p$, energy $E$) can be represented by waves describing their propagation through space.

2) The wavelength is $\lambda = h/p$ and frequency is $f = E/h$.

3) Simultaneous measurement of position and momentum is limited by Heisenberg’s uncertainty principle: $\Delta p \Delta x \geq h/4\pi$.

4) Stable orbits = standing waves.

5) If you build an apparatus that registers a localized interaction (atom excitation, electron emission, pixel activation...), you can only predict probability for particle (photon,...) to be at a given point $r$:
   \[ \text{Probability}(r) = \text{Wave Intensity}(r) = \text{Amplitude squared} \]

6) Interaction is “all or nothing” (quantized: 1 photon, or 1 electron, or...)

Atomic Physics

Stable “orbits” have fixed energies, are standing “electron waves” and have (average) circumference $2\pi r = n\lambda = nh/p$ ($n = 1,2,3...$).

Hydrogen atom: average radii are $r = \frac{h^2 n^2 \varepsilon_0}{\pi me^2} = n^2 \cdot 0.53 \cdot 10^{-10}$ m.

and corresponding energy levels (kinetic+electrostatic) of

\[ E = -\frac{me^4}{8\varepsilon_0^2 h^2} \frac{1}{n^2} = \frac{-13.6 \text{ eV}}{n^2} \]

Possible photon frequencies are $f = \Delta E/h$ where $\Delta E$ is the energy difference between two of these orbits; for hydrogen $\Delta E = 13.6 \text{ eV} \times (1/n_{\text{final}}^2 - 1/n_{\text{initial}}^2)$.

- Balmer series: $n_{\text{final}} = 1$ (all UV); $n_{\text{final}} = 2$ are visible.

Number of electrons/orbit limited (Pauli Exclusion Principle): any given orbit can have only 2 electrons (1 “spin up”, 1 “spin down”).

At higher energy, several orbits can have exact same energy (“degeneracy”).

$\Rightarrow$ the possible number of electrons in each energy level is 2 in 1$^{\text{st}}$, 2 + 6 in 2$^{\text{nd}}$, 2 + 6 + 10 = 18 in 3$^{\text{rd}}$, etc. (=> Periodic Table of Elements!)

Total number of electrons = total number of protons in nucleus = $Z$.

$\Rightarrow$ determines element (chemical properties).

The larger $Z$, the smaller each individual orbit.

Higher orbits cover larger distances from nucleus.

In reality, orbits are smeared-out 3-dimensional waves (“electron clouds”).
Nuclear Physics
Nucleus contains roughly 99.97% of the mass of an atom, but occupies only $10^{-15}$ of its volume (radius: $1 \ldots 7 \times 10^{-15}$ m = $1 \ldots 7$ fm)
Mass number $A = $ proton (element) number $Z +$ neutron number $N$;
$m_{\text{Nucleus}} < Z m_{\text{proton}} + N m_{\text{neutron}}$ (negative binding energy $\rightarrow \Delta m = \Delta E/c^2$ $\rightarrow$ Einstein!)
Most binding energy per nucleon for $A$=56 (iron); Smaller $A$: less binding energy (missing neighbors at surface $\rightarrow$ liquid drop model); Larger $A$: less binding energy because of electrostatic repulsion between protons

Transmutations of nuclei:
Change $Z$ (different element) and/or $N$ (different isotope)
Fusion: two light nuclei combine; energy gain if final $A \leq 56$
  Requires initial energy to overcome Coulomb repulsion between ingredients
Fission: one very heavy nucleus breaks into 2 pieces (initiated by a neutron); energy gain (due to electrostatic repulsion between fragments) if final pieces have $A \geq 56$; usually additional neutrons are liberated in the process
  Chain reaction if enough nuclei around (critical mass) for neutrons to initiate new fission
Alpha ($\alpha$) decay: Heavy nucleus emits $^4\text{He}$ nucleus ($Z \rightarrow Z-2$, $A \rightarrow A-4$)
Beta ($\beta$) decay: Isotope emits electron (and anti-neutrino) – $Z \rightarrow Z+1$, or positron (and neutrino) – $Z \rightarrow Z-1$ ($A$ stays constant, $N = A-Z$ changes, too)
Other nuclear reactions (“knockout”, n decay, p decay, transfer reactions…)

