motion along a straight line
“displacement” & “distance”

→ we need to be a bit pedantic here:

→ ‘distance’ = total ground covered while traveling, e.g. odometer reading

→ ‘displacement’ = vector from where you started to where you end up

\[ \Delta x = x_f - x_i \]

\[ \Delta x = 10 \text{ m} - 0 \text{ m} \]

\[ \Delta x = +10 \text{ m} \quad \text{sign indicates the direction} \]

\& distance = 10 m

\[ \Delta x = x_f - x_i \]

\[ \Delta x = 0 \text{ m} - 10 \text{ m} \]

\[ \Delta x = -10 \text{ m} \]

but distance = 10 m
“displacement” & “distance”

→ displacement and distance can be quite different

\[ \Delta x = (+8\, \text{m}) + (-6\, \text{m}) + (+8\, \text{m}) = +10\, \text{m} \]

\[ \Delta x = +10\, \text{m} \]

Distance = \(8\, \text{m} + 6\, \text{m} + 8\, \text{m} = 22\, \text{m} \)

‘vector’ sum (sum with signs)
average velocity

⇒ just the displacement divided by the time taken

\[ v_{av} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} \]

e.g. driving a car along a long straight stretch of road

\[ \Delta x = 272 \text{ m} - 52 \text{ m} = +220 \text{ m} \]
\[ \Delta t = 30.17 \text{ s} - 26.05 \text{ s} = 4.12 \text{ s} \]

\[ v_{av} = \frac{\Delta x}{\Delta t} = \frac{+220 \text{ m}}{4.12 \text{ s}} = +53.4 \text{ m/s} \]
average velocity

→ suppose we go the other way (but define the x-axis the same way)

\[ \Delta x = 52 \text{ m} - 272 \text{ m} = -220 \text{ m} \]

\[ \Delta t = 102.05 \text{ s} - 93.88 \text{ s} = 8.17 \text{ s} \]

\[ v_{av} = \frac{\Delta x}{\Delta t} = \frac{-220 \text{ m}}{8.17 \text{ s}} = -26.9 \text{ m/s} \]

negative sign means opposite the axis direction
average & instantaneous velocity

→ let’s make a plot of the position of an object as a function of time
compute the average velocity between $t = 1.00 \text{ s}$ & $t = 5.00 \text{ s}$

$x = 25.00 \text{ m}$

$x = 1.00 \text{ m}$

$\Delta t = 4.00 \text{ s}$

$\Delta x = 24.00 \text{ m}$

$v_{av} = \frac{\Delta x}{\Delta t} = \frac{24.00 \text{ m}}{4.00 \text{ s}} = 6.00 \text{ m/s}$
average & instantaneous velocity

compute the average velocity between $t = 2.00 \, \text{s}$ & $t = 5.00 \, \text{s}$

\[ \Delta t = 3.00 \, \text{s} \]

\[ \Delta x = 20.00 \, \text{m} \]

\[ v_{av} = \frac{\Delta x}{\Delta t} = \frac{20.00 \, \text{m}}{3.00 \, \text{s}} = 6.67 \, \text{m/s} \]
average & instantaneous velocity

→ compute the average velocity between $t = ?$ & $t = 5.00$ s

? is this getting closer to the slope of the curve at $t=5$s?
compute the average velocity between $t = ?$ & $t = 5.00$ s

as $\Delta t \to 0$, we approach the slope of the curve
instantaneous velocity

the velocity “at an instant in time” is defined to be

\[ v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \]

and we just saw that it corresponds to the slope of the \( x-t \) curve
instantaneous velocity

$v > 0$

“moving to larger x”

$v = 0$

“not moving”

$v < 0$

“moving to smaller x”
acceleration

→ if an object’s velocity changes, it has undergone an acceleration

→ we can define average acceleration

\[ a_{av} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i} \]

e.g. driving a car along a long straight stretch of road

\[ v = 0 \text{ m.p.h} \]

\[ v_f = 60 \text{ m.p.h} = 60 \frac{\text{mi}}{\text{hr}} \times \left( \frac{1609 \text{ m}}{1 \text{ mi}} \right) \times \left( \frac{1 \text{ hr}}{3600 \text{ s}} \right) = 27 \text{ m/s} \]

\[ v_i = 0 \text{ m/s} \]

\[ \Delta t = 5.50 \text{ s} \]

\[ a_{av} = +4.9 \text{ m/s}^2 \]

positive sign means velocity is increasing
acceleration - meaning of the sign

\[ a_{av} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i} \]

\[ v_f - v_i > 0 \quad a_{av} > 0 \]

\[ v_f - v_i < 0 \quad a_{av} < 0 \]
acceleration - meaning of the sign

\[ a_{av} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i} \]

\[ v_f - v_i < 0 \quad a_{av} < 0 \]

\[ v_f - v_i > 0 \quad a_{av} > 0 \]

so be careful, the sign of the acceleration doesn’t just mean speeding up / slowing down
we define instantaneous acceleration

\[ a = \lim_{{\Delta t \to 0}} \frac{\Delta v}{\Delta t} \]

which is the slope of the \( v-t \) curve
instantaneous acceleration
motion with constant acceleration

