30. (a) Prove that the Poisson bracket of two constants of the motion is itself a constant of the motion even when the constants depend upon time explicitly.

(b) Show that if the Hamiltonian and a quantity $F$ are constants of the motion, then the $n$th partial derivative of $F$ with respect to $t$ must also be a constant of the motion.

(c) As an illustration of this result, consider the uniform motion of a free particle of mass $m$. The Hamiltonian is certainly conserved, and there exists a constant of the motion

$$F = x - \frac{pt}{m}.$$ 

Show by direct computation that the partial derivative of $F$ with $t$, which is a constant of the motion, agrees with $[H, F]$. 