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QM thus far:
- A state vector (everything we know of a system)
  - Encodes probabilities and knowledge
- Most look @
  - What’s observable
    - What happens after observation? ("Collapse")
    - \( |\psi\rangle \rightarrow \psi(t) \): What happens as a function of time?
      \[ i\hbar \frac{\partial}{\partial t} |\psi\rangle = H |\psi\rangle \]
      \[ \Rightarrow \quad \frac{\partial}{\partial t} |\psi\rangle = \frac{\hbar}{i} \mathbf{H} |\psi\rangle \Rightarrow \text{Schrödinger equation} \]
  - \( t = \frac{\hbar}{2\pi} \)

- Complex #’s \( \mathbb{C} \), Real \( \mathbb{R} \)
  - at \( \mathbb{R} \)
- What does \( e^{i\phi} \) mean?
  \[ e^{i\phi} = \cos \phi + i \sin \phi \]
- What is a vector?
  - Written in components \( (a_1, a_2, a_3, \ldots, a_n) \)
  - Ex: \((x, y, z) (3D)\)
  - Can also make vectors in lesser or larger dimensions
  - QM: \( (x, y, z, P_x, P_y, P_z) \): Momentum & position
  - Can take an infinite number of components
- Must be able to add two vectors
  - \( \vec{r}_1 + \vec{r}_2 \Rightarrow \vec{r}_3 \) adding two vectors creates a new set of information
- Can multiply vectors when in QM
  - \( \vec{a} \cdot \vec{r} \) Can multiply vectors together to get a scalar product
- All vectors together are called vector space

Note: Multiplying a complex vector by a constant does not change the information within the system.
Two-Slit Experiment

When two vectors are added, interference pattern occurs, & changes where the probability distribution is. (Can no longer see where the photons come from.)

Can measure electromagnetic fields however could it just be the state vector of photons or an interference pattern?

Example for state vector:

\[ \begin{align*}
    \uparrow, & \quad \downarrow, & \quad \text{known,}\quad \text{can be used interchangelably.}
\end{align*} \]

Any linear combination is a state vector of...
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14> = \alpha \langle \uparrow | \uparrow > + \beta \langle \downarrow | \downarrow >

Most general state vector that can describe the system \( (C_1, C_2) \)

Example of probabilistic state where we can only find the probability

Consider \( \langle 14 \rangle_0 = \frac{1}{\sqrt{2}} \langle \uparrow > + \frac{1}{\sqrt{2}} \langle \downarrow > \)

Can be above or below with equal probability but they are still two different states of the system.

Note: These are terms in QM that can be described as "here or there." Example: Spin.

Example 2 of a state vector:
- Motion in 1D (electron along x-axis)
- How can we describe a state vector that can be anywhere on the x-axis?
- Cannot write it down by writing sequential numbers
- \( 00 < X < 00 \rightarrow C \)

Need to map to every real number between every real number
- Simply a function \( \psi(x) \) that returns a complex number
- \( \psi: \mathbb{R} \rightarrow \mathbb{C} \)

- Is a function a vector? Can it be treated as one?
- They can be added, they can multiply
- They are a vector & all together form a vector space

Rules:
- However, \( \psi \) (the function) must be continuous
- \( \psi(x) \) should exist & be continuous

Scalar product:
- \( \frac{\hbar}{i} \cdot \frac{\hbar}{i} = |\phi_1| |\phi_2| |\cos \theta| \) (Scalar product applied to different vector)
QM requires that there is a scalar product for state vectors.

- The norm of a vector tells about its probability.
- Complex conjugate $C = a + i\beta$.
  \[ \bar{C} = a - i\beta \]

**Example:**

\[ \langle \psi_1 | \psi_2 \rangle = C_1^* C_1 + C_2^* C_2 \]

Scalar product \( (\langle x_1, x_2 \rangle) \) of state vector \( \langle x_1, x_2 \rangle \)

\[ \langle x_1, x_2 \rangle = C_1^* x_1 + C_2^* x_2 \]

So an electron traveling along x-axis can be described by a wave function.

\[ \langle \psi_1 | \psi_2 \rangle = \int_{-\infty}^{\infty} \psi_1^* \psi(x) dx \]

Requires that \( \int_{-\infty}^{\infty} |\psi(x)|^2 dx = \langle \psi | \psi \rangle \) exists.