Stellar Structure
What have we learned?

• Can determine surface temperature via blackbody radiation, and absorption spectra
• Can determine relative magnitude, and after determining distance through parallax, absolute magnitude => Luminosity
• Relating black-body intensity to Luminosity yields surface and hence radius and volume
Hertzsprung-Russel Diagram

Note: Star sizes are not to scale.

In this histogram, bars rise from an H-R diagram to represent the frequency of stars in space. Red dwarfs and white dwarfs are the most common kinds of stars. O and B stars, supergiants, and giants are so rare their bars are not visible in this graph.
How do we find the mass?

- Use binary stars. Need a little relativity...

**4-vector:** Event \((ct, x, y, z) = (ct, \overline{r}) =: (x^0, x^1, x^2, x^3) = x^\mu, \mu = 0...3\)

For an inertial system \(S'\) moving along the x-axis of \(S\) with constant velocity \(v < c\), and with all axes aligned and the same origin \((x^\mu = (0,0,0,0) \Leftrightarrow \overline{x}^\mu = (0,0,0,0))\):

\[
\gamma = \frac{1}{\sqrt{1 - v^2 / c^2}}; \ x' = \gamma \left( x - \frac{v}{c} ct \right); \ ct' = \gamma \left( ct - \frac{v}{c} x \right); \ y = y'; \ z = z'
\]

Clocks in \(S'\) appear to \(S\) as if they were going slow by factor \(1/\gamma\), and vice versa. Length of object at rest in \(S'\) appears contracted by factor \(1/\gamma\) in \(S\).

**Velocity addition:**

\[
\frac{u_x}{c} = \frac{u_x + v}{c + \frac{u_x v}{c}}; \quad \frac{u_y}{c} = \frac{1}{\gamma} \frac{u_y}{c + \frac{u_x v}{c}}.
\]

**Doppler shift:**

\[
\frac{\lambda_{\text{obs}}}{\lambda_{\text{emitted}}} = (z + 1) = \frac{1 + \frac{v_{\parallel}}{c}}{\sqrt{1 - v^2 / c^2}} \quad (v\text{ is the relative velocity between emitter and observer and } v_{\parallel}\text{ is its component along the line of sight; } z > 0 \text{ is redshift, } z < 0 \text{ is blueshift})
\]
What have we learned?

• Can determine surface temperature via blackbody radiation, and absorption spectra
• Can determine relative magnitude, and after determining distance through parallax, absolute magnitude => Luminosity
• Relating black-body intensity to Luminosity yields surface and hence radius
• Binary stars: red- and blue shift of spectral lines determines absolute velocities; time dependence determines angular velocity of rotation around common center of gravity => distance of both stars from center of gravity
• Kepler’s law gives sum of masses and Newton’s 3rd law gives ratio of masses => individual masses
• Combining everything: average density
Hertzsprung-Russell Diagram

- Supergiants
- Main sequence
- Giants
- White dwarfs

Log$_{10}(L/L_\odot)$ vs. $T_e$ (K)

Log$_{10}(L/L_\odot)$ vs. $M/M_\odot$
Question: How do we deduce interior structure of stars from these observations?