Stars

PHYS323
Perfect Blackbody Spectrum

Intensity per lambda-interval [W/m^2]

Max wave length

lambda [nm]
Real Sun

Solar Radiation Spectrum

- Sunlight at Top of the Atmosphere
- 5250°C Blackbody Spectrum
- Radiation at Sea Level

Graph showing spectral irradiance (W/m²/nm) vs. wavelength (nm).
Quantum Mechanics in 20 min

1. Many observables are quantized (i.e., cannot change by an arbitrarily small amount)
   1. Light waves: Energy for a specific frequency $f$ can only be absorbed or emitted in chunks (photons) of $E = hf$
   2. Possible energy for hydrogen atom can only assume values $E_n = -\text{Ry}/n^2$ (see next slide)
   3. Angular momentum can only change by integer multiples of $\hbar = \hbar/2\pi$

2. All other observables are intrinsically uncertain
   1. Position: $x...x+\Delta x$
   2. Momentum: $p...p+\Delta p$
   3. Heisenberg: $\Delta x \Delta p \geq \hbar/2$

3. Picture: particle motion described by waves (“wave function” $\psi$) that cannot be located precisely. Quantization $\iff$ Standing Waves
Quantum Mechanics in 20 min

① Electron “motion” in hydrogen atom (nucleus = proton): standing wave described by wave function $\psi(r)$

② Schrödinger: Wave function is solution of the equation $H\psi(r) = E\psi(r)$, where $E$ is a possible energy “eigenvalue” and $H$ is a differential operator (“The Hamiltonian”)

③ Hydrogen atom: Only possible energies are $E_n = -Ry/n^2$ with $Ry = 13.6$ eV and $n = \text{integer}$. In general all atoms have a fixed series of possible energies $E_n$

Light can only be emitted with frequencies given by $hf = E_n - E_m$
Quantum Mechanics and Line Spectra

\[ \Delta E(n \rightarrow m) = Ry \left( \frac{1}{m^2} - \frac{1}{n^2} \right) \]

\[ \lambda = \frac{hc}{\Delta E} \]

Lyman series

Balmer series

Paschen series
Measuring Temperature

- Peak wavelength (blackbody spectrum)
- Total energy output (Stefan-Boltzmann law)
  - Works only if $R$ is known. Vice versa: $R$ if $T$ is known!

- Relative intensity of spectral lines:
  - Almost all hydrogen atoms in the ground state (electrons in the $n = 1$ orbit) => few transitions from $n = 2$ => weak Balmer lines
  - Most hydrogen atoms are ionized => weak Balmer lines

Boltzmann:

$$n(E) \propto e^{E/kT}$$

The lines of each atom or molecule are strongest at a particular temperature.
Hertzsprung-Russel Diagram
Jupiter is about 1 pixel in size
Earth is invisible at this scale
Hertzsprung-Russel Diagram
Question: How do we deduce interior structure of stars from these observations?
Interior Structure
What Powers the Sun?

• Gravitational energy (potential plus kinetic, according to virial theorem) as function of radius:
  \[ E_{\text{tot}} = -\frac{3}{10} \frac{GM^2}{R} \]

• Therefore, maximum amount of energy “generated” by Sun through contraction to present radius:
  \[ E_{\text{tot}} = -\frac{3}{10} \frac{6.674 \cdot 10^{-11} (1.989 \cdot 10^{30})^2}{6.955 \cdot 10^8} \text{ J} = 1.14 \cdot 10^{41} \text{ J} \]

• Energy actually radiated by Sun over last 4.5 B yrs
  \[ E_{\text{rad}} = L_{\text{Sun}} \cdot T = 3.84 \cdot 10^{26} \text{ W} \cdot 1.42 \cdot 10^{17} \text{ s} = 5.45 \cdot 10^{43} \text{ J} \]
  = 500 times more! (Or Sun must be less than 9 M yrs old!)

• Solution: Must be something else \( \rightarrow \) nuclear fusion
The Structure of Matter

- What is the Universe made of?
- What are the most fundamental objects in Nature?
- What particles were there in the beginning (right after the big bang)?
- How do they interact?
- How do they form composite objects?
Matter Particles

- Make up visible matter
- Pointlike (<10^{-18} m), Fundamental *)
- Have mass (from < \frac{1}{2} \text{ eV} to 178,000,000,000 \text{ eV} = 178 \text{ GeV})
- Distinct from their antiparticles *)
- Fermions (Spin \frac{1}{2}) \Rightarrow they “defend” their space (Pauli Principle) and can only be created in particle-antiparticle pairs
- Can be “virtual”, but make up matter being (nearly) “real”
- “stable” (against strong decays; lifetimes from \infty to 10^{-24} \text{ s})

