Core Collapse Supernovae

• Types Ib, Ic: no H lines due to shell loss (type Ia already discussed)
  Type II: Plenty of H left (standard end stage of SG)

• Onion (25 $M_\odot$):
  – H-He-C-O-Ne-Mg-Si shells
  – Final state: inert Iron/Nickel core
    -> no more energy available from nuclear fusion
    -> contraction, degeneracy pressure -> core collapse
       (1 $M_\odot$ collapses from Earth size to 10’ s of km in < 1 sec!)

• Core heats up, contracts (Chandrasekhar limit!) =>
  – photo-dissociation (nucleus -> nucleons)
  – e- capture: $e^- + p \rightarrow n + \nu_e$, n decay => large energy loss ($\nu$’ s)!
  – core collapse (seconds!), shockwave, neutrino death ray, sudden
    luminosity increase (3x10$^9$ $L_\odot$ for weeks), blown off outer layer
  – Light fall-off controlled by nuclear decays
  – Huge number of n’ s -> r process
Final Stage of Super Giants:

Supernovae Remnants

SN1994D (NGC4526)

Crab Nebula

SN 1006
Supernova remnant

• Neutron star:
  – nearly no p’ s, e’ s, just neutrons
  – Remember: \( R_{\text{white dwarf}} \propto \frac{1}{m_e} \frac{M}{M_\odot}^{-1/3} \)
  – \( m_n = 1840 \ m_e \Rightarrow R \ \text{1840 times smaller (really, about 500 times because only 1 e- per 2 neutrons)} \Rightarrow \text{of order 10 km!} \)
  – Density: few \( 10^{44}/m^3 = 1/fm^3 > \) nuclear density \( \Rightarrow \) nucleus with mass number \( A = 10^{57} \)
  – Chandrasekar limit: 5 solar masses (2-3 in reality?)
  – Lots depends on nuclear equation of state *, general relativity

*) Repulsive core / Nuclear superfluid / quark-gluon plasma / strange matter?
Supernova remnants

• Some new ideas:
  – Superfluid center
  – Partial deconfinement (cold plasma)
  – s quark matter (now ruled out?)
Pulsars

• Sources of periodic radio emission \((T = 0.001 - 1 \text{ s})\)
  – Example: Crab pulsar \(T = 33 \text{ ms}, \omega = 190/\text{s}\) (1/trillion precision!)
  – Frequency slowing down slowly over time
  – Rotation? Requires \(GM/R^2 > R \omega^2 \Rightarrow R < 3\sqrt[3]{\frac{GM}{\omega^2}} \approx 155 \text{ km}\)
    assuming 1 solar mass => neutron star!
  – Why so fast? Angular momentum conservation: \(5 \cdot 10^4\) times smaller radius -> \(25 \cdot 10^8\) times larger \(\omega\) => from months to ms
  – Source of radio waves: rotating magnetic dipole of order \(10^8\text{-}10^{10} \text{ T}\)
  – Why so huge? 2 arguments:
    • field@surface \(\propto\) magnetic moment/R\(^3\) \(\propto\) angular momentum (conserved)
    • \(-d\Phi/dt = \text{EMF}, \text{Lenz’ law},\) plenty of free charges (plasma) => \(\Phi\) remains constant during collapse => \(B\) increases like \(1/R^2\)
Introduction: Special Relativity

• Observation: The speed \( c \) (e.g., the speed of light) is the same in all coordinate systems (i.e. an object moving with \( c \) in \( S \) will be moving with \( c \) in \( S' \))

• Therefore: If \( |\Delta \vec{r}| = c \Delta t \Rightarrow (c \Delta t)^2 - (\Delta \vec{r})^2 = 0 \) is valid in one coordinate system, it should be valid in all coordinate systems!

