Solar Neutrinos

John N. Bahcall

How does the Sun shine? How well do we understand the evolution and ages of
stars? Does the neutrino have a mass? Can a neutrino change its lepton number
in flight? Are there weak interactions beyond those described by the standard
model of particle physics? These are some of the questions that motivate the
study of solar neutrinos.

A neutrino is a weakly interacting particle that travels at essentially the speed
of light and has an intrinsic angular momentum of $\frac{1}{2}$ unit ($\hbar/2$). Neutrinos are
produced on Earth by natural radioactivity, by nuclear reactors, and by high-
energy accelerators. In the Sun, neutrinos are produced by weak interactions
that occur during nuclear fusion. There are three known types of neutrinos,
each associated with a massive lepton that experiences weak, electromagnetic,
and gravitational forces, but not strong interactions. The known leptons are
electrons, muons, and taus (in increasing order of their rest masses).

Neutrino astronomy is difficult for the same reason it is interesting. Because
neutrinos only interact weakly with matter, they can reach us from otherwise
inaccessible regions where photons, the traditional messengers of astronomy, are
trapped. Hence, with neutrinos we can look inside stars and examine directly
energetic physical processes that occur only in stellar interiors. We can study
the interior of the Sun or the core of a collapsing star as it produces a supernova.

Large detectors, typically hundreds or thousands of tons of material, are
required to observe astronomical neutrinos. These detectors must be placed
deep underground to avoid confusing the rare astronomical neutrino events with
the background interactions caused by cosmic rays and their secondary particles,
which are relatively common near the surface of the Earth.

The nearest star, our Sun, supplies the largest known flux of neutrinos at the
Earth’s surface. Every second approximately a hundred billion solar neutrinos
cross every square centimeter on Earth. Quite naturally, the first attempt to
detect astronomical neutrinos began with an experiment to observe neutrinos
produced in the deep interior of the Sun.

For two decades, from 1968 to 1988, the only operating solar neutrino exper-
iment (carried out by Raymond Davis Jr. and his colleagues and using $^{37}\text{Cl}$ as
a detector) yielded results in conflict with the most accurate theoretical calcula-
tions of how many neutrinos are produced in the Sun. This conflict between
theory and observation became known as the ’solar neutrino problem.’

Both the theoretical and the observational results for the chlorine experiment
are expressed in terms of the solar neutrino unit, SNU, which is the product of a
characteristic calculated solar neutrino flux (units: $\text{cm}^{-2}\text{s}^{-1}$) times a theoretical
cross section for neutrino absorption (unit: $\text{cm}^2$). A SNU has, therefore, the
units of events per target atom per second and is chosen for convenience equal
to $10^{-36}\text{s}^{-1}$.

After two decades of critical examination of both the theory and the exper-
nent, both results were determined robustly. The predicted rate for capturing
solar neutrinos in a $^{37}$Cl target is (Bahcall and Ulrich, 1988; Bahcall, 1989)

$$\text{Predicted rate} = (7.9 \pm 0.9) \text{ SNU}.$$  \hspace{1cm} (1)

The rate observed by R. Davis, Jr. (1986) and his associates in their chlorine radiochemical detector is

$$\text{Observed rate} = (2.1 \pm 0.3) \text{ SNU}.$$ \hspace{1cm} (2)

Both the theoretical and the experimental uncertainties are quoted as 1σ errors.

The predictions used in Eqs. (1) and (2) are valid for the combined standard model, that is, the standard model of electroweak theory (of Glashow, Weinberg, and Salam) and the standard solar model.

Similar results to those shown in Eq. (1) and Eq. (2) were obtained in 1968. The most recent theoretical result is $8.5 \pm 1.8$ SNU (Bahcall and Pinsonneault 2004) and the final experiment value is $2.6 \pm 0.2$ SNU (Cleveland, Daily, Davis, et al. 1998). The robustness of the discrepancy between theory and observation stimulated the development two generations of increasingly more powerful and sophisticated detectors designed to find the reason why theory and observation differ.

More is known about the Sun than about any other star and the calculations of neutrino emission from the solar interior can be done with relatively high precision. Solar neutrino experiments test in a direct and rigorous way the theories of nuclear energy generation in stellar interiors and of stellar evolution. These tests are independent of many of the uncertainties that complicate the comparison of the theory with observations of stellar surfaces. For example, convection and turbulence are important near stellar surfaces but unimportant in the solar interior. Hence, the solar neutrino discrepancy puzzled (and worried) astronomers who want to use neutrino observations to understand better how the Sun and other stars shine. Prior to June 2001 (see discussion of SNO experiment below), the solar neutrino problem seemed to most (but not all) physicists to indicate that astronomers did not understand the details of the solar nuclear fusion reactions that produce neutrinos.

