Refractive phenomena in the shock wave dispersion with variable gradients

A. Markhotok and S. Popovic[a)]
Department of Physics, Old Dominion University, Norfolk, Virginia, 23529, USA

(Received 9 March 2010; accepted 15 April 2010; published online 16 June 2010)

In this article the refraction effects in the weak shock wave (SW) dispersion on an interface with a temperature variation between two mediums are described. In the case of a finite-gradient boundary, the effect of the SW dispersion is remarkably stronger than in the case of a step change in parameters. In the former case the vertical component of velocity for the transmitted SW (the refraction effect) must be taken into account. Results of comparative calculations based on the two-dimensional model corrected for the refraction effect show significant differences in the shapes of the dispersed SW fronts. © *2010 American Institute of Physics*. [doi:10.1063/1.3432565]

I. INTRODUCTION

Investigations of the shock wave (SW) interaction with a weakly ionized gas have been done intensively in the past decades and have shown promising results with respect to practical benefits for aircraft design and its impact on the environment. The results indicate broad possibilities to modify and control the SW front.[1–3] Any capability to modify the structure of a SW can reduce its strength and significantly decrease the shock induced drag, thus mitigating the effects of shocks in supersonic flight, heating problems during re-entry into the atmosphere, and reducing the effects of sonic booms. The changes in the SW front, such as its curvature, can possibly affect the structure of the detonation wave by affecting the ignition conditions through the dependence on the detonation speed.[4,5]

In Refs. 6–8 a new mechanism of the weak SW dispersion related to the curved shape of the incident SW front or a boundary between two mediums of different temperatures has been described. The need to bring up this mechanism was to explain the odd shapes of the deflection signals from the SW as it entered the hot discharge regions. The advantage of the proposed model is that, to some extent, it offers a rather simple method of calculations to predict the shape of the dispersed shock front. This cuts the computational cost substantially by eliminating the need to employ laborious and sophisticated numerical procedures. Because of the simplicity the results are free of computational ghosts or associated numerical oscillations which may substantially contaminate numerical solutions. The possibility that a shock front incident on a hot plasma region may have a shape that is not necessarily flat but rather has a definite curvature was considered. The curvature of the incident shock front itself, and/or curvature of the boundary between neutral gas and the discharge region, can lead to the change in the shock front shape. This happens because when a shock front crosses a boundary its front and rear parts propagate in areas with different temperatures and hence, different propagation velocities. As a result, the front part of the SW accelerates with respect to the rear. Since, this leads to a modification of the shock front, the described mechanism opens a broad possibility to control weak SW dispersion by employing this single mechanism or, for example, a few different thermal mechanisms.[8] In this article we consider the same effect specifically on a nonsharp interface, i.e., when parameters of the two mediums change across the interface not abruptly but rather over some distance of at least several lengths of the free mean pass of the gas molecules (finite gradient case).

Such a situation appears more realistic for most applications, and what is also important, the SW dispersion in this case becomes stronger. This reduction in strength occurs because when a SW is incident on a sharp interface, some part of the incident shock energy gets reflected back. Thus, the energy of the shock transmitted into the hot discharge area becomes reduced compared to the incident energy. However, when the interface smoothly extends over the final distance, the reflected wave vanishes, and no SW energy loss occurs during its transmission through the interface.[9] More strictly, the smoothness of the gradient can be defined based on the investigations.[10] A plane shock moves into a region of gas which is initially at rest under uniform pressure but with a nonuniform density was considered there. The applicability limits for the gradient smoothness (the characteristic length of the extended boundary) can be determined from the analyzing the solution for a constant shock strength. It was found that no solution exists for a density changing more rapidly than $1/x^2$ where x is the longitudinal coordinate.

The experimental evidence of such a situation can be found in Ref. 11, where no shock reflections were observed off the boundary with a weakly ionized plasma of a laser spark in air. Similarly, no reflected shock off the boundary was seen when a SW was passed through a heated layer of air around a hot tube. The boundary of the heated layer was still clearly observable.

