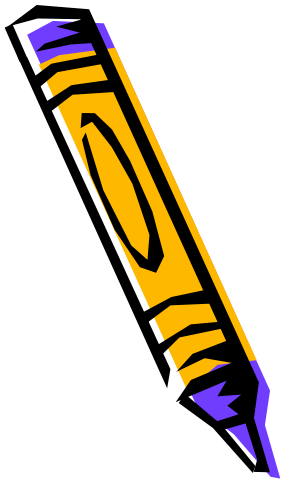
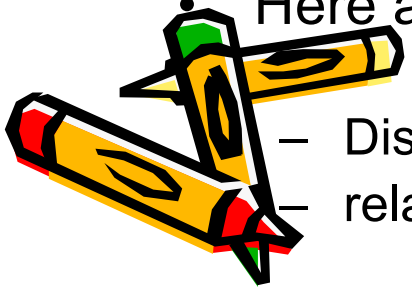
PHYSICS 102N
Spring 2022

Week 15
Relativity

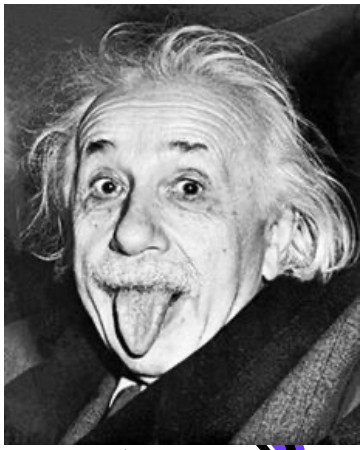
What is Relativity?



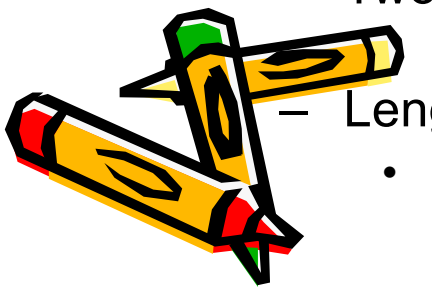
- Well known concept since Galilei's times:
  - Motion only makes sense ***relative*** to something (e.g., relative to an inertial coordinate system)
  - (Inertial) coordinate systems can move relative to each other
  - Laws of Physics should be the same in different inertial systems, even if they move relative to each other
    - E.g.: **F** = m**a** is true in **all** inertial systems
- But you have to make some assumptions. Galileo:
  - Time passes uniformly, with same "pace", in all inertial systems (IS)
    - Different observers agree on time elapsed between 2 events
    - In particular, they agree whether 2 events happened simultaneously or not
  - Length of an object is the same if measured from different IS
- Here are some consequences of these assumptions
  - Distance between 2 events is NOT universal (bouncing ball on train)
  - relative velocities add



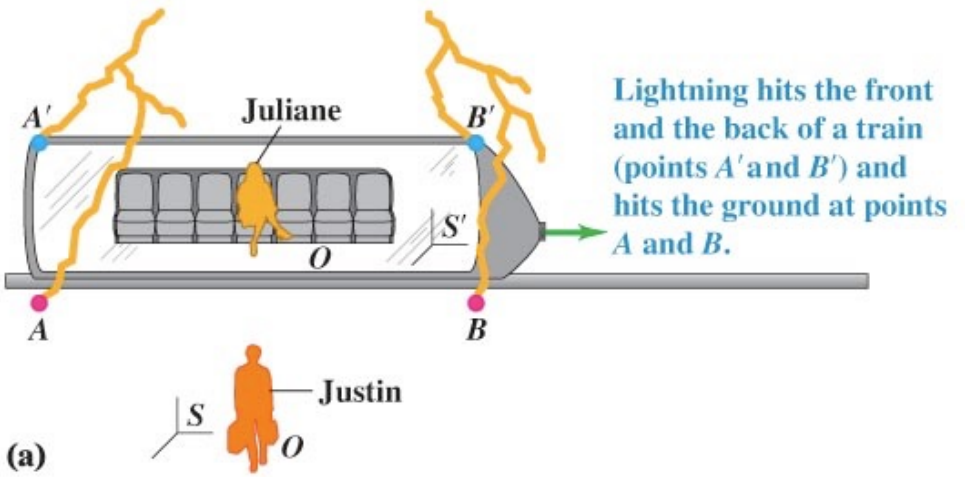
What does Einstein say?