Neutrinos from the Sun provide particle beams for probing the weak interactions on distance scales that cannot be achieved with traditional laboratory experiments. Since neutrinos from the Sun travel astronomical distances before they reach the Earth, experiments performed with these particle beams are sensitive to weak-interaction phenomena that require long path lengths in order for slow weak-interaction effects to have time to occur. The effects of tiny neutrino masses ($\geq 10^{-6}$ eV), unmeasurable in the laboratory, can be studied with solar neutrinos. Moreover, neutrinos traverse an enormous amount of matter, $10^{21}$ gm cm$^{-2}$, as they travel from the center of the Sun to detectors on Earth. The huge column density of matter that solar neutrinos traverse can give rise to 'matter effects' on neutrino propagation that have not yet been observed with terrestrial neutrinos.

The Sun shines by converting protons into $\alpha$ particles. The overall reaction
Table 1: The pp chain in the Sun. The average number of pp neutrinos produced per termination in the Sun is 1.85. For all other neutrino sources, the average number of neutrinos produced per termination is equal to the termination percentage/100.

<table>
<thead>
<tr>
<th>Reaction</th>
<th>Termination\textsuperscript{a}</th>
<th>(\nu) energy (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(p + p \rightarrow 2 \text{H} + e^+ + \nu_e)</td>
<td>1(a) 100</td>
<td>(\leq 0.42)</td>
</tr>
<tr>
<td>or (p + e^- + p \rightarrow 2 \text{H} + \nu_e)</td>
<td>1(b) (\textit{pep}) 0.4</td>
<td>1.44</td>
</tr>
<tr>
<td>(2 \text{H} + p \rightarrow ^3\text{He} + \gamma)</td>
<td>2 100</td>
<td></td>
</tr>
<tr>
<td>(^3\text{He} + ^4\text{He} \rightarrow \alpha + 2p)</td>
<td>3 85</td>
<td></td>
</tr>
<tr>
<td>or (^3\text{He} + ^4\text{He} \rightarrow ^7\text{Be} + \gamma)</td>
<td>4 15</td>
<td></td>
</tr>
<tr>
<td>(^7\text{Be} + e^- \rightarrow ^7\text{Li} + \nu_e)</td>
<td>5 15</td>
<td>(90%) 0.86</td>
</tr>
<tr>
<td>or (^7\text{Li} + p \rightarrow 2\alpha)</td>
<td>6 15</td>
<td>(10%) 0.38</td>
</tr>
<tr>
<td>(^7\text{Be} + p \rightarrow ^8\text{B} + \gamma)</td>
<td>7 0.02</td>
<td></td>
</tr>
<tr>
<td>(^8\text{B} \rightarrow ^8\text{Be}^* + e^+ + \nu_e)</td>
<td>8 0.02</td>
<td>&lt;15</td>
</tr>
<tr>
<td>(^8\text{Be}^* \rightarrow 2\alpha)</td>
<td>9 0.02</td>
<td></td>
</tr>
<tr>
<td>or (^3\text{He} + p \rightarrow ^4\text{He} + e^+ + \nu_e)</td>
<td>10 0.00002</td>
<td>(\leq 18.77)</td>
</tr>
</tbody>
</table>

\textsuperscript{a} The termination percentage is the fraction of terminations of the pp chain, \(4p \rightarrow \alpha + 2e^+ + 2\nu_e\), in which each reaction occurs. The results are averaged over the model of the current Sun. Since in essentially all terminations at least one \(pp\) neutrino is produced and in a few terminations one \(pp\) and one \(pep\) neutrino are created, the total of \(pp\) and \(pep\) terminations exceeds 100%.

can be represented symbolically by the relation

\[
4p \rightarrow \alpha + 2e^+ + 2\nu_e + 25\text{MeV}.
\]  

(3)

Protons are converted to \(\alpha\) particles, positrons, and neutrinos, with a release of about 25 MeV of thermal energy for every four protons burned. Each conversion of four protons to an \(\alpha\) particle is known as a termination of the chain of energy-generating reactions that accomplishes the nuclear fusion. The thermal energy that is supplied by nuclear fusion ultimately emerges from the surface of the Sun as sunlight. About 600 million tons of hydrogen are burned every second to supply the solar luminosity. Nuclear physicists have worked for half a century to determine the details of this transformation.