Thus the Mach number of the transmitted shock M_2 will be the same as for the incident SW M_1, i.e., their ratio M_2/M_1 in the dispersion relation[6–8] is equal to the unity. In this case the ratio of the shock velocities V_2 and V_1 in the hot and cold areas, respectively,

[a)]Electronic mail: spopovic@odu.edu.

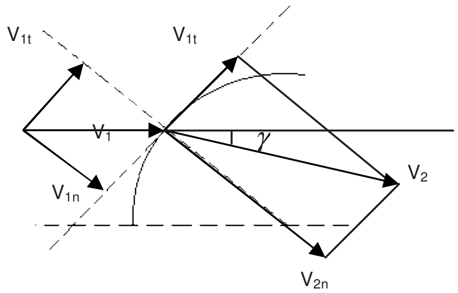

FIG. 1. Velocity components of the planar shock front incident on a plasma and the transmitted shock front. The temperature of plasma T_2 is higher than the temperature of the surrounding gas T_1.

$$V_2/V_1 = \sqrt{T_2/T_1}(M_2/M_1),$$

will be dependent on the ratio of the temperatures only. Since in the case of a sharp interface the Mach numbers ratio is always less than unity, for a smooth interface the dispersion effect becomes significantly stronger and the shape of the weak shock front will undergo greater changes in the longitudinal direction,[7] proportionally to this velocities ratio (V_2/V_1)

$$X_i = (V_2/V_1 - 1)(R - x_i),$$

here R is the radius of the spherical incident shock front or an interface, and X_i and x_i are coordinates of the transmitted and the incident shock fronts (or the interface), Fig. 2.

Stronger dispersion in this case suggests paying more attention to a secondary effect associated with the vertical velocity component arising as the shock penetrates through the interface. This vertical component is responsible for the change in the shock front in the transverse direction and is referred to here as the refraction. In the next part of this article we derive all relevant relations. Then, the model will be applied to numerically calculate the SW dispersion on a plasma sphere. Comparing the results of calculations for the cases of sharp and finite-gradient boundaries demonstrates the strength of the effect.

II. THE REFRACTION EFFECT

The refraction effect in the SW dispersion arises when there is a curvature difference between the incident shock front and the interface[7] between two mediums (1 and 2) with a temperature step T_1/T_2. Thus, the refraction effect is a consequence of the SW dispersion on the boundary. Consider a planar shock front traveling from left to the right and incident on a plasma sphere surrounded by a cold gas (Fig. 1). This is a two-dimensional (2D) problem since each point at the incoming flat shock front is incident on the plasma interface at different angles and only one point at the symmetry axis is incident normally to the surface. Consider a point i on the shock front that is incident on the plasma boundary under an angle α (between the incident velocity V_1 and the normal to the interface).

Since the tangential component of the total velocity of the shock front does not change through the boundary but the normal component does, the total velocity of the shock front changes its direction by an angle γ (Fig. 1)

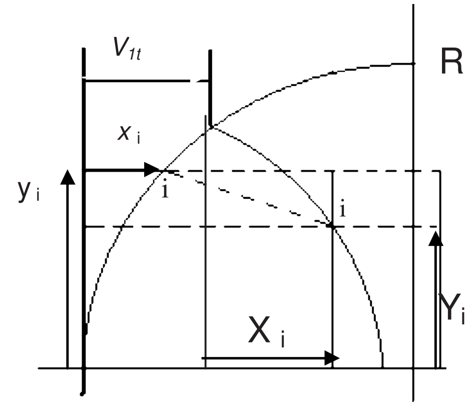

FIG. 2. Schematic diagram for the initially flat shock front change during its penetration of the spherical interface. The incoming flat shock front (in bold) moves from the left to the right. The coordinates (x_i, y_i) and (X_i, Y_i) are for a point i at the interface and the transmitted shock front correspondingly.

$$\gamma = \alpha - \tan^{-1}(V_t/V_{2n}). \tag{1}$$

This effect is stronger at larger incidence angles, so it will affect more far off-axis points on the sphere with no effect on the axis. If the temperature of the plasma region is higher than in the surrounding gas, then the transmitted normal velocity component is higher than the incident, and the total vector deflects toward the longitudinal symmetry axis of the sphere. Then the x-component of the shock front velocity in the plasma region is responsible for the change in the front in the longitudinal direction, and a small y-component induces changes in the vertical (transverse) direction. This y-component of the transmitted velocity is the origin of the phenomena referred to here as refraction, i.e., the deflection of the velocity vector from its initial, incident direction, in the SW dispersion on a curved boundary. This effect is very similar to the optical refraction of a light wave on a boundary between two media in which the light propagation speeds differ.