- Experimental fact: relative velocities do NOT add once you get close to the velocity of light (c = 299 792 458 m/s)
  - Light emitted from an object moving with 1000 m/s relative to "ground" travels with velocity c BOTH relative to ground AND relative to object!
    - Think about this one for 1 minute - or a few months (like Einstein did)…
- This has severe consequences:
  - Time does NOT pass with equal pace in all IS
  - Time difference between two events will come out different as measured from different IS
    - remember equivalent result for distance between 2 events even for Galileo
    - Shortest time difference is measured by system where both events happen at the same position (all other IS measure longer times = "time dilation")
  - Two events might be simultaneous in one IS and non-simultaneous in another! They may even occur in opposite order in different IS
  - Length of an object is measured differently from different IS
    - Longest distance is measured by IS where object is at rest; all other IS measure shorter length = "length contraction"



Examples

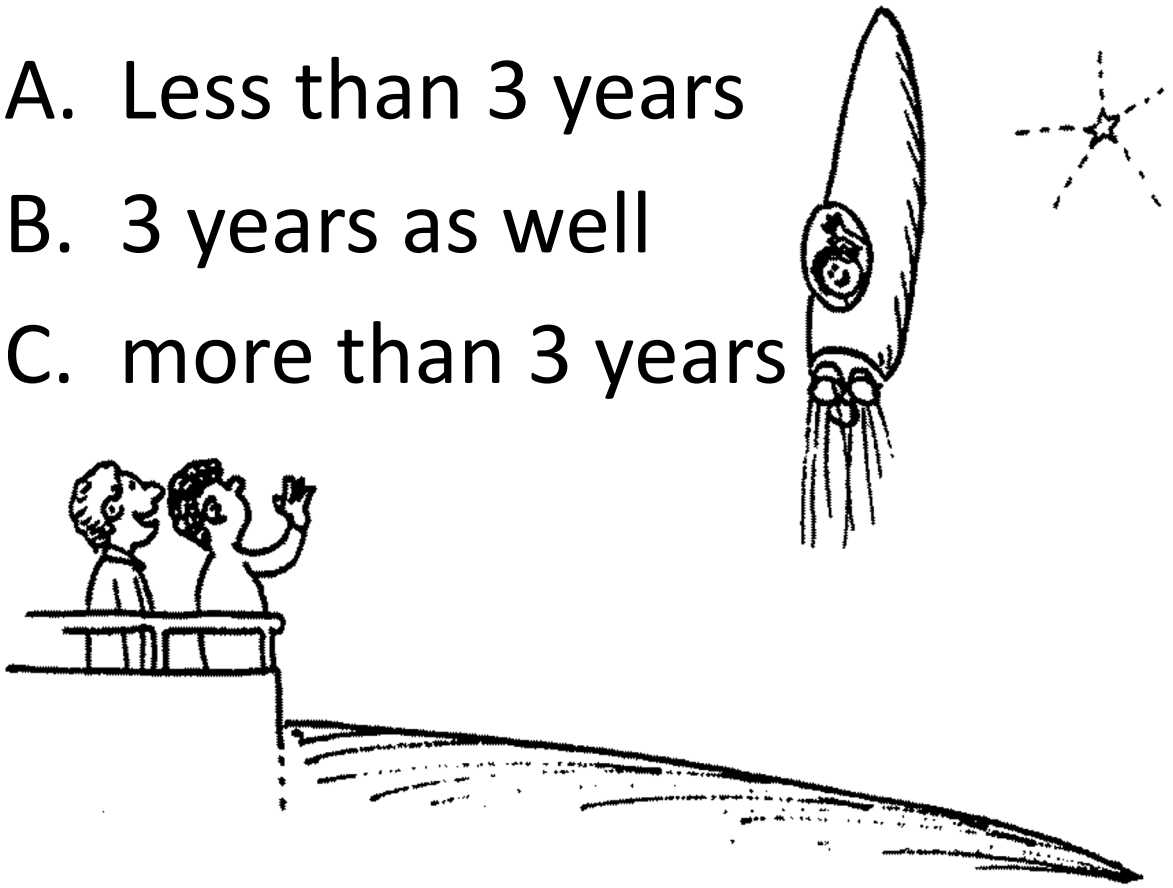


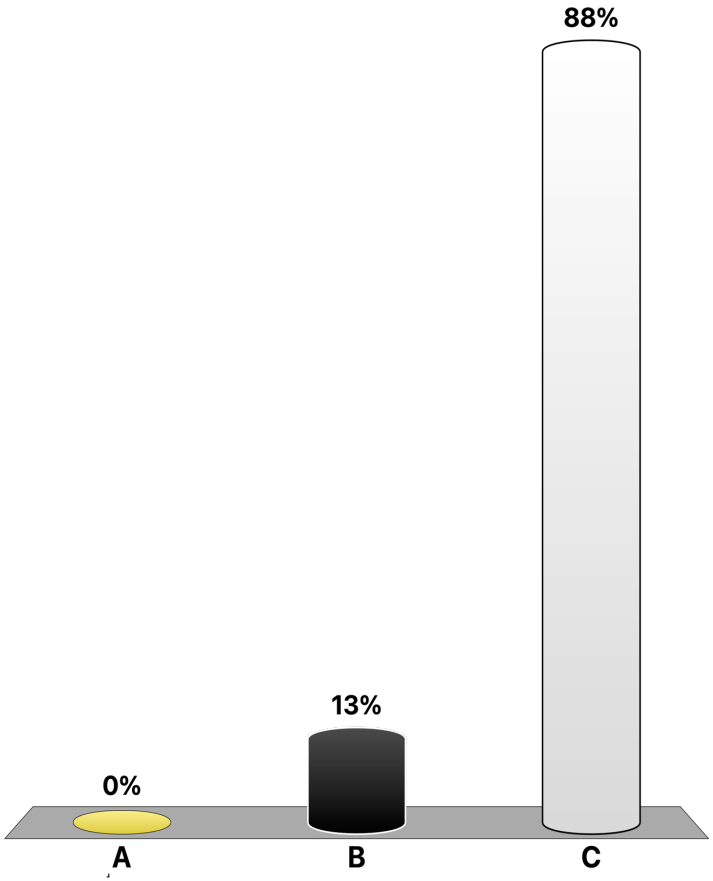
- Non-simultaneity
  - Justin sees both at once
  - Juliane sees B' before A'
  - BOTH are correct! answer depends on IS

- Twin Paradox
  - One twin travels with spaceship at 98% of c
  - other twin stays back on Earth
  - upon return, 1st one has aged 6 years but second one 30 years!
  - Note: 1st twin is NOT in an IS, so his description is NOT equivalent
- Length Contraction
  - 1st twin from above appears only 1/5 as long as 2nd twin when measured by 2nd twin
  - Total travel distance of 1st twin = 30 light years according to 2nd one, but only = 6 light year according to 1st one.

An astronaut ages 3 years when traveling at 99% of the speed of light to the star Procyon and back. The space officials who greet her on her return have aged...

A. Less than 3 years
B. 3 years as well
C. more than 3 years





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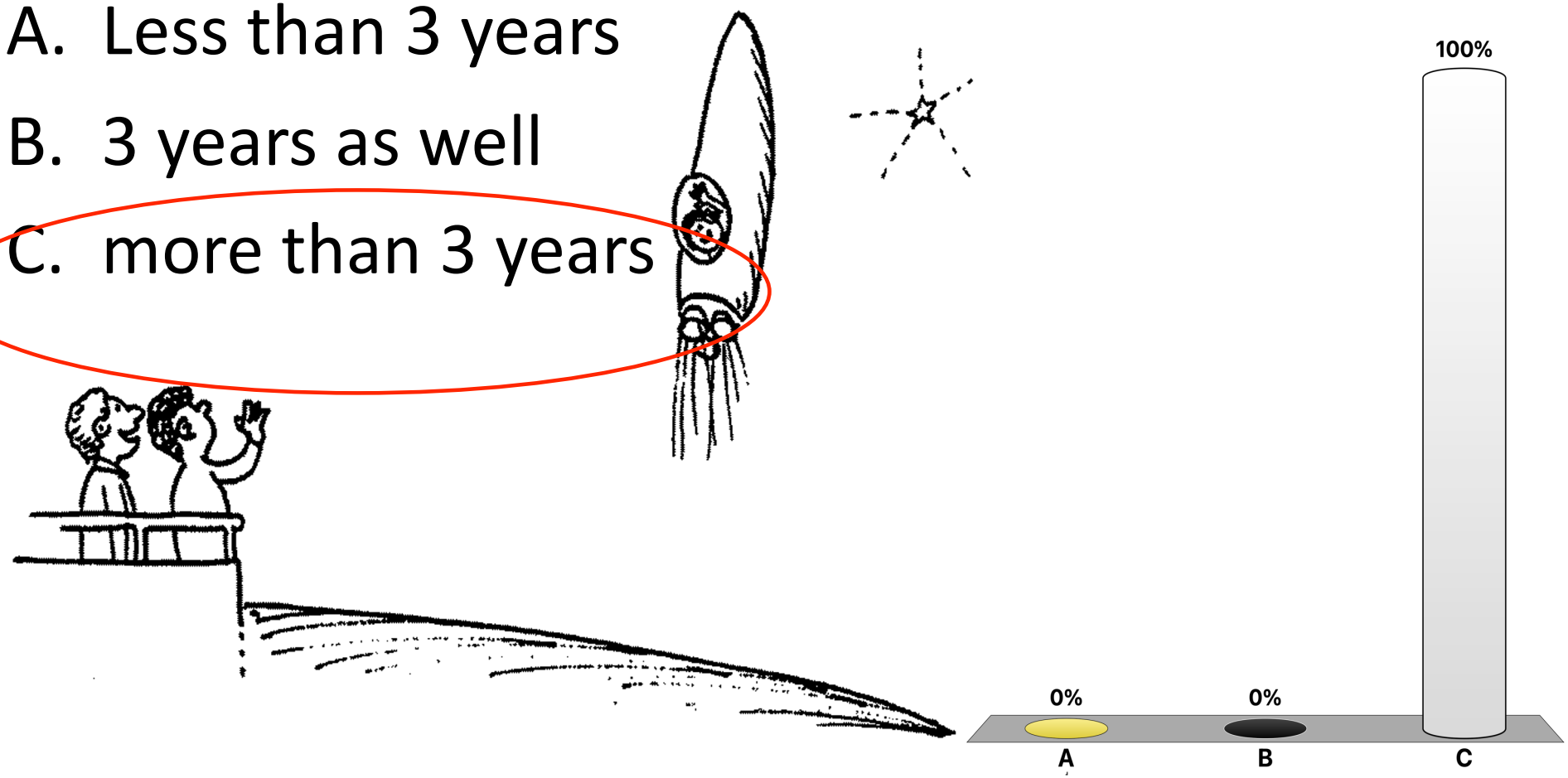
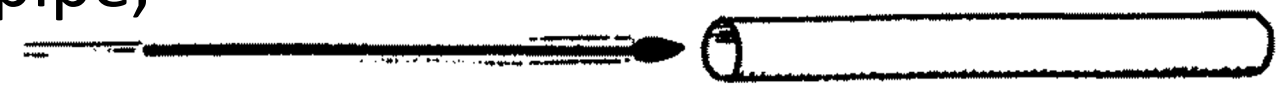