The main nuclear burning reactions in the Sun are shown in Table 1, which represents the energy-generating pp chain. This table also indicates the relative frequency with which each reaction occurs in the standard solar model. For
Fig. 1: Solar neutrino spectrum. This figure shows the energy spectrum of neutrinos predicted by the standard solar model (Bahcall and Pinsonneault 2004). The neutrino fluxes from continuum sources (like pp and $^8$B) are given in the units of number per cm$^2$ per second per MeV at one astronomical unit. The line fluxes (pep and $^7$B) are given in number per cm$^2$ per sec. The spectra from the pp chain (Table 1) are drawn with solid lines; the neutrino energy spectra from reactions with carbon, nitrogen, and oxygen (CNO) isotopes are drawn with dotted lines.

For simplicity, we do not include in Table 1 nuclear reactions that involve isotopes of carbon, nitrogen, and oxygen (CNO reactions). The CNO reactions contribute only about 1% of the solar luminosity and relatively small neutrino fluxes (see Figure 1).

The fundamental reaction in the solar energy-generating process is the proton-proton (pp) reaction, reaction 1a of Table 1, which produces the great majority of solar neutrinos. However, these $p - p$ neutrinos have energies below the detection thresholds for the $^{37}$Cl detector and all other solar neutrino experiments carried out so far except radiochemical experiments that use $^{71}$Ga as a detector (see below).

Most of the predicted capture rate in the $^{37}$Cl experiment comes from the rare termination in which $^7$Be captures a proton to form radioactive $^8$B (Bahcall 1964, see reaction 7 of Table 1). The $^8$B decays to unstable $^8$Be, ultimately producing two $\alpha$ particles, a positron, and a neutrino. The neutrinos from $^8$B decay have a maximum energy of less than 15 MeV. Although the reactions involving $^8$B occur only once in every 5000 terminations of the pp chain, the
Table 2: Calculated Solar Neutrino Fluxes and 1σ Uncertainties

<table>
<thead>
<tr>
<th>Source</th>
<th>Flux $(10^{10} \text{ cm}^{-2} \text{s}^{-1})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$pp$</td>
<td>$5.9(1 \pm 0.01)$</td>
</tr>
<tr>
<td>$pep$</td>
<td>$0.014(1 \pm 0.02)$</td>
</tr>
<tr>
<td>$hep$</td>
<td>$8(1 \pm 0.2) \times 10^{-7}$</td>
</tr>
<tr>
<td>$^7\text{Be}$</td>
<td>$0.49(1 \pm 0.12)$</td>
</tr>
<tr>
<td>$^8\text{B}$</td>
<td>$5.8 \times 10^{-4}(1 \pm 0.23)$</td>
</tr>
<tr>
<td>$^{13}\text{N}$</td>
<td>$0.06(1 \pm 0.4)$</td>
</tr>
<tr>
<td>$^{15}\text{O}$</td>
<td>$0.05(1 \pm 0.4)$</td>
</tr>
<tr>
<td>$^{17}\text{F}$</td>
<td>$6(1 \pm 0.4) \times 10^{-4}$</td>
</tr>
</tbody>
</table>

predicted event rates for the $^{37}\text{Cl}$, Kamiokande, Super-Kamiokande, and SNO experiments are dominated by this rare mode.

The neutrino energy spectrum predicted by the standard solar model is shown in Figure 1, where contributions from both line and continuum sources are included.

The solar neutrino fluxes at the Earth’s surface that are calculated from the most recent standard solar model (Bahcall and Pinsonneault 2004) are shown in Table 2. The 1σ uncertainties in the calculated neutrino fluxes are also shown in Table 2.