Radioactivity:
$\alpha$, $\beta$ decay, spontaneous fission, proton and neutron emission (more rare);
$\gamma$ decay: photon emission when nucleus goes from higher to lower energy state, e.g. after being created in another radioactive decay
Half life $T_{1/2}$: Time it takes for $1/2$ of initial radioactive nuclei to decay
One nucleus: 50/50 chance to survive after time $T_{1/2}$ passes
Number of nuclei left after time $t$ (start out with $N_0$): $N(t) = N_0 / 2^{t/T_{1/2}}$
(1/2, 1/4, 1/8, 1/16, ..., of original number left after 1, 2, 3, 4,... half lives)
Typical $\alpha$ emitter: $^{238}\text{U}$, $T_{1/2} = 4.5 \times 10^9$ years (age of Earth); decay delayed by barrier (alpha particle has to first move away far enough from rest nucleus)
Typical $\beta$ emitter: $^{14}\text{C}$, $T_{1/2} = 5700$ years (age of civilization); decay delayed because weak interaction takes a long time
Radioactive isotopes used for dating (ratio of number of nuclei left $N$ divided by initial number of nuclei $N_0 \rightarrow$ number of half lives)
Effects of radiation:
α: highly ionizing (large damage), but easy to shield (don’t inhale! Radon!!)
β: moderately ionizing (moderate damage), need thicker shielding (Al sheet)
γ: less ionizing, but has unpredictable range; most difficult to shield (lead!)
n: very penetrating, highly effective interacting with hydrogen.

Nuclear Force:
“Left-over” effect from strong force between quarks (1 ton!), attractive at 1-2 fm distance, repellant at shorter distance, disappears at large distance; only acts on near neighbors in nucleus -> liquid drop-like behavior of nuclei

Particle Physics

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<td>Spin 0,1,2, “massless”, can aggregate</td>
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<td>(e, \mu, \tau), 3 neutrinos</td>
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Protons, neutrons, hyperons, mesons,… made of quarks
Electrons in atoms, muons and taus in cosmic rays and accelerators
Neutrinos are siblings without charge (each lepton has a neutrino partner)
Each particle has an anti-particle partner with opposite charge
\(\gamma = \text{photon} = \text{carrier of electromagnetic interaction}\)
\(W, Z = \text{carrier of weak interaction}\)
Gluon = carrier of strong interaction
All particles gain mass from Higgs field (yet to be discovered at CERN)
Charge, energy, momentum and angular momentum absolutely conserved

Special Relativity
There is no absolute motion – all inertial reference frames are equivalent
The highest speed possible between 2 observers is \(c\) (speed of light), and it is the same as measured in any reference frame
2 events separated in space may appear simultaneous in one reference frame but one after the other in a different frame (moving relative to the first one)
Fast-moving\(^3\) yard sticks are measured to be shorter (length contraction: \(1/\gamma\))

\(^3\) Relative to an observer, as measured in that observer’s reference frame
Fast-moving clocks are measured to run slower (time dilation: $1/\gamma$)

Apparent mass (inertia) increases for a fast-moving object; this increase is equal to $T_{\text{kin}}/c^2$; all energy contributes to mass of object ($\Delta m = \Delta E/c^2$)

$\Rightarrow$ Mass at rest represents energy $E = mc^2$ (Energy can be converted into mass; vice versa conversion of mass into energy in particle-antiparticle annihilation), total energy $E = mc^2 + T_{\text{kin}} = \gamma mc^2$; total momentum $p = \gamma mv$

$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$ (all of these effects are tiny if $v$ is much slower than $c$)

**General Relativity**

Inertia and gravitational mass are equivalent $\Rightarrow$

Object in gravitational field behaves like object in accelerating reference frame; new definition of Inertial System: “Freely falling” $\Rightarrow F = ma$ holds

All gravitational effects can be expressed through geometric distortions of space and time (curvature proportional to mass density)

Results: Changes in planet’s orbits, bending of light rays passing stars, red-shifting (energy loss) of light traveling away from heavy masses, clocks on Earth running slow relative to GPS satellites… Most extreme case: black hole – spacetime curves in upon itself (no escape beyond “event horizon”)

**Cosmology**

Universe started 14 billion years ago at extremely high density $\Rightarrow$

$\Rightarrow$ rapid increase in size (inflation) $\Rightarrow$ cooling off – elementary particles, then nucleons, then nuclei (mostly $^4\text{He}$) form $\Rightarrow$

After 380,000 years, Universe is cold enough so that atoms can form (without being ionized right away) $\Rightarrow$ Universe becomes more transparent for light $\Rightarrow$ cosmic background radiation (initially as IR, UV and visible light)

Matter starts clumping together (driven by dark matter = 10x more than visible matter), galaxies and clusters of galaxies form, individual stars and planets form $\Rightarrow$ nuclear fusion gives more light and more nuclei (elements)

Universe keeps expanding and cooling off $\Rightarrow$ further away galaxies are “running away from us”; background radiation gets longer wave lengths (is now in radio wave spectrum)

**BIG PUZZLE:** expansion is accelerating! Dark Energy ??? And what exactly IS dark matter??? Stay tuned and read the science pages…