A 1-meter long spear is thrown at a relativistic speed through a pipe that is 1 meter long. Both these dimensions are measured when each is at rest. When the spear passes through the pipe,



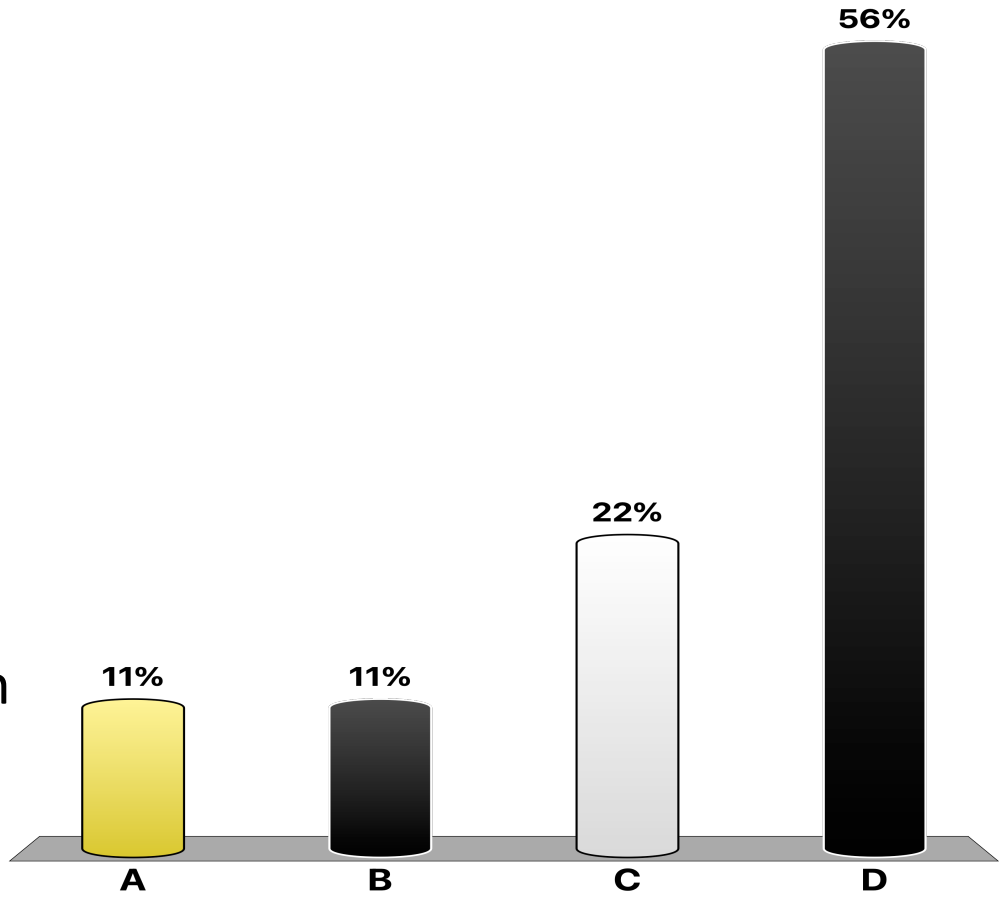
which of these statements best describes what is observed?