The beautiful $^{37}\text{Cl}$ experiment of Davis and his collaborators (Davis 1978, Cleveland et al. 1998) was for two decades the only operating solar neutrino detector. The reaction that was used for the detection of the neutrinos is

$$\nu_e + ^{37}\text{Cl} \rightarrow e^- + ^{37}\text{Ar},$$

which has a threshold energy of 0.8 MeV. The target was a tank containing $10^5$ gallons of $\text{C}_2\text{Cl}_4$ (perchloroethylene, a cleaning fluid), deep in the Homestake Gold Mine in Lead, South Dakota. The underground location was necessary in order to avoid background events from cosmic rays. Every few months, for almost three decades, Davis and his collaborators extracted a small sample of $^{37}\text{Ar}$, typically of order 15 atoms, out of the total of more than $10^{30}$ atoms in the tank. The $^{37}\text{Ar}$ produced in the tank is separated chemically from the $\text{C}_2\text{Cl}_4$, purified, and counted in low-background proportional counters. The typical background counting rate for the counters corresponds to about one radioactive decay of an $^{37}\text{Ar}$ nucleus a month! Experiments have been performed to show that $^{37}\text{Ar}$ produced in the tank is extracted with more than 90% efficiency.

The existence of the solar neutrino problem (see Eq. 1 and Eq. 2) sparked an intense debate about the origin of the problem. More importantly, the problem stimulated the construction and operation of five sophisticated new solar neutrino observatories. These observatories are: Kamiokande (a water Cherenkov detector of neutrino-electron scattering in Japan), SAGE and GALLEX (radio-
chemical detectors, in Russia and in Italy, that observe neutrino absorption by

\[ ^{71}\text{Ga}, \] Super-Kamiokande (a much larger version of the original Kamiokande
water Cherenkov detector), and the SNO detector (which detects neutrinos us-
ing heavy water, \(^{2}\text{H}_{2}\text{O}\)).

The Kamiokande experiment (Kamiokande Collaboration 1996), located in
the Japanese Alps, detected Cherenkov light emitted by electrons that are scat-
tered in the forward direction by solar neutrinos. The reaction by which the
neutrinos are observed is

\[ \nu + e \rightarrow \nu' + e', \] (5)

where the primes on the outgoing particle symbols indicate that the momentum
and energy of each particle can be changed by the scattering interactions. With
techniques that have been developed so far, only the higher-energy neutrinos
(\(> 5\text{ MeV}, \text{i.e., } ^{8}\text{B} \text{ and hep neutrinos only} \)) can be observed by neutrino-electron
scattering.

A much larger and more sensitive version of the Kamiokande experiment
, known as Super-Kamiokande (Super-Kamiokande Collaboration 1998, 2001),
first published new precision data in 1998. Neutrino–electron scattering experi-
ments furnish information about the incident neutrino energy spectrum (from
measurements of the recoil energies of the scattered electrons), determine the
direction from which the neutrinos arrive, and record the precise time of each
event. Super-Kamiokande detected so many neutrino events (about 15 events
per day, more than 5000 in total) that it inaugurated an era of precision meas-
urements of multiple aspects of solar neutrino interactions.

Two radiochemical solar neutrino experiments using \(^{71}\text{Ga} \) were performed,
one by a primarily Western European collaboration [with U.S. and Israeli par-
ticipation (see GALLEX/GNO collaboration 1992, 1999, 2000)] (GALLEX) and
the second by a group working in Russia (under conditions of hardship that
sometimes required exceptional ingenuity and even heroism) with US participa-
tion (SAGE, see SAGE Collaboration 1994, 2002). The GALLEX collaboration
used 30 tons of gallium in an aqueous solution; the detector is located in the
Gran Sasso National Laboratory in Italy. The Soviet experiment uses about
60 tons of gallium metal as a detector in a solar neutrino laboratory constructed
underneath a high mountain in the Baksan Valley in the Caucasus Mountains
of the Soviet Union. The amount of detector material used in each of these
experiments is impressive considering that, at the time the experimental tech-
niques were developed, the total world production of gallium was only 10 tons
per year!

The gallium experiments provide unique information about the most common
nuclear reaction fusion reaction in the Sun, the \( p - p \) reaction (see reaction 1a of
Table 1). The absorption reaction by which neutrinos are detected with gallium is

\[ \nu_{e} + ^{71}\text{Ga} \rightarrow e^{-} + ^{71}\text{Ge}. \] (6)

The germanium atoms are removed chemically from the gallium and the ra-
dioactive decays of \(^{71}\text{Ge} \) are measured in small proportional counters. The
threshold for absorption of neutrinos by \(^{71}\text{Ga} \) is 0.23 MeV, which is well below
the maximum energy of the \( p - p \) neutrinos. The independent GALLEX/GNO and SAGE experiments yield results that are in good agreement with each other. At this writing, no other solar neutrino experiment has a demonstrated capability to detect the low-energy neutrinos from the basic \( p - p \) reaction, although several detectors are being developed that could observe electrons produced by neutrino-electron scattering or by absorption of \( p - p \) neutrinos.