*) Until further notice

3 “colors” = 3 different charges: red, green, blue
Forces and Force Carriers

- Mediate Interactions (Forces) - form “Waves”
- Pointlike, Fundamental
- Massless *)
- Some are their own antiparticles (photon, $Z^0$, graviton)
- Spin 1, 2 -> Bosons (tend to cluster together, can be produced in arbitrary numbers)
- Can be real, but carry forces as virtual particles
- Some are absolutely stable ($\gamma$, gluons, gravitons)

*) See next slide

Note: gluons come in 8 possible combinations of color/anticolor (9th is “sterile” – doesn’t exist)
The Structure of Matter

In the center of the atom is a nucleus formed from nucleons—protons and neutrons. Each nucleon is made from three quarks held together by their strong interactions, which are mediated by gluons. In turn, the nucleus is held together by the strong interactions between the gluon and quark constituents of neighboring nucleons. Nuclear physicists often use the exchange of mesons to convey interactions among the nucleons.

In an atom, electrons range around the nucleus at distances up to 10,000 times the nuclear diameter. If the electron cloud were shown to scale, this chart would cover a small town.
**Periodic Table**

<table>
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<th>Hydrogen</th>
<th>Helium</th>
<th>Lutetium</th>
<th>Actinium</th>
<th>Thorium</th>
<th>Protactinium</th>
<th>Uranium</th>
<th>Plutonium</th>
<th>Americium</th>
<th>Curium</th>
<th>Berkelium</th>
<th>Californium</th>
<th>Lawrencium</th>
<th>Flerovium</th>
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<td>Am</td>
<td>Cm</td>
<td>Bk</td>
<td>Cf</td>
<td>Es</td>
</tr>
</tbody>
</table>

* Lanthanide series

** Actinide series
Nuclear Binding energies

Fig. 15.3

Binding energy per nucleon, as a function of the mass number, $A$. Some important isotopes are identified (Data from [6]).

Mass-energy of an atom: ($Z$ protons, $N$ neutrons, $A = Z+N$):

$$M_A c^2 = Z M_p c^2 + N M_n c^2 + Z m_e c^2 - E_b \text{ (Binding energy)}$$

Typical binding energies $B = 7-8 \text{ MeV}/A$ with a maximum for nuclei around iron ($A=56$). Light nuclei have significantly lower $BE$ per nucleon; beyond iron, the $BE$ per nucleon decreases slowly with $A$ (due to Coulomb repulsion). Energy liberated during a nuclear fusion reaction $1 + 2 \rightarrow 3$:

$$E_1 A_1 + M_2 c^2 - M_3 c^2 = (A_1 + A_2) \cdot E_b/A_3 - A_1 \cdot E_b/A_1 - A_2 \cdot E_b/A_2$$

$\rightarrow$ Energy gain only if $E_b/A_3 > E_b/A_1, E_b/A_2$ (up to Fe) Ex.: $^2\text{H}(Z=1, A=2) + ^2\text{H} \rightarrow ^4\text{He} + 23.85 \text{ MeV} = 3.82 \cdot 10^{-12} \text{ J}$

$$\Rightarrow 5.45 \cdot 10^{45} \text{ J} \approx 1.43 \cdot 10^{55} \text{ He atoms} = 9.48 \cdot 10^{28} \text{ kg} = 5\% \text{ of Sun's Mass} \sqrt{\text{}}$$
Nuclear Power Generation
The Life of Main Sequence Stars

Stars gradually exhaust their hydrogen fuel. In this process of aging, they are gradually becoming brighter, evolving off the zero-age main sequence.
- Requires temperatures above $10^8$ K (8.6 keV)
- $^8\text{Be}$ is unstable $\rightarrow \alpha\alpha$! Only at high temperature are there a few $^8\text{Be}$ in equilibrium with $^4\text{He}$ (energy sink!)
- $^8\text{Be} + \alpha \rightarrow ^{12}\text{C}$ would be too slow if not for $^{12}\text{C}$ excited state
- Predicted by Hoyle!
- Some $\alpha$ get eaten by $^{12}\text{C} \rightarrow ^{16}\text{O}$ admixture
• Expansion onto the Giant Branch

Expansion and surface cooling during the phase of an inactive He core and a H-burning shell.

The Sun will expand beyond Earth’s orbit!
Fusion into Heavier Elements

Fusion into heavier elements than C, O:

requires very high temperatures; occurs only in very massive stars (more than 8 solar masses)
Final Stages of Giants ($\approx M_\odot$)

- Final C core collapse
- Shock wave
- Outer layers ejected
- “Planetary” Nebulae
White Dwarfs

- Reminder: Last stages of sun and similar-sized stars

Last stage: Helium burning stops, core collapses and significant fraction of mass gets ejected as planetary nebula.