A. The spear shrinks so that the pipe completely covers it at some point in time

B. The pipe shrinks so that the spear sticks out from both ends at some point in time.

C. Both shrink equally so that the pipe completely covers it at some point in time.

D. Any of the above, depending on the motion of the observer relative to both pipe and spear.



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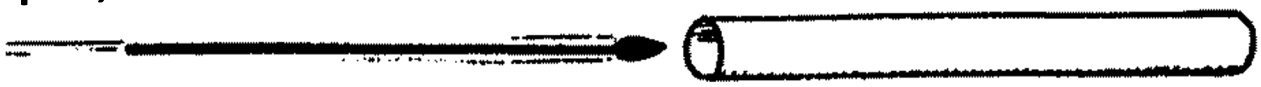

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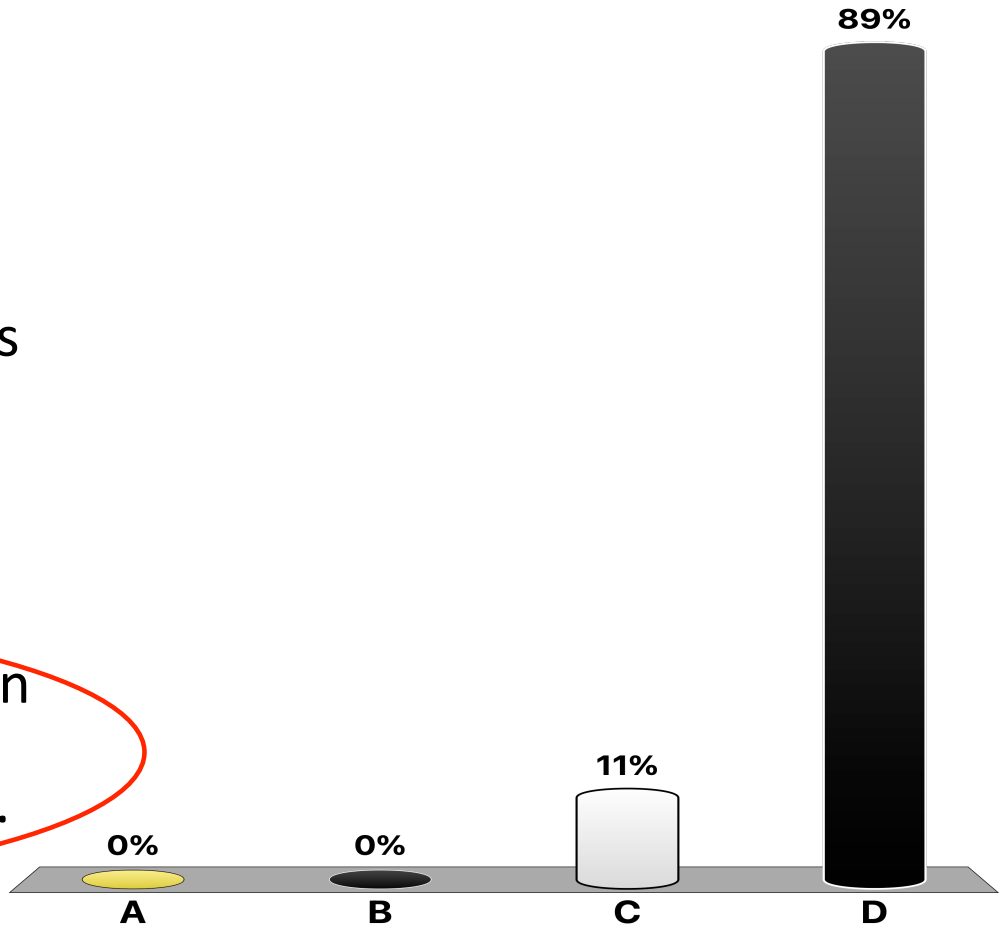