The Sudbury Solar Neutrino Observatory (see SNO Collaboration 2001, 2002, 2004) is a powerful 1-kiloton heavy water \((D_2O)\) experiment that is located in an INCO nickel mine near Sudbury, Ontario (Canada). The deuterium (denoted by \( D \) or by \(^2\)H) experiment is a collaboration between Canadian, American, and British scientists. Like the Kamiokande and Super-Kamiokande detectors, the SNO deuterium detector measures the energy and direction of recoil electrons by observing their Cherenkov light with photomultipliers. Thus SNO can also observe neutrino-electron scattering, see Eq. (5).

More importantly, SNO can observe two unique reactions. The first reaction detects only electron type neutrinos and can be written

\[
\nu_e + D \rightarrow e^- + p + p .
\]  

(7)

The second reaction is equally sensitive to neutrinos of all types, \( \nu_e, \nu_\mu \) and \( \nu_\tau \), and can be written

\[
\nu + D \rightarrow \nu' + n + p .
\]  

(8)

The SNO detector can measure the all-neutrino reaction, Eq. (8), (also called a ‘neutral current’ reaction) in several different ways.

Figure 2 compares the rates measured in all seven of the solar neutrino experiments with the rates predicted by the combined standard model: the standard solar model plus the standard model of electroweak interactions. With the exception of the neutral-current detection of SNO, all of the measurements disagree with the predictions of the combined standard solar and particle physics model.

The Kamiokande, Super-Kamiokande, and SNO experiments are sensitive to \(^8\)B and hep neutrinos, but the other solar neutrinos that are shown in Figure 1 are below the experimental energy thresholds. The thresholds are set at several MeV in order to avoid numerous lower-energy background events. Only the gallium experiments are sensitive to the fundamental \( p - p \) neutrinos and only the gallium and chlorine experiments are sensitive to the neutrinos from \(^7\)Be and from the CNO sources of neutrinos (\(^{13}\)N, \(^{15}\)O, and \(^{17}\)F).

Neutrino absorption, exemplified by reactions Eq. (4), Eq. (6), and Eq. (7), is sensitive only to electron-type neutrinos, \( \nu_e \), whose type (flavor) is unchanged in transit to the Earth. For neutrino–electron scattering, Eq. (5), the cross section for \( \nu_\mu \) or \( \nu_\tau \) at the energies of interest is about one-seventh the cross section for \( \nu_e \). Neutrino–electron scattering is primarily sensitive to \( \nu_e \) but has a small sensitivity to \( \nu_\mu \) and \( \nu_\tau \). The SNO experiment includes a detection mode that is equally sensitive to all three types of neutrinos, Eq. (8). In this neutral-current mode, deuterium nuclei are disintegrated into their constituent neutrons and protons without changing the charge of the nucleons. The measurement of
Fig. 2: Comparison of measured rates and standard-model predictions for seven solar neutrino experiments. The unit for the radiochemical experiments (chlorine and gallium) is SNU ($10^{-36}$ interactions per target atom per sec); the unit for the water-Cerenkov experiments (Kamiokande, Super-Kamiokande, and SNO) is the rate predicted by the standard solar model plus standard electroweak theory.

The neutral-current disintegration of deuterium provides a determination of the total flux of solar neutrinos above the energy threshold, about 2.2 MeV, for the reaction shown in Eq. (8).

On June 18th, 2001 at about 12 noon EDT the SNO collaboration announced the first scientific results of their epochal experiment. Combining the SNO measurements of $\nu_e$ (Eq. 7) from $^8$B neutrinos produced in the Sun with the precise Super-Kamiokande measurement of neutrino-electron scattering (Eq. 5), the SNO collaboration solved the 33 year old solar neutrino problem. About two-thirds of the $^8$B $\nu_e$ produced in the Sun are transformed into the more difficult to detect $\nu_\mu$ and $\nu_\tau$ on their way from the center of the Sun to detectors on Earth. Moreover, the total number of neutrinos of all types ($\nu_e$, $\nu_\mu$ and $\nu_\tau$) is equal, within the uncertainties, to the value predicted by the standard solar model.