• => Introduce 4-dimensional “space-time” coordinates:
  \( x^0 = ct; (x^1, x^2, x^3) = \vec{r} \)

• => Introduce “metric” \( g \) that defines the “distance” between any 2 space-time points (using Einstein’s summation convention) as
  \( (\Delta s)^2 = g_{\mu \nu} \Delta x^\mu \Delta x^\nu \); \( g_{00} = 1, g_{11} = g_{22} = g_{33} = -1, \) all others = 0

• Postulate that all products between 2 vectors and the metric is invariant (the same in all coordinate systems)

• Meaning? If \( |\Delta ct| > \Delta r \), for a moving object, then there is one system \( S_0 \) where \( \Delta r = 0 \Rightarrow \) rest frame for that object. => \( \sqrt{(ds)^2} \) is the time elapsed in \( S_0 \) between the 2 points (“Eigentime”)

Examples

• Object moving (relative to $S$) with speed $v$ along $x$. “Distance” between point $1 = (0,0,0,0)$ (origin) and point $2 = (ct, vt, 0, 0)$:

$$(\Delta s)^2 = g_{00}(ct)^2 + g_{11}(vt)^2 + g_{22}0^2 + g_{33}0^2 = (ct)^2 - (vt)^2 = (c\tau)^2$$

where $\tau$ is the “eigentime” (time elapsed between the two points in the frame $S_0$ where the object is at rest - i.e. the system moving with $v$ along $x$-axis) => $\tau = t \cdot \sqrt{1 - v^2 / c^2} = \gamma^{-1}t$

• Consequence: As seen from $S$, the clock in $S_0$ is “going slow”!
  - From point of view of $S_0$, it is the clock in $S$ that is going slow!
  - With similar arguments, one can prove “length contraction”, “relativity of synchronicity” and all the other “relativity weirdness”

• Argument can be extended to other quantities: All must come as 4-vectors or as invariant scalars (or tensors…), and the same metric applies to calculate the “invariant length” of each 4-vector

$$p^\mu = \left( \frac{E}{c}, \vec{p} \right); g_{\mu\nu} p^\mu p^\nu = \left( \frac{E}{c} \right)^2 - \vec{p}^2 = m^2 c^2$$

  - Example: 4-momentum
Now a bit more General…

- Equivalence Principle: Motion in a gravitational field is (locally) indistinguishable to force-free motion in accelerated coordinate system $S'$: $y = -\frac{1}{2}gt^2$
  - Example: Free fall in elevator
  - 2nd example: Clock moving around circle with radius $r$, angular velocity $\omega$, speed $v = r\omega$ => goes slow by factor
    \[
    \sqrt{1 - r^2\omega^2/c^2} = \sqrt{1 - r^*g^*/c^2}
    \]
    where "$g" = r\omega^2 is the centripetal acceleration. If we replace this with a gravitational force, we must choose $\Phi = U_{pot}/m$ such that $d\Phi/dr = -r\omega^2$ => $\Phi = -\frac{1}{2}r^2\omega^2$
    \[
    \Rightarrow \tau_{clock} = \sqrt{1 + 2\Phi(r)/c^2} t
    \]
- => New metric: $g_{00} = 1 + 2\Phi/c^2$
  - Example: clock at bottom of 50 m tower is slower by $5 \cdot 10^{-15}$ than clock at top. Can be measured using Mößbauer effect!
  - More general: Curved space-time!
General Relativity

- Einstein’s idea: Space-time is curved, with a metric determined by mass-energy density.
- Force-free objects move along geodetics: paths that maximize elapsed eigentime (as measured by metric).
  - Example: twin paradox – it is really the stay-at-home twin that ages more.
- Most general equations complicated (differential geometry), but special manageable case: spherically symmetric mass $M$ at rest => Schwarzschild metric
  \[
  (ds)^2 = \left(1 - \frac{2GM}{rc^2}\right)(cdt)^2 - \left(1 - \frac{2GM}{rc^2}\right)^{-1} dr^2 - r^2 \left[ (d\theta)^2 + \sin^2 \theta (d\phi)^2 \right].
  \]
- Examples: Falling, redshifting, bending, gravitational lensing, Event horizon (Schwarzschild radius).
Calculation: Falling

