Fundamental constants:

- Speed of light: $c = 2.9979 \times 10^8$ m/s (roughly a foot per nanosecond)
- Planck constant: $\hbar = 6.626 \times 10^{-34}$ J s; $\hbar = h / 2\pi = 197.33$ eV/c nm = 0.658 eV/PHz
- Fundamental charge unit: $e = 1.602 \times 10^{-19}$ C
- Electron mass: $m_e = 9.109 \times 10^{-31}$ kg
- Hydrogen atom (1H) mass: $m_H = 1.6735 \times 10^{-27}$ kg ($A = 1.0078$)
- Helium atom (4He) mass: $m_{\text{He}} = 6.6465 \times 10^{-27}$ kg ($A = 4.0026$)
- Coulomb’s Law constant: $k = 1/4\pi\epsilon_0 = 8.988 \times 10^9$ N m$^2$/C$^2$
- Gravitational constant: $G = 6.674 \times 10^{-11}$ N m$^2$/kg$^2$
- Avogadro constant: $N_A = 6.022 \times 10^{23}$ particles per mol

Special Relativity:

Lorentz Transformation from S to S’:

$$\gamma = \frac{1}{\sqrt{1 - v^2 / c^2}}; \quad x' = \gamma \left( x - \frac{v}{c} ct \right); \quad ct' = \gamma \left( ct - \frac{v}{c} x \right); \quad y = y'; \quad z = z', \text{ replace } v \text{ with } -v \text{ for } S' \rightarrow S$$

Velocity Addition:

$$u_x = \frac{u_x' + \frac{v}{c}}{\gamma \frac{c}{c}}; \quad u_y = \frac{u_y'}{\gamma \frac{c}{c}} = \frac{1}{\gamma \frac{c}{c}} \frac{u_y'}{\gamma \frac{c}{c}}$$

Doppler Shift:

$$\frac{\lambda_{\text{obs}}}{\lambda_{\text{emitted}}} = (z + 1) = \frac{1 + \frac{v}{c}}{\sqrt{1 - v^2 / c^2}}$$

Four-vectors: $x^\mu = (ct, x, y, z)$; $x_\mu = (ct, -x, -y, -z)$ ($\mu = 0, 1, 2, 3$ for $ct, x, y, z$).

Invariant (squared) Interval:

$$\Delta x'^\mu = (\Delta ct, \Delta x, \Delta y, \Delta z) \Rightarrow \Delta s^2 = \Delta x'^\mu \Delta x_\mu = \Delta ct^2 - \Delta x^2 - \Delta y^2 - \Delta z^2$$

Positive $\Delta s^2$: “time-like separation”, $\Delta s^2$ = square of elapsed time $ct$ in a system that travels from the start point (event) to the end point (event) of the interval.

Negative $\Delta s^2$: “space-like separation”, $-\Delta s^2$ = square of distance between the two events in a system (which always exists!) where they occur simultaneously.

$\Delta s^2 = 0$: “light-like separation”; a light ray could travel from one event to the other.

Four-momentum:

$$P^\mu = (E / c, P_x, P_y, P_z) = (\Gamma mc, \Gamma m\vec{u}); \quad \Gamma = \frac{1}{\sqrt{1 - u^2 / c^2}}. \quad u = \text{velocity.}$$

Transformation of $P^\mu$ to coordinate system $S'$ is analog to $x^\mu$ (see above).

$$E = P \cdot c, \quad E_\mu = mc, \quad T_\mu = (\Gamma - 1)^m c^2 (\approx m/c, w \text{ only if } u << c); \quad \text{Photons: } u = c, E = \mid P \mid c.$$
Quantum Mechanics – Motion in 1D:

Scalar Product: \( \langle \psi_1 | \psi_2 \rangle = \int_{-\infty}^{\infty} \psi_1^*(x) \psi_2(x) \, dx \).

