Here is a better picture for the lens case:
Lenses focus wide beam in y to smaller spot in y.

- **Instantaneous kick in py**
  - Thin lens

- **Drift**

At focal point:
- Narrow in y, but overall phase space volume conserved.

Initial distribution before lens: wide in y.

After lens: **After lens**

After drift: **Final state**
Liouville’s Theorem:

1) $N$ particles all described by the same $H(q^i, p^i, t)$ for each $i$

2) distributed over a phase space volume $V$

3) $V(t) = \text{const.}$

\[
\frac{\partial \mathbf{v}_k^i}{\partial t} = \frac{\partial H}{\partial q^i} \quad \frac{\partial \mathbf{v}_k^i}{\partial t} = 2 \left( \frac{\partial H}{\partial p^i} \right)
\]

\[\Delta V = \left( \oint_{\text{volume}} \mathbf{v} \cdot d\mathbf{A} \right) dt = \oint_{\text{volume}} \mathbf{v} \cdot d\mathbf{v} dt\]