Astrophysics - Problem Set 1 – Solution

Problem 1

Please answer the following questions with "Y" or "N":

- 1a) Is there any fundamental difference between the infrared "light" emitted by a warm object (e.g. a hot cookpot) and the x-rays used by a dentist, apart from their frequencies?
 N [Both are examples of electromagnetic waves only distinct by their frequencies]
- 1b) The apparent magnitude m of a star at 10 parsec distance is the same as its absolute magnitude M. True? **Y** [By definition of M]
- 1c) If you double the distance to some star, it's apparent magnitude m will also double. True? **N** [m depends logarithmicly on the brightness]
- 1d) In the absence of any external influences, any closed orbit of a satellite around a massive body is an ellipse. True? **Y** [Kepler's Law]
- 1e) Kepler's Law about the "radius vector sweeping out equal areas in equal times" applies only for gravitational attraction, no other interactions. True? N
 [This follows from conservation of angular momentum and therefore is true for any central force]
- 1f) The virial theorem states that the kinetic energy of the solar system (due to the motion of all its parts) is equal to twice its potential energy (due to all gravitational attraction between its parts). True? N [It's ½ of the absolute value of the potential energy]

Problem 2

The following is a set of multiple choice questions. Answer each with one single digit:

- 2a) Which of the following properties does NOT distinguish between different types of electromagnetic plane waves in vacuum? 2
 - 1 Their different frequencies or wavelengths
 - 2 Their different phase velocities [No, the phase velocity is always c]
 - 3 Their direction of propagation
 - 4 Their direction of polarization
- 2b) Compare the radiation emitted by the surface of a very hot star (13,000 K) with that from the surface of a colder one (3600 K). Which statement is correct?: **1**
 - 1 Both stars emit visible light.

2 - The hot star appears much more reddish to the eye than the colder one. [No, it's more bluish!]

3 – Both stars emit about the same amount of energy per unit time and surface area [No, the radiated energy goes like the 4th power of the temperature]

4 – The hot star emits no infrared ($\lambda > 800$ nm) radiation. [Wrong; blackbody radiation goes over all wave lengths, and in particular there is no cutoff at long wave lengths. In fact, a hotter object radiates more power at any wave length than a colder one.]

Problem 3

For the following, assume a star is at a distance of 20 parsec from Earth. Calculate the following (only numbers and units are needed):

- 3a) By how much (what angle) does the apparent direction (relative to the fixed background of faraway stars) to this star change over exactly half a year? 0.1" = 1/10 arcsec [= 2 A.U./d; note that Earth moves by 2 A.U. over 1/2 year, and 1 parsec = 1 A.U./1" with 1" = 1/206265 rad]
- 3b) If the star has the same luminosity as Sun, what is its **absolute** magnitude *M*? **4.83** [same as sun]
- 3c) What is its apparent magnitude m? 6.34

[This follows from $m = M + 5 \lg \left(\frac{r}{10 \text{ parsec}} \right)$]

3d) How far away would a 100 W light bulb have to be to have the same apparent brightness (magnitude) as this star? **315 km** (10⁻¹¹ parsec)

[E.g., using $m = 66.29 - 2.51 \text{g} \left(\frac{L}{1 \text{ W}}\right) + 51 \text{g} \left(\frac{d}{1 \text{ pc}}\right)$ from the formula sheet and solving

for *d*, or using the fact that if the light bulb is 100 W/3.84 $\cdot 10^{26}$ W times dimmer than that star, it must be $(100/3.84 \cdot 10^{26})^{1/2}$ times closer to have the same apparent brightness].

Note:
$$L_{sun} = 3.84 \cdot 10^{26} \text{ W}, m_{sun} = -26.74, M_{sun} = +4.83 \text{ and } m_2 \cdot m_1 = 2.5 \lg(F_1/F_2) \text{ with } F = \frac{L}{4\pi r^2}$$

Problem 4

The International Space Station has a mass of m = 420,000 kg and orbits planet Earth 250 km above its surface. Calculate the following numbers:

[Note: the ISS has a distance of 6,371 km + 250 km = 6621 km from the center of Earth]

4a) Assuming a circular orbit, what is the speed of the space station? 7.757 km/s

[Using $v = \sqrt{GM/R}$ from our first class.]

- 4b) What is its kinetic energy? $1.26 \cdot 10^{13} J$
- 4c) What is its potential energy? -2.52[·]10¹³ J [twice the kinetic energy, but with opposite sign]
- 4d) What is its total mechanical energy? -1.26 \cdot 10¹³ J
- 4e) How much energy was needed to lift it from rest on Earth's surface to its present orbit? Before launch, on the surface of Earth, its potential (and therefore total ^{*)}) energy was -2.63[·]10¹³ J, slightly more negative (because of the lower height) than in orbit. So the energy required was -1.26[·]10¹³ J (-2.63[·]10¹³ J) = 1.37[·]10¹³ J. This corresponds to roughly 3.3 kilo-ton of an explosive like TNT.

You may ignore all other celestial bodies. Set the potential gravitational energy of Earth to zero at infinite distance.

^{*)} Strictly speaking, the kinetic energy on the surface of Earth is NOT zero, due to Earth's rotation. This is one reason why space ship are preferrably launched from Florida. I find a local velocity of 407 m/s at Cape Canaveral, which would give the ISS an initial kinetic energy of 3.5^{-10¹⁰} J. But this is of course only a slight rounding error on the result above.