→ simple, but very important, example of a particle being accelerated \( a = \text{const} \)

\[
a = \frac{v - v_0}{t - 0} = \frac{v - v_0}{t}
\]

\( a = \text{const} \)

@ \( t=0 \), velocity = \( v_0 \)

@ \( t \), velocity = \( v \)

\[ v = v_0 + at \]

→ the constant (positive) acceleration is causing the velocity to increase at a constant rate

e.g. if \( a=+10 \text{ m/s}^2 \),

in 1 sec, \( v \) increases by 10 m/s

in another 1 sec, \( v \) increases by another 10 m/s ...
suppose you’re driving on the highway at 17 m/s and you press the accelerator to accelerate at a constant 3.0 m/s$^2$. After 3.0 seconds, what is your speed?

\[
v = v_0 + at
\]

\[
= 17 \text{ m/s} + (3.0 \text{ m/s}^2 \times 3.0 \text{ s})
\]

\[
= 17 \text{ m/s} + 9.0 \text{ m/s}
\]

\[
v = 26 \text{ m/s}
\]
we’d like to get an equation for the position as a function of time
we can figure it out (“derive” it)

first a simpler example - constant velocity

\[ v = \text{const} = \frac{x - x_0}{t - 0} \]

\[ x = x_0 + vt \]

notice that the change in \( x \) is
the area under the \( v-t \) graph

area = \( v \times t \)
it is generally true that the change in position is the area under the $v$-$t$ graph

constant acceleration

\[ x = x_0 + \frac{1}{2} (v + v_0) t \]

\[ v = v_0 + at \]

\[ x = x_0 + v_0 t + \frac{1}{2} at^2 \]
it is generally true that the change in position is the area under the \(v-t\) graph.

\[
x = x_0 + v_0 t + \frac{1}{2} a t^2
\]

\(x-t\) curve is a parabola.
probable useful for you to remember the following equations

\[ v = v_0 + at \]
\[ x = x_0 + v_0 t + \frac{1}{2} at^2 \]
\[ v^2 = v_0^2 + 2a(x - x_0) \]
\[ x = x_0 + \frac{1}{2} (v_0 + v) t \]
entering the freeway

A sports car is sitting at rest on a freeway entrance ramp. The driver sees a break in traffic and floors the gas pedal, so that the car accelerates at a constant 4.9 m/s\(^2\) as it moves in a straight line onto the freeway. What distance does the car travel in reaching a freeway speed of 30 m/s?

\[
v_0 = 0 \\
v = 30 \text{ m/s} \\
a = 4.9 \text{ m/s}^2
\]

Define \(x_0 = 0\)

\[
x = ? \quad \text{solve for this}
\]

\[
t = ?
\]

\[
x - x_0 = \frac{v^2 - v_0^2}{2a}
\]

\[
x = \frac{(30 \text{ m/s})^2}{2 \times 4.9 \text{ m/s}^2} = 92 \text{ m}
\]
A motorist traveling at a constant velocity of 15 m/s passes a school-crossing corner where the speed limit is 10 m/s. A police office on a motorcycle stopped at the corner starts off in pursuit with constant acceleration of 3.0 m/s². How much time elapses before the officer catches up with the car?

We want to know when the POLICE and the CAR are at the same location.

\[ x_{P0} = 0 \]
\[ v_{P0} = 0 \]
\[ a_P = 3.0 \text{ m/s}^2 \]
\[ x_{C0} = 0 \]
\[ v_{C0} = 15 \text{ m/s} \]
\[ a_C = 0 \]

\[ x_P = x_C \]
\[ t = ? \]
we want to know when the POLICE and the CAR are at the same location

\[ x_P = x_C \]

\[ t = ? \]
pursuit 2 - the panicking motorist

A motorist traveling at a constant velocity of 15 m/s passes a school-crossing corner where the speed limit is 10 m/s. A police office on a motorcycle stopped at the corner starts off in pursuit with constant acceleration of 3.0 m/s².

This time the motorist sees the cop and applies the brakes as he passes the corner, causing a constant acceleration of -2.0 m/s².

How much time elapses before the officer catches up with the car?

\[
\begin{align*}
\text{POLICE} \\
x_{P0} & = 0 \\
v_{P0} & = 0 \\
a_P & = 3.0 \text{ m/s}^2 \\
\end{align*}
\]

\[
\begin{align*}
\text{CAR} \\
x_{C0} & = 0 \\
v_{C0} & = 15 \text{ m/s} \\
a_C & = -2.0 \text{ m/s}^2 \\
\end{align*}
\]

we want to know when the POLICE and the CAR are at the same location

\[
\begin{align*}
x_P & = \frac{1}{2} a_P t^2 \\
x_C & = v_{C0} t + \frac{1}{2} a_C t^2 \\
\frac{1}{2} a_P t^2 & = v_{C0} t + \frac{1}{2} a_C t^2 \\
0 & = v_{C0} t + \frac{1}{2} (a_C - a_P) t^2 \\
t & = 0, \frac{2v_{C0}}{a_P - a_C} \\
t & = 3 \text{ s}
\end{align*}
\]
free fall

if we neglect the effect of air, objects dropped or thrown vertically up or down **accelerate at a constant rate**

objects accelerate toward the center of the Earth due to gravity, which we’ll explore later in this course

now just because I tell you this, doesn’t mean it is true! - it’s a **theory** that needs to be tested by doing **experiments**
Free fall experiment - is the acceleration constant?

