Quiz 2

Assume $S'$ contains a rod 1 m in length, aligned with the $x'$ axis. It has a (rest) mass $m = 1$ kg.
The origin of $S'$ is identical with the origin of $S$. Please answer the following questions.

A. What is the velocity $v$ of $S'$ relative to $S$ in units of $c$?
B. At what point $(ct, x)$ in $S$ does the event $E_1 = \text{“clock in } S' \text{ strikes 1m”}$ occur?
C. What is the invariant interval $(\Delta s)^2$ between the origin and that event in $S$?
D. What is the invariant interval $(\Delta s)^2$ between the origin and that event in $S'$?
E. If $S$ measures both ends of the rod simultaneously (at $ct = 0$), what length does it measure?
F. What are the coordinates of the event $E_2 = \text{“} S' \text{ measures the far side end of the rod, simultaneously with its origin } (ct', x') = (0,0) \text{”}$?
G. What is the invariant interval $(\Delta s)^2$ between the origin and that event in $S$?
H. What is the total 4-momentum of the rod in $S'$? $(P_0, P_1, P_2, P_3)$
I. What is the total 4-momentum of the rod in $S$? $(P_0, P_1, P_2, P_3)$
J. The rod collides with an identical one at rest in $S$. What is the invariant mass of the combined object after the collision?
Assume $S'$ contains a rod 1 m in length, aligned with the $x'$ axis. It has a (rest) mass $m = 1$ kg. The origin of $S'$ is identical with the origin of $S$. Please answer the following questions.

A. What is the velocity $v$ of $S'$ relative to $S$ in units of $c$? **Answ.:** $v/c = \Delta x/\Delta ct = \frac{1}{2}$; $v = 0.5 \, c$

B. At what point $(ct, x)$ in $S$ does the event $E_1 =$ “clock in $S'$ strikes 1m” occur?
   **Answ.:** $ct = \gamma ct' = (1 + 0.25)^{-1/2} \cdot 1 \, m = 1.155 \, m$ (time dilation); $x = v \, ct = 0.577 \, m$

C. What is the invariant interval $(\Delta s)^2$ between the origin and that event in $S$?
   **Answ.:** $1 \, m^2$ - either by direct calculation $(ct^2 - x^2)$ or by observing that it must be the same as in $S'$ (see below)

D. What is the invariant interval $(\Delta s)^2$ between the origin and that event in $S'$?
   **Answ.:** $1 \, m^2$ by definition for a time-like interval or by direct calculation

E. If $S$ measures both ends of the rod simultaneously (at $ct = 0$), what length does it measure?
   **Answ.:** $1/\gamma \, m = 0.866 \, m$ (length contraction or use Lorentz transformation with $ct = 0$)

F. What are the coordinates of the event $E_2 =$ “$S'$ measures the far side end of the rod, simultaneously with its origin $(ct', x') = (0,0)$”? **Answ.:** $ct = \gamma ct' + \gamma v/c \, x' = 0 + 0.577m$; $x = \gamma v/c \, ct' + \gamma x' = 0 + 1.155 \, m$
Quiz 2

Assume $S'$ contains a rod 1 m in length, aligned with the $x'$ axis. It has a (rest) mass $m = 1$ kg.
The origin of $S'$ is identical with the origin of $S$. Please answer the following questions.

F. What are the coordinates of the event $E_2 = "S' measures the far side end of the rod, simultaneously with its origin (ct', x') = (0,0)"$? \textbf{Answ.}: $ct = \gamma ct' + \gamma v/c x' = 0 + 0.577m$; $x = \gamma v/c ct' + \gamma x' = 0 + 1.155$ m

G. What is the invariant interval $(\Delta s)^2$ between the origin and that event in $S$? \textbf{Answ.}: Again, 1 m$^2$ by definition (space-like interval), direct calculation or invariance

H. What is the total 4-momentum of the rod in $S'$? $(P^0, P^1, P^2, P^3)$
\textbf{Answ.}: $P^0 = E/c = mc = 2.9979 \cdot 10^8$ kg m/s; $P^1 = P^2 = P^3 = 0$ ($u = 0$!)

I. What is the total 4-momentum of the rod in $S$? $(P^0, P^1, P^2, P^3)$
\textbf{Answ.}: $P^0 = E/c = \Gamma mc = 3.462 \cdot 10^8$ kg m/s; $P^1 = \Gamma mv = 1.731 \cdot 10^8$ kg m/s; rest = 0 ($u = \nu$!)

J. The rod collides with an identical one at rest in $S$. What is the invariant mass of the combined object after the collision?
\textbf{Answ.}: $P_{tot}^0 = mc + \Gamma mc = 6.46 \cdot 10^8$ kg m/s; $P_{tot}^1 = \Gamma mv = 1.731 \cdot 10^8$ kg m/s
$m^2 c^2 = (P_{tot}^0)^2 - (P_{tot}^1)^2 = [(1+\Gamma)^2 - 0.5^2 \Gamma^2 ](1$kg c$)^2 \Rightarrow m = 2.076$ kg