Then, the derivation for the X-coordinate and Y-coordinate of the dispersed shock front surface at the point i can be done in the same way as it was done in Ref. 7. One can get a relationship between the coordinates of the transmitted shock front (X_i, Y_i) and the boundary surface (x_i, y_i), (Fig. 2)

$$X_i = x_i(V_1 - V_{2x})/V_1 + t(V_{2x} - V_1), \quad Y_i = y_i - V_{2y}\left(t - \frac{x_i}{V_1}\right). \tag{2}$$

In this figure, the shock front (the most left line in bold) is coming from the left to the right and incident on the spherical plasma boundary (one quarter of the boundary is shown). A part of the shock that propagates through the plasma gets curved (the second curved line inside the plasma sphere and crossing the plasma boundary) while the rest of the shock that continues traveling outside the plasma stays flat (the bold line outside of the boundary connected with the curved part of the shock).

Here time t starts at the first contact point between the incident shock front and the plasma boundary. Then, for example, for the time $t=R/V_1$ needed to propagate the SW into the cold area a distance equal to the radius R, the penetrated shock front coordinates are

$$X_i = \left(\frac{V_{2x}}{V_1} - 1\right)(R - x_i) = \left(\frac{V_2}{V_1}\cos\,\gamma - 1\right)(R - x_i),$$

$$Y_i = y_i - \frac{V_{2y}}{V_1}(R - x_i) = y_i - \frac{V_2}{V_1}\sin\,\gamma(R - x_i). \tag{3}$$

To keep further derivations more general, we first derive expressions for the case of a sharp boundary that can be extended for a more simple case of a nonsharp boundary. The ratio of velocities V_2/V_1 in the system (2) can be easily obtained by expressing the velocities components in terms of the incidence angle α, the ratios of the Mach numbers, and the temperatures in two media

$$V_{2n}/V_{1n} = \sqrt{T_2/T_1}(M_{2n}/M_{1n}),$$

$$V_2 = \sqrt{V_{2n}^2 + V_t^2}, \quad V_t = V_1 \sin\,\alpha,$$

$$V_2/V_1 = \sqrt{\cos^2\,\alpha\frac{T_2}{T_1}\left(\frac{M_{2n}}{M_{1n}}\right)^2 + \sin^2\,\alpha},$$

$$\gamma = \alpha - \tan^{-1}\left(\sqrt{\frac{T_1}{T_2}}\cdot\frac{M_{1n}}{M_{2n}}\cdot\tan\,\alpha\right). \tag{4}$$

The ratio of Mach numbers for normal incidence can be obtained using transcendental equation[4]

$$\frac{1}{M_{1n}(k-1)}\{[2kM_{1n}^2-(k-1)][(k-1)M_{1n}^2+2]\}^{1/2} \times\left\{\left[\frac{2kM_{2n}^2-(k-1)}{2kM_{1n}^2-(k-1)}\right]^{\gamma-1/2\gamma}-1\right\} = M_{1n}\left(1-\frac{1}{M_{1n}^2}\right) - M_{2n}\left(1-\frac{1}{M_{2n}^2}\right)\left(\frac{T_2}{T_1}\right)^{1/2}$$

$$M_1^2 = M_{1n}^2 + M_{1t}^2, \quad M_{1n} = M_1\cos\,\alpha, \quad M_{1t} = M_1\sin\,\alpha,$$

$$M_2^2 = M_{2n}^2 + M_{2t}^2, \quad M_{2t} = M_{1t}\sqrt{T_1/T_2}, \tag{5}$$

For a nonsharp interface, the Mach number does not change across the boundary, so the ratio of normal components of the velocities is dependent on the temperature ratio only. This eliminates the solution of the transcendental equation for finding M_{2n}, and slightly simplifies the Eqs. (1)–(4) by correcting them with $M_2/M_1=1$.

