Stellar Structure
What have we learned?

• Can determine surface temperature via blackbody radiation, and absorption spectra
• Can determine relative magnitude, and after determining distance through parallax, absolute magnitude => Luminosity
• Relating black-body intensity to Luminosity yields surface and hence radius
• Binary stars: red- and blue shift of spectral lines determines absolute velocities; time dependence determines angular velocity of rotation around common center of gravity => distance of both stars from center of gravity
• Kepler’s law gives sum of masses and Newton’s 3rd law gives ratio of masses => individual masses
• Combining everything: average density
Hertzsprung-Russel Diagram

- Supergiants
- Main sequence
- Giants
- White dwarfs

Log$_{10}$ ($L/L_\odot$) vs. $T_e$ (K)
Question: How do we deduce interior structure of stars from these observations?
What do we need to know?

• Where does radiated energy ultimately come from?
• Need to figure out $\rho$, $P$, $T$ as function of $r < R$
• 3 ingredients:
  – energy transport from center to surface $\rightarrow T(r)$
  – hydrostatic equilibrium (what keeps the star from further collapse $\rightarrow \rho$, $P$
  – Equation of state to relate $\rho$, $P$, $T$
Energy Transport

• 3 mechanisms:
  – Heat conduction (can be ignored given the humongous sizes of stars)
  – Heat convection: Does play a significant role in many stars (see later)
  – Radiation (electromagnetic)
    • Propagation (in particular net outward flow)
    • Also need to account for radiation sinks and sources
    • Also contributes to pressure!
Interaction of photons with matter

• Absorption
  – excitation of atoms from lower to higher energy eigenstates
  – ionization of atoms (electrons kicked out)

• Emission
  – Atoms going from higher to lower energy eigenstate
  – Electrons get “caught” by ions

• Scattering
  – Bremsstrahlung
  – Thompson (Compton) scattering
Opacity in Photosphere

Random Walk of photons through sun

$6310K \quad 7080K$
Propogation of electromagnetic waves

Energy per volume $dV$ in wave length interval $d\lambda$:
\[
\frac{dE(\lambda \ldots \lambda + d\lambda)}{dV} = u_\lambda d\lambda ; u_\lambda = \text{specific energy density}.
\]

Example: black-body
\[
\frac{dn_\gamma}{d\lambda} \frac{hc}{\lambda} = \frac{8\pi hc}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda\lambda T}} - 1}
\]

Total (integral over all wave lengths): $dE_{\text{tot}}/dV = 4\sigma/c T^4$

Power emitted per area $dA$ into solid angle $d\Omega$ in wave length interval $d\lambda$:
\[
\frac{dE(\lambda \ldots \lambda + d\lambda)}{dt dA d\Omega} = \cos \theta \cdot I_\lambda(\theta, \varphi) d\lambda ; I_\lambda = \text{specific intensity}.
\]

Average:
\[
\langle I_\lambda \rangle = \frac{1}{4\pi} \int \int I_\lambda(\theta, \varphi) d\Omega = \frac{c}{4\pi} u_\lambda.
\]
Ex.: black-body:
\[
\langle I_\lambda \rangle = \frac{2hc^2}{\lambda^5} \frac{d\lambda}{e^{\frac{hc}{\lambda\lambda T}} - 1}
\]

Average:
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\langle I_\lambda \rangle = \frac{1}{4\pi} \int \int I_\lambda(\theta, \varphi) d\Omega = \frac{c}{4\pi} u_\lambda.
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Ex.: black-body:
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\]

Power emitted in positive (neg.) $z$-direction per area $dA$ perpendicular to $z$ and per $d\lambda$:

Radiation flux density
\[
F_\lambda = \int d\varphi \int_{0}^{\pi/2} \cos \theta \cdot I_\lambda(\theta, \varphi) \sin \theta d\theta
\]

For isotropic specific intensity (for top hemisphere):
\[
\Rightarrow F_\lambda = \pi \langle I_\lambda \rangle
\]

For black-body radiation:
\[
F_\lambda d\lambda = \frac{2\pi hc^2}{\lambda^5} \frac{d\lambda}{e^{\frac{hc}{\lambda\lambda T}} - 1} = 2\pi hc \frac{f^3}{c^3} \frac{df}{e^{\frac{hf}{kT}} - 1}
\]

Radiation pressure in $z$-direction:
\[
dP^z_\lambda = \frac{2}{c} \int d\varphi \int_{0}^{\pi/2} \cos^2 \theta \cdot I_\lambda(\theta, \varphi) \sin \theta d\theta
\]

For isotropic specific intensity in top hemisphere:
\[
\Rightarrow dP^z_\lambda = \frac{4\pi}{3c} \langle I_\lambda \rangle d\lambda = \frac{1}{3} u_\lambda d\lambda
\]
Temperature [K]

Stellar Model

Computer Model

Simple Fit

\( T_{\text{eff}} = 5500 \text{ K} \)

Surface 22,000 km in
Interior Structure

- Convective zone
- Radiative zone
- Core energy generation