circular motion
& gravitation
uniform circular motion

an object moving around a circle at a constant rate

must have an acceleration always perpendicular to the velocity (else the speed would change)

the velocity is clearly tangent to the circle (or it would move off the circle)

hence the acceleration points always toward the center of the circle - “centripetal acceleration”

\[ a_{\text{rad}} = \frac{v^2}{r} \]
circular motion

→ velocity is tangent to the curve - can see it by cutting the rope
uniform circular motion

→ acceleration is of constant magnitude and directed toward the circle’s center

→ something must provide the force

e.g. ball moving on a frictionless plane tethered by a string to a fixed point

viewed side on: tension in the string provides a force always pointing toward the center of the circle
uniform circular motion

- acceleration is of constant magnitude and directed toward the circle’s center

- something must provide the force

  e.g. a conical pendulum

  the horizontal component of tension in the string provides a force always pointing toward the center of the circle
uniform circular motion

→ acceleration is of constant magnitude and directed toward the circle’s center

→ something must provide the force

e.g. car rounding a curve

\[ \text{friction} \] between tires & road provides a force pointing toward the center of the circle

(a) Car rounding flat curve

(b) Free-body diagram of car
the Earth is in an orbit around the Sun that is very close to a circle
but there is no string joining the Earth to the Sun
nor is there anything to have friction against
what force is holding the Earth in a circular orbit?

gravity
Newton’s law of Gravitation

From astronomical observations and precise lab measurements we infer that the force of gravity between two bodies of mass $m_1$ and $m_2$ whose centers are separated by a distance $r$ is

$$F_g = G \frac{m_1 m_2}{r^2}$$

where there is a universal constant that controls the strength of gravitational attraction

$$G = 6.674 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$$

notice Newton’s third law at work here
the scale of gravitation

Consider two spheres having mass 5.00 g and 1.00 kg whose centers are separated by 10.00 cm. Two such spheres could be used in a lab experiment called a Cavendish balance.

\[ F_g = G \frac{m_1 m_2}{r^2} \]

\[ G = 6.674 \times 10^{-11} \text{ N m}^2/\text{kg}^2 \]

\[ F_g = 6.674 \times 10^{-11} \text{ N m}^2/\text{kg}^2 \times \frac{5.00 \times 10^{-3} \text{ kg} \times 1.00 \text{ kg}}{(10.00 \times 10^{-2} \text{ m})^2} \]

\[ = 33.37 \times 10^{-11-3+2} \text{ N} \]

\[ = 3.34 \times 10^{-11} \text{ N} \]

the force is extremely small!

Gravity is a very weak force, but it always adds up - the more mass a body has, the larger the gravitational pull it can exert on other masses.

Thus gravity becomes important if at least one of the two bodies under consideration is very massive:

- e.g. the Earth & you
- e.g. the Sun and the Earth
weight

Recall that earlier we said that all objects free-fall with an acceleration of \( g = 9.80 \, \text{m/s}^2 \) due to their weight of magnitude \( w = mg \).

But now we have a more complete formalism for gravitation. Can we see where this comes from?

Gravitational force from the Earth on an object of mass \( m \) located close to the surface of the Earth,

\[
F_g = G \frac{m_1 m_2}{r^2}
\]

where

\[
G = 6.674 \times 10^{-11} \, \text{Nm}^2/\text{kg}^2
\]

But this is the weight force \( w = mg = F_g \).

Thus

\[
g = \frac{G m E}{R_E^2}
\]

which is independent of the mass of the body and depends only on the mass and radius of the Earth.

But now we see that as we get further away from the surface of the Earth, the weight will get smaller!
the Earth orbits the Sun, the Moon orbits the Earth, GPS, TV, spy... satellites orbit the Earth...

they are all examples of satellites where the only important force is gravitational attraction

so why doesn’t the moon plummet toward the Earth given that it is accelerating toward it?
consider a ‘thought’ experiment where the moon got into its orbit by being launched from a huge platform

suppose we ‘dropped’ the moon with negligible tangential velocity

the moon would fall straight ‘down’ to the Earth
satellites

consider a ‘thought’ experiment where the moon got into its orbit by being launched from a huge platform

suppose we fire the moon with increasing tangential velocity

Earth

Moon
suppose a satellite is found to be in a circular orbit, what do Newton’s laws say about the motion?

forces: (just gravity)

\[ \sum \vec{F} = \vec{F}_g \] directed radially inward

\[ F_g = G \frac{mM}{r^2} \]

acceleration: (in circular motion)

\[ a_{rad} = \frac{v^2}{r} \]

\[ G \frac{mM}{r^2} = m \frac{v^2}{r} \quad \implies \quad v = \sqrt{\frac{GM}{r}} \]

relation to the orbital period:

\[ v = \frac{2\pi r}{T} \quad \implies \quad T = \frac{2\pi r^{3/2}}{\sqrt{GM}} \]
weightless in the ISS

the International Space Station orbits the Earth every 91 minutes at a distance of 353 km above the surface of the Earth

\[ F_g = G \frac{m_1 m_2}{r^2} \]

\[ G = 6.674 \times 10^{-11} \text{ Nm}^2/\text{kg}^2 \]

\[ R_E = 6380 \text{ km} \]

\[ m_E = 5.98 \times 10^{24} \text{ kg} \]