III. NUMERICAL RESULTS FOR SW DISPERSION ON A PLASMA SPHERE AND COMPARISON WITH THE EXPERIMENT

The goal of the calculations below is to see how the effect of the Y-component of the transmitted velocity (the refraction effect) influences the shape of the shock front dispersed on an interface. For comparison, we present curves obtained using a 2D approach, first accounting and then not accounting for the refraction effect in the case of a sharp boundary. We also made calculations for the case of a nonsharp boundary, when the dispersion effect becomes stronger. In this case the angle γ also gets larger, so the refraction effect is expected to have a greater influence on the curves.

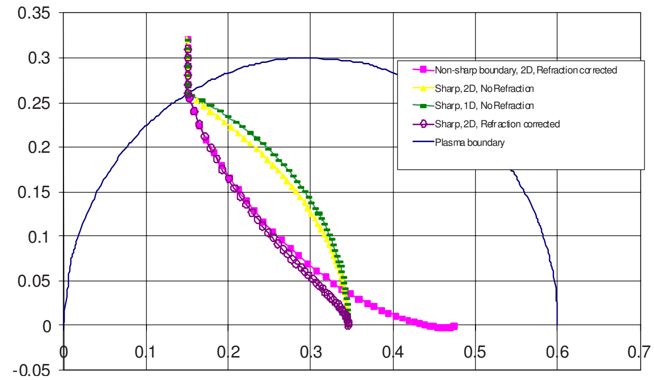

FIG. 3. (Color online) SW dispersion on a sharp and nonsharp boundary. $M=1.9$, $T_2=300$ K, $T_2/T_1=10$.

We apply our calculations to the conditions of the experiment described in Ref. 12. The scheme for these experimental results also can be found in Ref. 7 where we used it to validate our model. In this experiment, a spherically shaped plasma region was generated in air by focusing a laser beam with pulse duration of 20 ns. Planar SWs generated by an electrically exploded dielectric film were incident on the plasma region. The incident SW's Mach number used in these calculations was taken to be $M_1=1.9$, which corresponds to the range of the experimental values. We also intentionally used highest temperature achieved in the experiment for the plasma ($T_2=3000$ K), and a moderate temperature for the surrounding gas ($T_1=300$ K) to obtain a more pronounced effect. The adiabatic coefficient in the Eqs. (5) was $k=1.4$.

The results for the change in the SW front as it crosses the spherical interface between the cold gas and the hot plasma are presented in the Fig. 3. The presented curves correspond to the instant of time when the shock front traveled in the cold gas a distance equal to the half of the radius of the sphere.

As it is seen from the figure, the correction for the refraction effect, as it is defined in the introduction to this paper, makes a principal difference in the curve shapes. Instead of spherical, the front is now bent to the opposite side displaying the presence of the inflection points on its surface (2D sharp refraction corrected and 2D sharp refraction noncorrected curves, pink squares, and yellow triangles, respectively). We also present a curve obtained using a one-dimensional (1D) approach [using 1D version of formula (5) for M_2], which does not account for the refraction effect. The 1D and 2D curves which do not account for the refraction effect show rather slight differences, even though considering that the temperature step used in the calculations is rather high.

For a nonsharp temperature change across the boundary (finite gradient), the stronger effect of SW dispersion is easily seen in the more stretched shock front (2D sharp and 2D nonsharp, both refraction corrected, pink squares, and purple circles, respectively) and the remarkably different shape of the front. The refraction effect is more pronounced, so the inflection point disappears and the sign of the curvature of the front turns to the opposite. Thus the sign of the curvature of the dispersed front makes a transition from positive to negative through the curve with the inflection points. When at the very front points of the nonsharp interface curve, it is seen that they cross the longitudinal axis of the symmetry, effectively collapsing the front. As the interaction time increases, its further evolution will follow the usual scheme of the shock–shock interaction with following crossed fronts. Such a situation is possible if the temperature step is high enough and if the interaction time is long enough to fully develop the front. It should be noted that when the shock front exits the sphere, the temperature step flips to the opposite, so the total vector of the transmitted velocity deflects off the symmetry axis. Further development of the front shape will proceed in the opposite direction, accounting for other types of interactions at the interface, which depend on the shape of the transmitted shock front.[7]