Normalization: \( \langle \psi | \psi \rangle = \int_{-\infty}^{\infty} \psi^*(x) \psi(x) \, dx < \infty \implies |\psi_{\text{normalized}}\rangle = |\psi\rangle / (\langle \psi | \psi \rangle)^{1/2} \).

Probability of locating a particle in an interval \( x \ldots x + dx \):

\[
dPr(x \ldots x + dx) = |\psi(x)|^2 \, dx = \psi(x)^\ast \psi(x) \, dx
\]

Operator \( O \) with eigenvalue \( \omega \) and eigenvector \( |\varphi_\omega\rangle \):

\( O |\varphi_\omega\rangle = \omega |\varphi_\omega\rangle \)

Position Operator \( X \): \( X \psi(x) = x \cdot \psi(x) \rightarrow \text{eigenvectors} \quad \psi_{x_0}(x) = \delta(x - x_0) \), eigenvalue \( x_0 \)

Momentum Operator: \( P \psi(x) = \frac{\hbar}{i} \frac{\partial}{\partial x} \psi(x) \rightarrow \text{eigenvectors} \quad \psi_{p_0}(x) = e^{ip_0x/\hbar} \), eigenvalue \( p_0 \)

Hamiltonian: \( H \psi(x) = \left( \frac{P^2}{2m} + V(x) \right) \psi(x) = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \psi(x) + V(x) \psi(x) \).

Heisenberg: \( \sigma_x \sigma_y \geq \hbar/2 \).

Schrödinger Equation: \( |\psi\rangle(t) ; \quad \frac{\partial}{\partial t} |\psi\rangle(t) = \frac{1}{i\hbar} H |\psi\rangle(t) \)

Eigenstates of Hamiltonian: \( H |\varphi_E\rangle = E |\varphi_E\rangle \implies |\psi_E(t)\rangle = |\varphi_E\rangle e^{-iEt/\hbar} \)

Motion in 1-D, eigenstates of the Hamiltonian: \( -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \psi(x) + V(x) \psi(x) = E \psi(x) \)

Eigenstate of the free Hamiltonian \((V(x)=0)\): \( \psi_p(x,t) = Ae^{i(px - pt^2/2m)/\hbar} \)

Gaussian Wave Package: \( \psi_{\text{GWP}}(x,t=0) = \sqrt{\frac{1}{\sqrt{2\pi}\sigma_p}} \int_{-\infty}^{\infty} e^{ipx/\hbar} e^{i\pi \sigma_p^2 / 2m} \, dp = \frac{1}{\sqrt{2\pi}\sigma_p} e^{i\pi \sigma_p^2 / 2m} ; \sigma_x = \frac{\hbar}{2\sigma_p} \)

Eigenstates for a 1-dim square well: \((V(x)=0, 0 \leq x \leq L, \infty \text{ else})\)

\( \varphi_0(x) = 0 \text{ for } x < 0, x > L; \text{ else } \varphi_n(x) = \sqrt{\frac{2}{L}} \sin \left( \frac{n\pi x}{L} \right) ; E_n = \frac{n^2 \pi^2 \hbar^2}{2mL^2} , n = 1, 2, \ldots \)

Eigenstates of Harmonic Oscillator: \( H = \frac{P^2}{2m} + \frac{m\omega^2}{2} X^2 \implies \)

\( \varphi_n(x) = AH_n \left( \frac{m\omega}{\hbar} x \right) e^{-m\omega x^2 / 2\hbar} ; E_n = (n + \frac{1}{2})\hbar \omega , n = 0, 1, \ldots \)

\( H_0(y) = 1, H_1(y) = 2y, H_2(y) = 4y^2 - 2; \)

\( A_0 = \left( \frac{m\omega}{\pi\hbar} \right)^{1/4} , A_1 = \frac{1}{\sqrt{2}} \left( \frac{m\omega}{\pi\hbar} \right)^{1/4} , A_2 = \frac{1}{\sqrt{8}} \left( \frac{m\omega}{\pi\hbar} \right)^{1/4} . \)
Quantum Mechanics in 3D:

**Cartesian coordinates:** \((x,y,z); \Delta \tau = \Delta x \Delta y \Delta z\)

\[\psi(x,y,z); H = -\frac{\hbar^2}{2m} \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) + V(x,y,z); \Delta \Pr(\Delta \tau) = |\psi(x,y,z)|^2 \Delta \tau\]

Infinite square well:

\[\varphi_{nk}(x,y,z) = \left( \frac{8}{L^3} \sin \frac{n\pi x}{L} \sin \frac{j\pi y}{L} \sin \frac{k\pi z}{L}, E_{nk} = (n^2 + j^2 + k^2) \right) \frac{\hbar^2 \pi^2}{2mL^2}\]

**Spherical coordinates:** \(r, \theta, \phi; \Delta \tau = r^2 \sin \theta \Delta \theta \Delta \phi\)

\[H = -\frac{\hbar^2}{2m} \left( \frac{1}{r^2} \frac{\partial}{\partial r} r^2 \frac{\partial}{\partial r} + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \sin \theta \frac{\partial}{\partial \theta} + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2}{\partial \phi^2} \right) + V(r)
= -\frac{\hbar^2}{2mr^2} \frac{\partial}{\partial r} r^2 \frac{\partial}{\partial r} + \frac{1}{2mr^2} \mathbf{L}^2 + V(r)\]

\(\mathbf{L}^2\) is the squared orbital angular momentum operator with eigenfunctions

\[Y_{lm}(\theta, \phi); \mathbf{L}^2 Y_{lm} = \hbar^2 \ell(\ell + 1)Y_{lm}; \ell = 0, 1, 2 \ldots; \mathbf{L}_z Y_{lm} = \hbar m Y_{lm}; m = -\ell, -\ell + 1, \ldots, \ell\]

Examples:

\[Y^{-1}_{1,0}(\theta, \phi) = \frac{1}{2} \sqrt{\frac{3}{\pi}} e^{-i\phi} \sin \theta\]

\[Y^{0}_{1,0}(\theta, \phi) = \frac{1}{2} \sqrt{\frac{3}{\pi}} \cos \theta\]

\[Y^{1}_{0,0}(\theta, \phi) = \frac{1}{2} \sqrt{\frac{3}{2\pi}} e^{i\phi} \sin \theta\]

**Separation of Variables:** \(\psi_{Elm}(r, \theta, \phi) = R_{E,l}(r)Y_{lm}(\theta, \phi)\) with

\[-\frac{\hbar^2}{2mr^2} \frac{\partial}{\partial r} r^2 \frac{\partial}{\partial r} R_{E,l}(r) + \frac{\hbar^2 \ell(\ell + 1)}{2mr^2} R_{E,l}(r) + V(r)R_{E,l}(r) = E \cdot R_{E,l}(r)\]

Probability to find particle in volume \(\Delta \tau\) at position \((r, \theta, \phi) = |\psi_{Elm}(r, \theta, \phi)|^2 \Delta \tau\)

Radial probability distribution: \(\Delta \Pr(r \ldots r + \Delta r) = |R_{E,l}(r)|^2 \Delta r\)

**Hydrogen-like atoms:**

(Nucleus of mass \(m_2\) and charge \(Ze\), bound particle of mass \(m_1\) and charge \(-e\))

\[V(r) = -\frac{Ze^2}{4\pi \epsilon_0 r} = -\frac{Z\alpha \hbar c}{r} \quad \alpha = e^2 / 4\pi \epsilon_0 \hbar c\]

Reduced mass of 2-body system with masses \(m_1\) and \(m_2\): \(\mu_r = \frac{m_1 m_2}{m_1 + m_2}\)
Energy Eigenvalues: \( E_n^\ell = -\frac{\mu_e Z^2}{m_e n^2} \frac{R_y}{m} (n = 1, 2, \ldots) \); \( R_y = \frac{m_e c^2 \alpha^2}{2} = 13.6 \text{ eV} \).