The fact that most of the neutrinos that come to us from the Sun are transformed in flight from $\nu_e$ to $\nu_\mu$ and $\nu_\tau$ explains why the radiochemical experi-
ments, chlorine and gallium see less than the predicted total number of neutrinos. The radiochemical experiments only detect $\nu_e$. The metamorphosis from $\nu_e$ to $\nu_\mu$ and $\nu_\tau$ also explains why the neutrino-electron scattering experiments, Kamiokande and Super-Kamiokande, see a deficit of neutrinos. The neutrino-electron scattering experiments are primarily, but not entirely, sensitive to $\nu_e$.

The SNO and Super-Kamiokande measurements together established two extraordinarily important conclusions. 1) Physics not included in the standard model of particle physics occurs. Neutrinos change their type. 2) The neutrino measurements confirm the theoretical model of how the Sun shines. The measured flux of neutrinos from $^8\text{B}$ beta-decay, which depends approximately on the 25th power of the central temperature of the Sun, is in good agreement with the theoretical calculations.

In short, the solar neutrino experiments showed that the standard model of particle physics is incomplete and the standard solar model is vindicated.

Subsequent measurements by the SNO and other solar neutrino experimental collaborations have confirmed and refined the original inferences announced in June, 2001.

Let’s step back in time for a moment to establish the theoretical particle physics context. The physics community was electrified in 1985 when an elegant theoretical solution for the solar neutrino problem was proposed that is consistent with expectations from Grand Unified Theories (GUT) of neutrino mass. According to this solution, a $\nu_e$ created in the solar interior is almost completely converted into $\nu_\mu$ or $\nu_\tau$ as the neutrino passes through the Sun. This conversion reflects the enhancement by the matter in the Sun of the probability that a neutrino of an electron type oscillates into a neutrino of a different type; it is universally referred to as the Mikheyev–Smirnov–Wolfenstein (MSW) effect in honor of its discoverers.

In order for the MSW effect to occur, the flavor eigenstates $\nu_e$, $\nu_\mu$, and $\nu_\tau$ must be different from the mass eigenstates. The flavor eigenstates are created in weak decays and have weak interactions with their associated charged leptons (electron, muon, and tau) that can be written in a simple (diagonal) form. The mass eigenstates, which have diagonal mass matrices, are the states in which neutrinos propagate in a vacuum. The mass eigenstates are often denoted by $\nu_1$, $\nu_2$, and $\nu_3$. For a simplified description in terms of two eigenstates, the relation between flavor and mass eigenstates in vacuum is described by a single mixing angle $\theta_{12}$, where $\tan \theta_{12}$ is the relative amplitude of $\nu_2$ and $\nu_1$ in the $\nu_e$ wave function ($\nu_e = \cos \theta_{12} \nu_1 + \sin \theta_{12} \nu_2$). The difference in the squares of the masses of the two neutrinos is denoted by $\Delta m^2_{21} = m_2^2 - m_1^2$.

All of the available data on solar, atmospheric, and reactor neutrino masses are consistent with an MSW description of neutrino propagation. A recent determination of neutrino parameters using all the available data yields (Bahcall, Gonzalez-Garcia, and Peña-Garay 2004):

$$\Delta m^2_{21} = 8.2^{+0.3}_{-0.3} \times 10^{-5} \text{eV}^2,$$ (9)
and
\[ \tan^2 \theta_{12} = 0.39^{+0.05}_{-0.04}. \] (10)
The same solution analyzing all of the data yields the values given below for the total flux of pp neutrinos, \( \phi(pp) \), and the total flux of \(^8\)B neutrinos, \( \phi(^8\text{B}) \), both expressed in terms of the values predicted by the standard solar model.
\[ \phi(pp) = 1.01 \pm 0.02(\text{experimental}) \pm 0.01(\text{theory}), \] (11)
\[ \phi(^8\text{B}) = 0.87 \pm 0.04(\text{experimental}) \pm 0.23(\text{theory}). \] (12)
The uncertainties indicated in Eq. (9)-Eq. (12) are \( \pm 1\sigma \) uncertainties.

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L. Wolfenstein, Phys. Rev. D, 17, 2369 (1978); Phys. Rev. D 20, 2634 (1979). Presented the fundamental equations for neutrino propagation in matter, the basis for the MSW effect. It took seven years for the physics community to recognize the significance of Wolfenstein’s brilliant insight. (A)