Radial motion $\Delta r$ in 2 steps (each taking time $\Delta t$): $\Delta r_1, \Delta r_2 = \Delta r - \Delta r_1$

$$\Delta s \approx \left[ \frac{2\Phi_1}{c^2} (\Delta ct)^2 - (\Delta r_1)^2 \right] + \left[ \frac{2\Phi_2}{c^2} (\Delta ct)^2 - (\Delta r_2)^2 \right] = \Delta ct \left[ \frac{2\Phi_1}{c^2} - \left( \frac{\Delta r_1}{\Delta ct} \right)^2 + \frac{1}{2} \left( \frac{1}{\Delta ct} \right)^2 \right]$$

$$= \Delta ct \left[ 1 + \frac{\Phi_1}{c^2} - \frac{1}{2} \left( \frac{\Delta r_1}{\Delta ct} \right)^2 + 1 + \frac{\Phi_2}{c^2} - \frac{1}{2} \left( \frac{\Delta r_2}{\Delta ct} \right)^2 \right] \approx \Delta ct \left[ 2 + \frac{\Phi_0 + \frac{d\Phi}{dr} \Delta r}{c^2} - \frac{1}{2} \left( \frac{\Delta r_1}{\Delta ct} \right)^2 + \frac{\Phi_0 + \frac{d\Phi}{dr} \Delta r + \Delta r_1}{c^2} - \frac{1}{2} \left( \frac{\Delta r_2}{\Delta ct} \right)^2 \right]$$

Find extremum of $\Delta s$ w.r.t. $\Delta r_1$ (for which $\Delta r_1$ does $\Delta s$ become max.?):

$$\frac{d\Delta s}{d\Delta r_1} = 0 \Rightarrow 0 = \frac{\frac{d\Phi}{dr}}{c^2} \frac{\Delta r_1}{(\Delta ct)^2} - \frac{\frac{d\Phi}{dr}}{c^2} \frac{\Delta r_2 (-1)}{(\Delta ct)^2} = \frac{1}{\Delta ct} \left[ \frac{\Delta r_2}{\Delta ct} - \frac{\Delta r_1}{\Delta ct} \right] + \frac{1}{c^2} \frac{d\Phi}{dr}$$

$$= \frac{1}{c^2} \left[ \frac{1}{\Delta t} (v_2 - v_1) + \frac{d\Phi}{dr} \right] \Rightarrow a = -\frac{d\Phi}{dr} \text{ q.e.d.}$$
= Black Holes

• Beyond a certain density, NOTHING can prevent gravitational collapse!
  – If there were a new source of pressure, that pressure would have energy (see HW), which causes more gravitation => gravity wins over
  – Singularity in space-time (infinitely dense mass point, infinite curvature; no classical treatment possible)
• For spherical mass at rest, Schwarzschild metric applies and we have an event horizon at \( r = r_S = 2GM/c^2 = 3\text{km} \ M/M_{\text{sun}} \) (Schwarzschild radius)
  – as object approaches \( r_S \) from outside, clock appears to slow to a crawl and light emitted gets redshifted to \( \infty \) long wavelength
  – along light path, \( ds = 0 \Rightarrow dr = \pm(1-r_S/r)\cdot dt \Rightarrow \) light becomes \( \infty \) slow and never can cross from inside \( r_S \) to outside
  – From outside, it takes exponential time for star surface to reach \( r_S \)
  – Rate of photon emission decreases exponentially (less than 1/s after 10 ms)
  – All material that falls in over time “appears” frozen on the surface of event horizon but doesn’t emit any photons or any other information
  – Co-moving coordinate system: will cross event horizon in finite time => no return!
Black holes in the Wild

- Smallest black holes likely > 3 \( M_{\text{sun}} \) (supernovae of 25 \( M_{\text{sun}} \) star followed by complete core collapse)
  - mostly detectable as invisible partner in binary system
  - some radiation from accretion disks (esp. X-ray)
    smallest radius about 3 \( r_S \);
    about 5-10% of gravitational pot. energy gets converted into luminosity (much more than fusion in stars in case of ns/bh)
  - White dwarf: \( L \approx L_{\text{sun}} \) (UV); ns/bh 1000’s times more (X-ray, gamma-ray)

- Gigantonomorous black holes in center of galaxies (see later in semester)

- Primordial black holes?