Degenerate in \( \ell \) and \( m; \ell = 0, 1, \ldots, n-1, m = -\ell \ldots + \ell; \) also degenerate in electron spin \( m_s = \pm 1/2 \) => total degeneracy \( 2n^2 \).

Characteristic radius: \( a = \frac{m_e}{\mu_r Z a_0} \) \( a_0 = \frac{\hbar c}{(m_e c^2 a)} = 0.53 \text{ Å} = 0.053 \text{ nm} \).

Eigenstates: \( \psi_n^\ell, m(r, \theta, \phi) = R_n^\ell(r) Y_{lm}(\theta, \phi) \). \( R_n^\ell(r) \) (examples):

\[
R_{1,0}^r(r) = \frac{2}{\sqrt{\pi a}} e^{-r/a}, \quad R_{2,0}^r(r) = \frac{2 - r/a}{\sqrt{8a^3}} e^{-r/2a}, \quad R_{2,1}^r(r) = \frac{r/a}{\sqrt{24a^3}} e^{-r/2a}
\]

Energy of a photon: \( E_\gamma = h f = \frac{h c}{\lambda} \)

Momentum of a photon: \( p_\gamma = h/\lambda \)

Light emitted or absorbed in transition with energy difference \( \Delta E = E_{\text{init}} - E_{\text{final}} \):
\[
f = \frac{\Delta E}{h} = \frac{hc}{\lambda} = 2\pi \hbar c/\Delta E
\]

Pauli principle: No two identical Fermions (spin-1/2, 3/2, \ldots particles) can be in the same exact quantum state.

**Molecules and Condensed Matter**

**Ionic Bond:** One atom gives up 1 (or more) electron(s), the other picks it (them) up; binding through electrostatic attraction.

**Covalent Bond:** Electron(s) shared between two atoms. Example: Let \( \psi_1(r_e) = \) wave function for hydrogen ground state with proton at position 1, and \( \psi_2(r_e) \) for proton at position 2. Symmetric superposition \( \psi_s(r_e) = \frac{1}{\sqrt{2}} \psi_1(r_e) + \frac{1}{\sqrt{2}} \psi_2(r_e) \) is attractive (net charge between protons), antisymmetric superposition \( \psi_a(r_e) = \frac{1}{\sqrt{2}} \psi_1(r_e) - \frac{1}{\sqrt{2}} \psi_2(r_e) \) is non-binding (zero net charge between protons).

**Metallic Bond:** Many electrons (one or more per atom) shared between a large number \( N \) of atoms \( \rightarrow \) positively charged “rest atoms” in “Fermi gas” of electrons. Electron energy eigenstates are clustered in “bands”; highest (partially or totally unoccupied) band = conduction band, next lower (filled) band = valence band. Each band contains of order \( N \) eigenstates. Interaction between electron gas and oscillation modes (=phonons) of the “rest atoms” gives rise to conductive heating, \( V = RI \), and superconductivity (Bose-Einstein condensation of “Cooper pairs” of electrons).

**Conductors:** partially filled conduction band and/or overlapping conduction and valence bands.

**Isolators:** Completely empty conduction band, completely filled valence band, large band gap.

**Semi-conductors:** Similar to isolators, but with smaller band gap. Can conduct at finite temperatures (see Fermi-Dirac distribution below). Conductivity increased through electron donor (n-doped) or electron acceptor (p-doped) impurities. \( \text{pn-junction} \) = diode.
Particle Physics

**Fundamental Fermions** (spin-1/2 particles obeying Pauli Exclusion Principle): quarks (up, down, charm, strange, top, bottom) and leptons (electron, muon, tau, electron-neutrino, muon-neutrino, tau-neutrino) and their antiparticles.

**Force-mediating Gauge Bosons** (spin-1 particles obeying Bose-Einstein statistics): Photon $\gamma$ (electromagnetic interaction), $W^+$, $W^-$, $Z^0$ (weak interaction), gluons (strong interaction) [graviton (gravity) only conjectured]. All except weak interaction bosons are massless; the latter gain mass (80-91 GeV/c$^2$) through interaction with the Higgs field.