High-speed photography of a ball falling in a vacuum chamber - a shot every $\Delta t$ seconds.

Measure how far the ball has travelled in each $\Delta t$ seconds.

For constant acceleration, should increase linearly with $t$,

$$y(t) = y_0 + \frac{1}{2} at^2$$

$$y(t + \Delta t) = y_0 + \frac{1}{2} a(t + \Delta t)^2$$

$$\Delta y = y(t + \Delta t) - y(t)$$

$$= (\frac{1}{2} a \Delta t^2) + (\frac{1}{2} a \Delta t) t$$

The data suggests constant acceleration.

With precise measurements, we find $|a| = 9.80 \text{ m/s}^2$. 

Physics 111N
free fall

turns out all objects accelerate at the same rate
e.g. drop an apple versus drop a feather

“no way!”, you’d say, “a feather will float downwards, an apple will drop”

true, but this is a property of the air surrounding the feather
remove the issue of air resistance - do the experiment in vacuum

or a somewhat more expensive experiment ...
free fall - hammer & feather on the moon
Suppose you were to drop a pumpkin from the top of a 40m high building. Neglecting air resistance, how long does it take for the pumpkin to reach the ground and how fast is it moving when it gets there?

**Moment of release**
- \( t = 0 \)
- \( y_0 = 40 \text{ m} \)
- \( v_0 = 0 \text{ m/s} \)

**Reaches ground**
- \( t = ? \)
- \( y = 0 \text{ m} \)
- \( v = ? \)

At all times
- \( a = -9.80 \text{ m/s}^2 \)

\[
y = y_0 + v_0 t + \frac{1}{2} a t^2
\]

\[
0 = y_0 + \frac{1}{2} a t^2
\]

\[
t = \sqrt{-\frac{2y_0}{a}} \quad t = 2.9 \text{ s}
\]
pumpkin throw

Suppose you were to throw a pumpkin vertically upward from the top of a 40m high building at 10 m/s. Neglecting air resistance, what is the maximum height above the ground reached by the pumpkin and how long after release does it reach this point? When does the pumpkin reach the ground?

moment of release
\[ t = 0 \]
\[ y_0 = 40 \text{ m} \]
\[ v_0 = 10 \text{ m/s} \]

highest point
\[ v = 0 \text{ m/s} \]
\[ t = ? \]
\[ y = ? \]

at all times
\[ a = -9.80 \text{ m/s}^2 \]

\[ v^2 = v_0^2 + 2a(y - y_0) \]
\[ 0 = v_0^2 + 2a(y - y_0) \]
\[ y = y_0 - \frac{v_0^2}{2a} \]

\[ y = 45 \text{ m} \]
Suppose you were to throw a pumpkin vertically upward from the top of a 40m high building at 10 m/s. Neglecting air resistance, what is the maximum height above the ground reached by the pumpkin and how long after release does it reach this point? When does the pumpkin reach the ground?

\[
\begin{align*}
\text{moment of release} & \quad t = 0 \\
& \quad y_0 = 40 \text{ m} \\
& \quad v_0 = 10 \text{ m/s} \\
\text{highest point} & \quad v = 0 \text{ m/s} \\
& \quad t = ? \\
& \quad y = ? \\
& \quad a = -9.80 \text{ m/s}^2 \\
& \quad v = v_0 + at \\
& \quad 0 = v_0 + at \\
& \quad t = \frac{-v_0}{a} \\
& \quad t = 1.0 \text{ s}
\end{align*}
\]
pumpkin throw

Suppose you were to throw a pumpkin vertically upward from the top of a 40m high building at 10 m/s. Neglecting air resistance, what is the maximum height above the ground reached by the pumpkin and how long after release does it reach this point? When does the pumpkin reach the ground?

moment of release
$t = 0$
$y_0 = 40 \text{ m}$
$v_0 = 10 \text{ m/s}$

reaches the ground
$y = 0 \text{ m}$
$t = ?$
$v = ?$

at all times
$a = -9.80 \text{ m/s}^2$

$y = y_0 + v_0 t + \frac{1}{2} a t^2$

$0 = y_0 + v_0 t + \frac{1}{2} a t^2$

solve a quadratic!

\[ t = \frac{1}{a} \left( -v_0 \pm \sqrt{v_0^2 - 2ay_0} \right) \]

\[ t = -2.0, 4.1 \text{ s} \]