Consequences for Mass, Energy and Momentum

- Q: Why can't we accelerate something beyond the speed of light (F = ma)?
- A: Because inertia ("m") becomes infinitely big!
  - Kinetic energy contributes to inertia of an object: Inertia = m + K.E./c^2
  - ALL kinds of energy contribute to inertia of an object!
  - Mass is just one special contribution to inertia => can define total energy E = m*c^2 + other types of energy => inertia = E /c^2
  - Correct form of energy due to mass and kinetic energy alone:

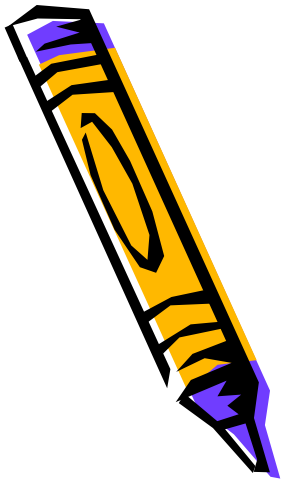
$$E = \gamma mc^2 = \frac{mc^2}{\sqrt{1 - v^2/c^2}}$$

  - Correct reformulation of momentum: inertia times velocity =>

$$\vec{p} = \frac{m\vec{v}}{\sqrt{1 - v^2/c^2}} = \gamma\, m\vec{v}$$

Note: $(E/c^2)^2 - (p/c)^2 = m^2$; m is also called "rest mass"

Examples



- Fast particles are “heavier” (have more inertia)
  - Protons at the Large Hadron Collider in Geneva (CH) need a humongous magnetic field to be bent around a humongous circle, although they “only” travel with 99.999999% of the speed of light
- Fast particles live longer (time dilation)
  - Cosmic ray muons can travel through 30 km of atmosphere (0.1 ms at speed of light) even though their lifetime is only 0.0022 ms)
- Energy can be converted into mass (pair creation)
  - “Atom smashers”: We routinely produce new particles (e.g., pions with mass 1/7 of proton mass) by smashing high-energy electrons (1/1836 of proton mass) into a target: m(pion) < K.E.(electron)/c^2
- Mass can be converted into energy (annihilation, nuclear decay)
  - Antimatter bomb in “Angels and Demons”: $e^+ + e^-$ -> light (photons)

    E(light) = {$m(e^+) + m(e^-)$}*c^2
  - Nuclear Fission Energy: Mass of all fission products combined < initial mass of fissioning nucleus

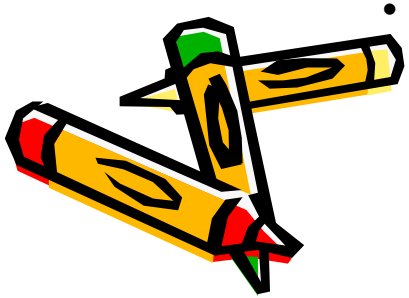
Not weird enough yet? => General Relativity

- Special Relativity:
  - Time and space are related - time elapsed and distances measured depend on frame of reference
  - Introduce “4-dimensional Space-Time Continuum”
    - Every event has 4 coordinates: *ct* and **r**
- General Relativity: Describe gravity as a DISTORTION of this space-time continuum
  - distances are altered near masses
  - time elapsed depends on nearby masses
  - straight lines get bent
  - parallel lines can converge
- Basic idea: distortion (“local curvature”) of space-time fabric is proportional to mass (energy) density

Mathematical formulation crazy complicated - involves differential geometry, non-Euclidean metric, tensors, and Einstein’s field equations

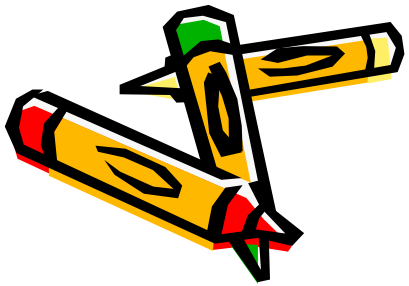
Guidance: Equivalence Principle

- Remember: All objects fall with the SAME acceleration independent of their mass!
  - **a** = **F**/*m*, but **F** = -*mg* => **a** = -*g*
  - THIS is the reason why we can describe gravity as a property of space and time itself (which are the only things entering acceleration)
  - ANY freely falling system is (locally) INDISTINGUISHABLE from an IS!
    - Objects inside a falling elevator or a circling space station follow all of Newton's laws - in particular Newton's FIRST law!
  - Vice versa, the laws inside a stationary system WITH gravity are the same as those governing motion in an accelerated system with NO external force
    - "Force free" objects accelerate -> drop to the floor or follow curved trajectories
    - Consequences:
      - even light must follow a curved path around a massive object
      - time dilation (remember twin paradox) => clocks go slower close to massive objects
      - "it isn't the falling that hurts, but the sudden stop at the end!"

