Astrophysics – Midterm Exam - Solution

Problem 1

Please answer the following questions by writing either “Y” or “N” behind each statement, depending on whether it is true or not.

1a) If a given star were a perfect black-body radiator, we could predict its apparent brightness (magnitude $m$) from its surface area, its distance, and its surface temperature. True? **Y**

1b) The fine details of the spectrum of the light emitted by a star is solely determined by its surface temperature. True? **N** [It also depends on the chemical composition of the surface.]

1c) There is no direct way of determining the masses of two stars that are gravitationally bound to each other. True? **N** [In fact, such binary systems are how we know about star masses]

1d) Is there a limit to how bright a star of given mass $M$ and opacity $\kappa$ can shine? **Y** [The Eddington Limit]

1e) Do proto-stellar clouds take most of the energy they radiate away from gravitational contraction? **Y**

1f) Is there any other time in the history of some stars (after they leave the Main Sequence) when gravitational contraction also supplies most of their radiated energy? **Y** [During the final collapse, in particular into a neutron star or a black hole, of a very massive star]

1g) Gas clouds, like planetary nebulae and the interstellar medium, can both emit and absorb electromagnetic radiation. True? **Y**

1h) White dwarfs are rather dim because of their low temperatures. True? **N** [They are dim because they are really small, but they can be quite hot]

1i) Neutron stars do not emit any detectable electromagnetic radiation at all. True? **N** [They do emit some light, but also radiowaves as pulsars.]

1j) Isolated black holes are hard to detect because they pretty much emit no radiation at all. True? **Y**
**Problem 2**
The following is a set of multiple choice questions, each with only ONE correct answer. Circle (the number in front of) the correct answer in each case!

2a) Which of the following nuclear reactions cost more energy than gets liberated?
   1 – Fusing two protons into a deuterium nucleus. [This is the main energy source of the sun at present.]
   2 – Fusing a helium nucleus with an oxygen nucleus to make Neon. [This is a fusion reaction that occurs in late-stage giants or supergiants and provides energy for their luminosity]
   3 – Fusing two iron nuclei to make Tellurium. [Correct – iron is more tightly bound per nucleon than Tellurium so this reaction would actually consume energy.]
   4 – The CNO cycle. [This is an alternative to the pp cycle in 1) and a major energy source for stars slightly more massive than sun.]

2b) Which of the following fusion processes provides presently most of sun’s energy?
   1 – Fusing two protons into a deuterium nucleus.
   2 – Fusing a helium nucleus with an oxygen nucleus to make Neon.
   3 – Fusing three helium atoms into Carbon.
   4 – The CNO cycle.

2c) Which of the following fusion processes will be going on (mostly) in the center of the sun during its late (asymptotic) red giant stage?
   1 – Fusing two protons into a deuterium nucleus.
   2 – Fusing a helium nucleus with an oxygen nucleus to make Neon.
   3 – Fusing three helium atoms into Carbon.
   4 – The CNO cycle.

2d) Which of the following processes could be responsible for the existence of elements heavier than iron and nickel?
   1 – Fusing two protons into a deuterium nucleus in main-sequence stars.
   2 – Fusing two iron nuclei to make Tellurium.
   3 – Rapid accumulation of neutrons on medium-heavy nuclei in supernova explosions.
   4 – The CNO cycle.
Problem 3

For the following questions, you only need to supply a numerical answer. However, you may attach your work (on a separate sheet) for partial credit.

a) Calculate how much energy is liberated if a star (really, a stellar core) the size of sun, but double its mass, collapses to a sphere of 10 km radius:\n\[ \Delta E = \Delta V_{grav} = \frac{3}{5} G (2M_{\text{sun}})^2 \left( \frac{1}{10 \text{ km}} - \frac{1}{R_{\text{sun}}} \right) = 6.34 \cdot 10^{46} \text{ J} \]

b) Repeat your calculation for the case where the same star collapses into a sphere with its Schwarzschild radius \( R_s \):\n\[ \Delta E = \Delta V_{grav} = \frac{3}{5} G (2M_{\text{sun}})^2 \left( \frac{1}{R_s} - \frac{1}{R_{\text{sun}}} \right) = \frac{3}{5} G (2M_{\text{sun}})^2 \left( \frac{1}{5908 \text{ m}} - \frac{1}{R_{\text{sun}}} \right) \approx 3.5 \cdot 10^{47} \text{ J} \]

c) How much total energy was “stored” initially as rest energy in the mass of this star, following Einstein’s famous equation?\n\[ E = 2M_{\text{sun}}c^2 = 3.58 \cdot 10^{47} \text{ J} \] (only 3.5 - 5 times more)

d) XC: An astronaut is visiting a planet only 1.25 times further from the center of a black hole than its Schwarzschild radius \( (r = 1.25R_s) \). How much time elapses according to a far-away observer during the time when the astronaut’s own watch ticks of one second?\n\[ \Delta \tau = \frac{1}{\sqrt{1 - \frac{R_s}{r}}} \Delta t = \frac{1}{\sqrt{1 - \frac{1}{1.25}}} \Delta t = \frac{1}{\sqrt{5}} \Delta t \Rightarrow \Delta t = 5 \Delta \tau = 2.24 \text{ s} \]

*) Here, you are asked to count ALL of the energy, including whatever may go into kinetic energy or heat of the remnant neutron star.

† ) Of course it strictly makes no sense to use the non-relativistic form of the energy here, but use it anyway
**Problem 4 - Extra Credit**

In your own words, describe what determines whether a given star ends up as a white dwarf, a neutron star, or a black hole. Give some numerical details (no derivations - you can quote from the formula sheet).

**Ans:**
The ultimate fate of a star is determined by its mass and, in particular, the mass of its residual core. Stars with masses up to a few times that of the sun end up as white dwarfs, since their degenerate carbon-oxygen cores can stabilize due to the Fermi gas pressure of the electrons while their outer hulls are blown off as planetary nebulae. As long as the core stays below the Chandrasekar limit of 1.4 solar masses, these white dwarfs are stable.

More massive stars will have cores that undergo further fusion reactions until they consist of iron, which will be instable against gravitational collapse at masses greater than 1.4 solar masses (Chandrasekar limit). This is due to the fact that the electrons become relativistic, and the pressure of a relativistic Fermi gas is not sufficient to stop collapse against gravitational attraction. These stars will end by erupting in a supernova explosion, and their cores will either collapse into neutron stars (extremely dense objects with only about 10 km radius that consist mostly of densely packed neutron matter that stabilizes them through the neutrons’ Fermi pressure) or, if the core mass exceeds about 2 solar masses, into black holes (singularities surrounded by an event horizon). The distinction is again due to the Chandrasekar limit, beyond which the Fermi pressure of the neutrons becomes insufficient to balance gravitational attraction (nuclear binding and special and general relativity play a role in determining this limit).