Graduate Quantum Mechanics II - Problem Set 5

Problem 1)

Given the well-known fact that for a classical bound state orbit in a $1/r$ potential (Coulomb or gravitational) the relationship between the total (binding) energy and the average kinetic and potential energy is given by $|E| = \langle T\rangle = -\frac{1}{2} \langle V\rangle$, express the radius for a classical circular orbit in terms of the angular momentum $L$ and the binding energy $|E|$ of the orbit. Compare with the most probable value for $r$ if measured on a quantum mechanical eigenstate of the same potential with very large $\ell = m$ and main quantum number $n = \ell + 1$ (see lecture notes). Comment?

Problem 2)

Calculate the energy difference between the qu.m. state defined above in Problem 1 and the state directly below it, with quantum numbers $n - 1$, $\ell - 1$ and $m - 1$. Express your result in terms of $E_n$ and $\ell$.

Find the angular velocity of a classical particle in a circular orbit ($1/r$ potential) in terms of the same quantities, $E$ and $L$. Compare your results. Comment?

Problem 3)

Write down the full time-dependent wave function (in coordinate space) of a particle in a one-dimensional constant potential of magnitude $V_0$. Assume the particle is in an (“improper”) eigenstate of the Hamiltonian. What is the phase velocity of the corresponding de-Broglie wave? If instead I had a superposition of waves (over a narrow range of energies) forming a traveling “wave packet”, what would be the group velocity (the velocity of the amplitude “envelope”) of that packet? Could the two ever be equal?

Problem 4)

Evaluate the Wigner function, $W(x, p) = \frac{1}{2\pi\hbar} \int_{-\infty}^{\infty} dx' e^{-ipx'/\hbar} \psi^*(x - x')\psi(x + x')$, explicitly for the 1-dimensional Gaussian wave packet $\psi(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{ipx/\hbar} e^{-x^2/4\sigma^2}$ and show that it indeed describes the “joint probability density” for momentum $p$ and position $x$. 