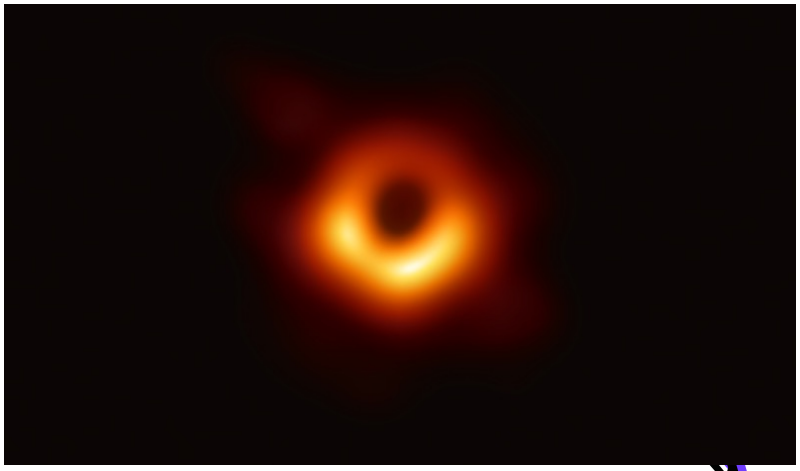


Examples

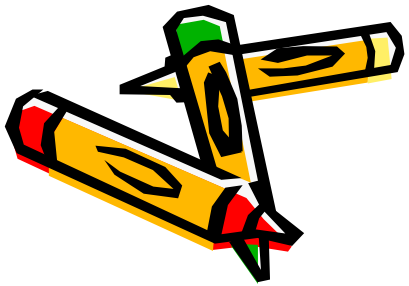
- Ordinary gravity: Things fall to the ground, planets orbit the sun…
  - "Old" explanation: Force of gravity pulls them
  - "New" explanation: They follow most "straightforward" paths possible in space-time, which itself is curved because of the presence of masses (earth, sun,…)
  - By the same token, light follows curved paths near massive objects
    - observable during total eclipse of the sun: stars seem to be "moved"
- Clocks run slow deep inside a gravitational field
  - Clock on surface of Earth runs more slowly than clock circling at high altitude
    - Important practical effect you have to account for to make your GPS work!
    - Gravitational "redshift" - photons lose energy ($E = hf$!)
    - Most extreme: near a "black hole" (photons from INSIDE would lose more energy than they have -> they never come out!)



Large-scale structure of the Universe

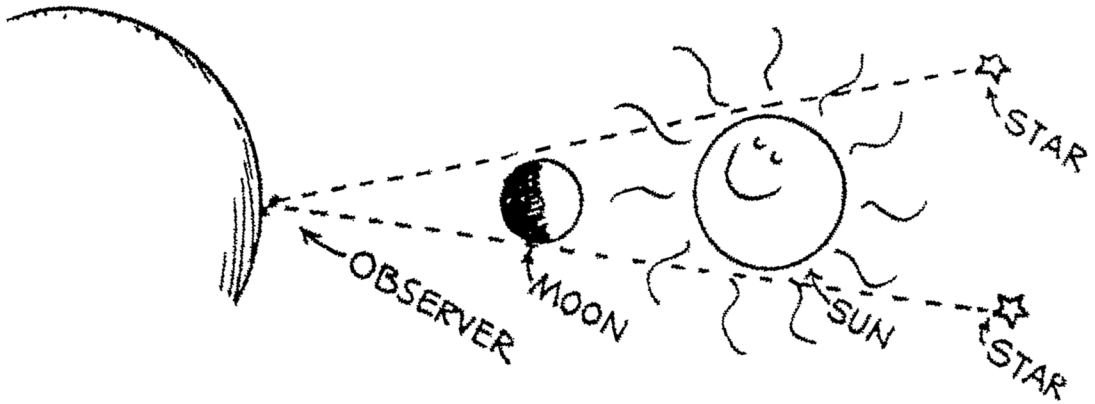


- Is the Universe as a whole curved?
  - If there is “too much” mass in the Universe, ALL “straight” paths lead back to themselves (the curvature is positive, like on a sphere)
    - You could see (in principle) the back of your head
    - The present expansion would ultimately come to a halt, reverse and then end in a “big crunch”
  - If there is “too little” mass in the Universe, all “straight” paths diverge from each other (the curvature is “negative”, like a saddle)
    - The present expansion continues forever, only slightly slowed down
  - If there is exactly the right amount of mass (“critical density”), the Universe is “flat” (ordinary geometry applies, parallel lines never meet…)
    - Expansion goes on forever, but becomes slower and slower and *nearly* stops
    - Not enough visible mass for that. Dark Matter?



