momentum
momentum

→ useful quantity for analysing situations where objects collide

→ a particle of mass $m$ moving with velocity $\vec{v}$ has momentum $\vec{p} = m\vec{v}$

→ Newton’s 2nd law can be rewritten in a more general form using momentum

$$\sum \vec{F} = \lim_{\Delta t \to 0} \frac{\Delta \vec{p}}{\Delta t}$$

so the action of forces is to change the momentum of a particle

(a constant force changes the momentum at a constant rate)
total momentum

\[ \vec{P} = \vec{p}_A + \vec{p}_B + \vec{p}_C \]

- the total momentum of a system of particles is the vector sum
conservation of momentum

→ momentum is useful because of the following conservation law

The total momentum of a system is constant whenever there is no net external force acting on it

→ it is easy to see that this is true using Newton’s laws

\[
\sum \vec{F} = \lim_{\Delta t \to 0} \frac{\Delta \vec{p}}{\Delta t}
\]

applied to a system with no net external force \(\Rightarrow\) \(\Delta \vec{P} = \vec{0}\)

internal forces “cancel each other out”

suppose particle A exerts a force \(\vec{F}\) on particle B - will change B’s momentum

but Newton’s third law says that B exerts an equal and opposite force \(-\vec{F}\) on A

and this will change A’s momentum at an equal rate & in the opposite direction

so overall \(\vec{p}_A + \vec{p}_B\) will be unchanged
A marksman holds a 3.00 kg rifle loosely, allowing it to recoil freely when fired, and fires a bullet of mass 5.00 g horizontally with speed $v_B = 300$ m/s. What is the recoil speed of the rifle?

Before firing: $P = 0$

After firing: $P = m_B v_B + m_G v_G$

No external forces
⇒ Conservation of momentum holds

“Total momentum before = total momentum after”

$$0 = m_B v_B + m_G v_G$$

$$v_G = - \frac{m_B}{m_G} v_B$$

$$v_G = \frac{-5.00 \times 10^{-3} \text{ kg}}{3.00 \text{ kg}} \times 300 \text{ m/s}$$

$$v_G = -0.500 \text{ m/s}$$
two gliders (carts) move toward each other on a linear air-track, which we assume to be frictionless. Glider A has a mass of 0.50 kg, and glider B a mass of 0.30 kg. Both gliders move with an initial speed of 2.0 m/s. After they collide, glider B moves away with a final velocity of +2.0 m/s. What is the final velocity of A?

\[ P = m_A v_A + m_B v_B \]

before collision:
\[ P_i = (0.50 \text{ kg})(+2.0 \text{ m/s}) + (0.30 \text{ kg})(-2.0 \text{ m/s}) \]
\[ P_i = +0.40 \text{ kg m/s} \]

after collision:
\[ P_f = (0.50 \text{ kg})(v_A') + (0.30 \text{ kg})(+2.0 \text{ m/s}) \]

with no external forces acting (e.g. no friction), momentum is conserved:
\[ P_f = P_i \]

\[ v_A' = -0.40 \text{ m/s} \]
a head-on collision again

two gliders (carts) move toward each other on a linear air-track, which we assume to be frictionless. Glider A has a mass of 0.50 kg, and glider B a mass of 0.30 kg. Both gliders move with an initial speed of 2.0 m/s. After they collide, glider B moves away with a final velocity of +2.0 m/s. What is the final velocity of A?

\[ v'_A = -0.40 \text{ m/s} \]

is kinetic energy conserved in this collision?

\[ E = K_A + K_B \]
\[ E = \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2 \]

before collision: \[ E_i = 1.6 \text{ J} \]

after collision: \[ E_f = 0.64 \text{ J} \]

\[ E_f < E_i \]

this is an ‘inelastic’ collision, some energy has been transferred away from kinetic (heat, sound ...)
inelastic & elastic collisions

the largest amount of energy ‘loss’ occurs in a ‘completely inelastic collision’
in such a collision, the objects stick together

(a) Before collision

The gliders stick together

(b) Completely inelastic collision

The system of the two gliders has less kinetic energy after the collision than before it.

(c) After collision

collisions in which no energy is ‘lost’ are known as ‘elastic’

(a) Before collision

Kinetic energy is stored as potential energy in compressed springs.

(b) Elastic collision

The system of the two gliders has the same kinetic energy after the collision as before it.

(c) After collision