<table>
<thead>
<tr>
<th>Name</th>
<th>Symbol</th>
<th>Mass (MeV/c$^2$)</th>
<th>J</th>
<th>B</th>
<th>Q (e)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Up</td>
<td>u</td>
<td>2.30 +0.7 -0.5</td>
<td>$\frac{1}{2}$</td>
<td>$^+\frac{1}{2}$</td>
<td>$^+\frac{3}{2}$</td>
</tr>
<tr>
<td>Down</td>
<td>d</td>
<td>4.80 -0.3</td>
<td>$\frac{1}{2}$</td>
<td>$^+\frac{1}{2}$</td>
<td>$^+\frac{3}{2}$</td>
</tr>
<tr>
<td>Charm</td>
<td>c</td>
<td>1275 ± 25</td>
<td>$\frac{1}{2}$</td>
<td>$^+\frac{1}{2}$</td>
<td>$^+\frac{3}{2}$</td>
</tr>
<tr>
<td>Strange</td>
<td>s</td>
<td>95 ± 5</td>
<td>$\frac{1}{2}$</td>
<td>$^+\frac{1}{2}$</td>
<td>$^+\frac{3}{2}$</td>
</tr>
<tr>
<td>Top</td>
<td>t</td>
<td>173 210 ± 510 ± 710</td>
<td>$\frac{1}{2}$</td>
<td>$^+\frac{1}{2}$</td>
<td>$^+\frac{3}{2}$</td>
</tr>
<tr>
<td>Bottom</td>
<td>b</td>
<td>1410 ± 30</td>
<td>$\frac{1}{2}$</td>
<td>$^+\frac{1}{2}$</td>
<td>$^+\frac{3}{2}$</td>
</tr>
</tbody>
</table>

All interactions proceed via gauge bosons coupling to various charges:
- electromagnetic interaction: electric charge (+ or -) (all charged Fermions plus W bosons)
- weak interaction: weak charges (“weak isospin and weak hypercharge”) – all particles except gluons
- strong interaction: color charges (“red”, “green”, “blue”) – all quarks and gluons.

**Nuclear Physics**

**Mass-energy of an atom**: $(Z$ protons, $N$ neutrons, $A = Z+N)$:

\[ M_A c^2 = Z M_p c^2 + N M_n c^2 + Z m_e c^2 - BE \] (Binding energy)

Typical binding energies $BE = 7$-$8$ MeV A with a maximum for nuclei around iron ($A=56$). Light nuclei have significantly lower $BE$ per nucleon; beyond iron, the $BE$ per nucleon decreases slowly with $A$ (due to Coulomb repulsion).

Energy liberated during a nuclear fusion reaction $1 + 2 \rightarrow 3$: $\Delta E = M_1 c^2 + M_2 c^2 - M_3 c^2$

Energy liberated during a nuclear decay $1 \rightarrow 2 + 3$: $\Delta E = M_1 c^2 - M_2 c^2 - M_3 c^2$

**Density**: roughly constant $\rho = 0.16$ Nucleons / fm$^3 = 2 \times 10^{17}$ kg/m$^3$

**Radioactive nuclei**:
- alpha-decay: $(Z,A) \rightarrow (Z-2,A-2) + ^4\text{He} + \text{energy}$
- beta-plus decay: $(Z,A) \rightarrow (Z-1,A) + e^+ + \nu_e$
- beta-minus decay: $(Z,A) \rightarrow (Z+1,A) + e^- + \bar{\nu}_e$

Decay probability in time $\Delta t$: $\Delta Pr(\Delta t) = \Delta t/\tau$ ($\tau =$ lifetime $= T_{1/2} / \ln 2$)

Number of undecayed nuclei at time $t$ (starting with $N_0$): $N(t) = N_0 e^{-t/\tau}$