- The truth is even weirder!
  - The universe SEEMS to be flat (“critical density”)
  - BUT: the expansion seems to be accelerating
  - Reason: Dark Energy (Huh?)

Q3 If the Sun passes between the Earth and a pair of stars as shown, and the Moon passes in front of the Sun and totally eclipses it so the stars are visible, then according to general relativity, the 2 stars will appear to be



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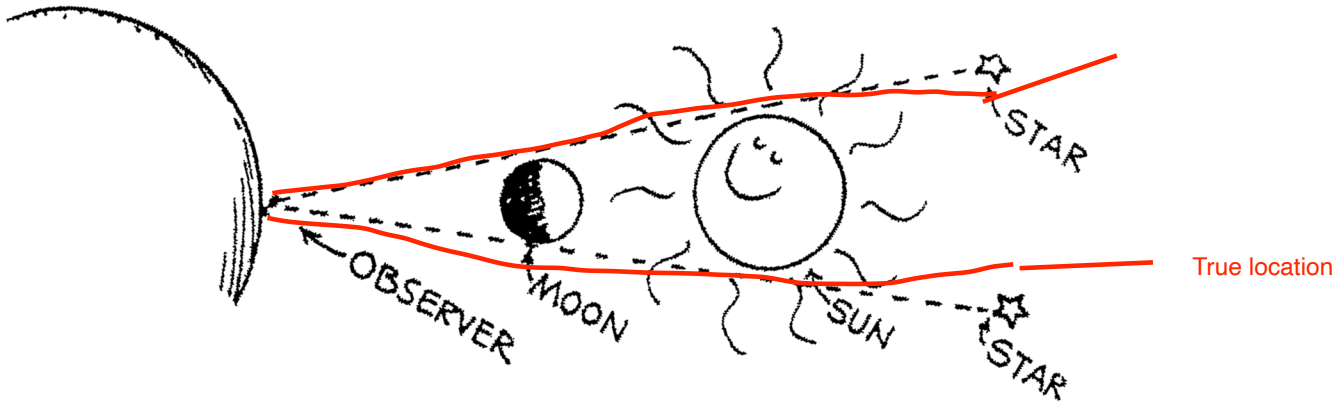

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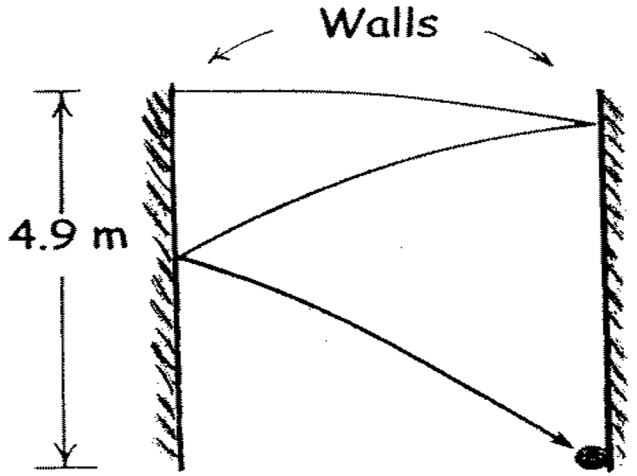

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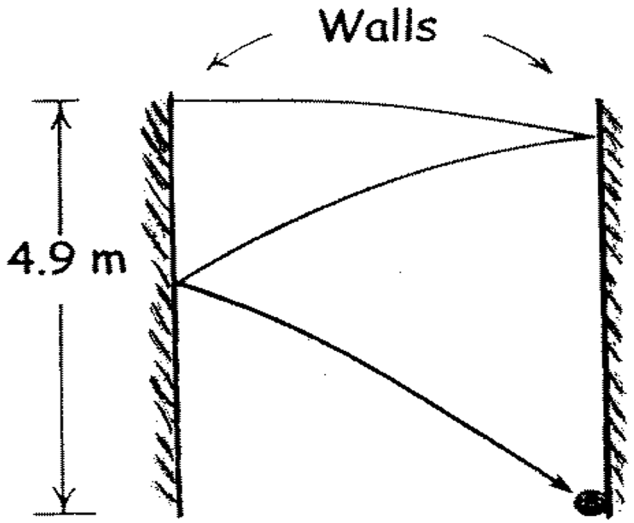

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