Such a collapse of the shock front shape was not observed in the experiment.[12] The reasons could be that the temperatures were rather overestimated, or the measured shock shape inside the plasma region of Ref. 12 is not clearly resolved. In principle, with better resolved pictures, a conclusion about the actual sharpness of the interface can be implicitly derived from the strength of the observed refraction effect. In the case of Ref. 12, the refraction effect could have been reduced because of sharp boundaries, to the point where the front did not develop enough to see the changes.

It is known that the sign of the temperature step across two media (slow-fast or fast-slow) and the sort of gases involved, can cause the SW refraction on an interface to follow the so called irregular path. In this case, the formation of a precursor as a part of a four-wave system can occur, with the following distortions of the inhomogeneous volume, development of instabilities, and vortex formation. But in the case of a finite gradient no reflections off the interface will be present and hence, no irregularities of the above mentioned type should occur during the SW dispersion.

IV. CONCLUSION

The refraction effect in the SW dispersion is a direct consequence of the SW dispersion on an interface. It happens on both sharp and finite-gradient interfaces. The refraction phenomena in the SW dispersion consist of the deflection of the total transmitted velocity component toward the longitudinal axis of symmetry or off this axis, depending on the sign in the temperature difference across the boundary. This effect represents a direct analogy to the optical refraction of a light wave on a boundary between two media in which light propagation velocities differ, though the nature of the phenomena is different.

The finite-gradient boundary makes the dispersion effect stronger and consequently the refraction effect too. The calculations become even easier, mostly because of no need to solve for the transmitted Mach number M_2 in the transcendental equation. Thus the equations for the SW front shape can be derived explicitly to its final expressions.

Comparative calculations show significant difference in the shock front shapes if for the refraction effect is considered. The SW refraction leads to variable effects including not only the stretching of the front surface but also its collapse or spreading off the axis.

The findings of this article could remarkably improve the possibility of controlling the SW dispersion. Accounting for the refraction effect can significantly improve the predictions for the shock front shapes, as well as allow getting indirect information about the degree of sharpness of a boundary in experiments.

[1]F. Marconi, AIAA Pap. **0333**, 1998 (1998).
[2]S. P. Kuo and D. Bivolaru, Phys. Plasmas **8**, 3258 (2001).
[3]V. P. Gordeev, A. V. Krasilnikov, V. I. Lagutin, and V. V. Otmennikov, Fluid Dyn. **31**, 313 (1996).
[4]R. Klein and D. Stewart, SIAM J. Appl. Math. **53**, 1401 (1993).
[5]G. Sharpe, Combust. Flame **123**, 68 (2000).
[6]A. Markhotok, S. Popovic, and L. Vuskovic, Proceedings of the 15th International Conference on Atomic Processes in Plasmas, NIST, Gaithersburg, MD, 19–22 March, 2007 (unpublished).
[7]A. Markhotok, S. Popovic, and L. Vuskovic, Phys. Plasmas **15**, 032103 (2008).
[8]A. Markhotok, S. Popovic, and L. Vuskovic, AIAA Pap. **3896**, 2008 (2008).
[9]B. Shirinzadeh, G. C. Herring, and R. J. Exton, AIAA J. **39**, 1210 (2001).
[10]S. W. Jones, Proc. R. Soc. London, Ser. A **228**, 82 (1955).
[11]E. M. Barkhudarov, V. R. Berezovskii, M. O. Mdivnishvili, M. I. Taktakishvili, N. L. Tsinadze, and T. Y. Chelidze, Sov. Tech. Phys. Lett. **10**, 498 (1984).
[12]A. F. Alexandrov, N. G. Vidiakin, V. A. Lakutin, M. G. Skvortsov, and I. B. Timofeev, Sov. Phys. Tech. Phys. **31**, 468 (1986).