- What happens with the core after the final collapse? => White Dwarf! (Example: Sirius B)
  - Core contracts until “Fermi pressure” of electrons balances gravitational attraction
  - Final size typically <1% of present solar radius => Density $10^6$ times larger than that of the sun! Temperature $10^7$ K at center
SuperGiant; $M=20M_\odot$, $R = 1200R_\odot = 5.5$ AU(!), $10^{-8}$ g/cm$^3$
Super Giant Stars

- Last stage of superheavy (>10 M☉) stars after completing Main Sequence existence
- Initially: Very hot, UV radiation
- Move mostly horizontally on H-R diagram (decreasing temperature, constant luminosity
  - Heaviest (100M☉) never go beyond blue SG stage
  - Others: red SGs
Fusion for Supergiants

- Onion (25 $M_\odot$):
  - H burning: 5 Mio yr
  - $^4$He burning: 500,000 yr
  - $^{12}$C burning: 500 yr
  - Ne burning: 1 yr
  - Si burning: 1 day
  - Final state: inert Iron/Nickel core -> no more energy available from nuclear fusion (nor from fission!)

Liquid drop model:

$$E_B = a_V A - a_S A^{2/3} - a_C \frac{Z(Z - 1)}{A^{1/3}} - a_A \frac{(A - 2Z)^2}{A} + \delta(A, Z) + a_{\text{Grav}} \frac{A^2}{A^{1/3}}$$
=> Chandrasekhar Limit

- For less massive, larger white dwarfs:
  - $R \approx 5600 \text{ km } (M/M_{\odot})^{-1/3} \Rightarrow V \propto 1/M; \rho \propto M^2$
  - $p_i = 670 \text{ keV/c } (n/n_{\text{Sirius B}})^{1/3} = 670 \text{ keV/c } (M/M_{\odot})^{2/3}$

- As mass increases, gas becomes more and more relativistic and radius becomes even smaller $\Rightarrow$ runaway collapse ($R \propto M^{-\infty}$)

- Mass limit $M_{ch} = 1.4 \ M_{\odot}$

- Above that mass (for a stellar remnant after blowing off outer hull) electron Fermi gas pressure not sufficient for stability $\Rightarrow$ neutron Fermi gas (see later)
Type Ia Supernova

- White dwarf accumulates mass from (Giant) companion
- Exceeds Chandrasekar limit
- Goes supernova
Supernova remnant

- Neutron star:
  - nearly no p’s, e’ s, just neutrons
  - Remember: $R_{\text{white dwarf}} \propto \frac{1}{m_e} M^{-1/3}$
  - $m_n = 1840 \ m_e \Rightarrow R$ 1840 times smaller (really, about 500 times because only 1 e- per 2 neutrons) $\Rightarrow$ of order 10 km!
  - Density: few $10^{44}/m^3 = 1/fm^3 >$ nuclear density $\Rightarrow$ nucleus with mass number $A = 10^{57}$
  - Chandrasekar limit: 5 solar masses (2-3 in reality?)
  - Lots depends on nuclear equation of state *),
    general relativity

*) Repulsive core / Nuclear superfluid / quark-gluon plasma / strange matter / pasta?
=> Black Holes

• Beyond a certain density, NOTHING can prevent gravitational collapse!
  – If there were a new source of pressure, that pressure would have energy
    \( P = 1/3 \ldots 2/3 \frac{E}{V} \), which causes more gravitation => gravity wins over
  – Singularity in space-time (infinitely dense mass point, infinite curvature; no
classical treatment possible)

• For spherical mass at rest, Schwarzschild metric applies and we have an
event horizon at \( r = R_S = 2GM/c^2 = 3\text{km} \) \( M/M_{\text{sun}} \) (Schwarzschild radius)
  – as object approaches \( r_S \) from outside, clock appears to slow to a crawl and light
    emitted gets redshifted to \( \infty \) long wavelength
  – along light path, \( ds = 0 \) => \( dr = \pm (1-r_S/r)\,dt \) => light becomes \( \infty \) slow and never
can cross from inside \( r_S \) to outside
  – From outside, it takes exponential time for star surface to reach \( r_S \)
  – Rate of photon emission decreases exponentially (less than 1/s after 10 ms)
  – All material that falls in over time “appears” frozen on the surface of event
    horizon but doesn’t emit any photons or any other information
  – Co-moving coordinate system: will cross event horizon in finite time => no return!
Gravitational Waves

Binary Black Hole Evolution:
Caltech/Cornell Computer Simulation

Top: 3D view of Black Holes and Orbital Trajectory

Middle: Spacetime curvature:
Depth: Curvature of space
Colors: Rate of flow of time
Arrows: Velocity of flow of space

Bottom: Waveform (red line shows current time)