Problem Set 9 - Solution

1) The probability is \( \langle \Phi | \mathcal{H} | \Phi \rangle \), where \( \mathcal{H} = \frac{1}{2m} \left( \frac{p^2}{2m} + V_\gamma \right) \), with

\[
\langle \Phi | \mathcal{H} | \Phi \rangle = \langle \Phi | \mathcal{H} | \Phi \rangle = V_\gamma \Theta(t) \delta(p-p')
\]

we see that for all final states \( p \neq p' \)

this probability is zero, implying zero.

The interpretation in the "sudden approximation" is that due to the fact that initially \( \langle \Phi(0) | \mathcal{H} | \Phi(0) \rangle \) the wave function does not have any time to change. However, after \( t = 0 \) it would change (slowly) with time if it were no longer an eigenstate of \( \mathcal{H} = H_0 + V_\gamma \). But it is:

\[
\frac{-\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V_\gamma e^{ipx} = \left( \frac{p^2}{2m} + V_\gamma \right) e^{ipx}
\]

so it is an eigenstate with different energy eigenvalue, but an eigenstate nevertheless - it will remain unchanged.

2) The functional dependence on time does not change; the fact that \( \langle \Phi | \mathcal{H} | \Phi \rangle = 0 \) at all times, so once again the probability is ZERO!

The argument is only slightly different: in the adiabatic approximation, the wave function will have time to continuously adjust, remaining an eigenstate of \( \mathcal{H}(t) \). But as we showed already, it is an eigenstate of \( \mathcal{H}(t) \) for any value of \( V_\gamma(t) \), so it can remain unchanged indefinitely. (The classical analogy is a marble rolling on an elevator floor - as long as the floor is flat, its energy will increase as the elevator rises, but its motion will not change.)
\[ d_{210} = \frac{eE}{\hbar} \int \left[ e^{i \frac{T}{\hbar}} - e^{-i \frac{T}{\hbar}} \right] e^{i \frac{2\pi}{\hbar} \left( 21w_c | z | 2\theta_0 \right)} e^{i \frac{2\pi}{\hbar} \left( 21w_c | z | 2\theta_0 \right)} dt' \]

First we observe that \( Z \) is the \( T_z^0 \) component of a spinorial rank \(-1\) tensor and therefore \( \Delta m = m_\pm - m_\pm = 0 \). Therefore we only need to calculate for \( W_2 = 0 \); the other \( Z \) probabilities are 0.

From PS 8 we know that \( <210 | z | 200> = -3a_0 \).

Furthermore, \( W_1 = 0 \) since the \( 2 \) states are degenerate under \( H_0 \) (Hydrogen atom, Hamiltonian). Therefore

\[ d_{210} = \frac{-3eEa_0}{\hbar} \frac{T^2}{2\pi} \int_0^T 0 = -\frac{3eEa_0}{\hbar} \frac{T^2}{2\pi} \]

and the probability is \( \frac{9}{4} \left( \frac{Ea_0T}{\hbar} \right)^2 \).

The first order approximation breaks down if \( T \gg \frac{2\pi}{3eEa_0} \).

In fact, we know that the system will reach a new eigenstate if \( d_{210} = \frac{1}{\sqrt{2}} \).

\[ | \psi > = \frac{1}{\sqrt{2}} \left( | 10 > - e^{i\theta} | 01 > \right) \]

\[ = \frac{-i\hbar}{\hbar} <10| \psi > <10| 01 > e^{i\theta} \int \frac{e^{i(2\pi w_c \hbar)/\hbar} e^{-\frac{2\pi}{\hbar} t'}}{\hbar} dt' \]

where \( \omega_c = \frac{\gamma}{\hbar} \beta \).

As shown before, the integral is equal to \( \sqrt{\pi} e \left( \frac{\gamma^2}{\hbar} \right)^{1/2} \) e\left(-\frac{\gamma^2}{\hbar} \right)^{1/2} \).

The transition matrix element is \( \frac{\hbar}{2} \Rightarrow \)

\[ d_{\psi} = \frac{-i\hbar}{\hbar} \left( \frac{\gamma^2}{\hbar} \right)^{1/2} e \left( \frac{\gamma^2}{\hbar} \right)^{1/2} \]

\[ \theta(i \rightarrow A) = \frac{-i\hbar}{\hbar} \left( \frac{\gamma^2}{\hbar} \right)^{1/2} e \left( \frac{\gamma^2}{\hbar} \right)^{1/2} \]

This is (from Eq. 2) zero for very long \( T \) unless \( \omega_p = \gamma \beta \) (classical precession frequency).