Modern Physics - Problem Set 1 – SOLUTION

First, here are the solutions in the expected email format:
Problem 1
1a N
1b N
1c Y

 Problem 2
2a 3
2b 1
2c 3

 Problem 3
3a $3.44 \times 10^{-6}$ s
3b 218 m

Problem 4
From the numbers given, $v/c = 0.8$ and $\gamma = 1.667$. As seen from Earth, the travel time is 5 years (4 light-years divided by $v = 0.8c$). As we saw in lecture, “moving clocks run slow” which means that during that same 5 years, the “internal clocks” of the spacecraft and the astronauts themselves tick off only 5 years/$\gamma = 3$ years. So they age considerably less (by only 3 years) than expected from the trip duration (5 years). But how can they explain to themselves that it took them only 3 years to cover a distance of 4 light-years? Of course, their own reference frame (that of the moving spaceship) is perfectly equivalent to Earth, so there must be a logical and consistent explanation. It comes from the fact that in their own reference frame, the whole (local) universe, including Sun, Earth and Alpha Centauri, are moving with velocity $-0.8c$. But moving lengths contract – by a factor $1/\gamma$ as well. That means that the distance they have to travel LOOKS to be only 4 light years / $1.667 = 2.4$ light years. This distance can indeed be traveled in only 3 years when going with velocity $0.8c$, so there is no contradiction.

Problem 5
[Attachment, scanned in or submitted handwritten solution, etc. – see below]

Below you can find a more complete solution with explanations (only for your benefit – you did not have to submit the additional material, except for Problem 5)
Problem 1
Please answer the following questions with “Y” or “N”:

1a) N [because it is a DEFINED quantity]
1b) N [People like Copernicus, Gallilei and of course Newton already knew that]
1c) Y [e.g., the speed of light!]

[Explanations in square brackets are NOT part of the required solution, just for your benefit!]

Problem 2
The following is a set of multiple choice questions. Answer each with a single digit number.

2a) 3

1 – Because Earth is moving only slowly (relative to the speed of light) X [That would depend on the reference frame]
2 – Because “relativity” affects only light X [No, it affects “everything”]
3 – Because our senses only record objects moving slowly relative to ourselves √
4 – Because “relativity” is just an abstract concept that doesn’t apply in reality X [See above]

2b) 1 [the only FALSE statement of the 4]
2c) 3 [The only CORRECT statement of the 4]

Problem 3
A 500 m long spaceship is moving with 90% of the speed of light relative to Earth. Every second (according to the ship’s clock) two lasers send a flash of light back to Earth – one from the front tip of the spaceship and one from the rear end. Calculate the following:

3a) According to Earth’s measurement, and correcting for the motion and length of the spaceship, what is the apparent time interval between the emission from the tip and the rear end?

\[ \Delta ct = \frac{v/c \Delta x'}{\sqrt{1 - v^2 / c^2}} = 1032 \text{ m} \Rightarrow \Delta t = 3.44 \times 10^{-6} \text{ s} = 3.44 \mu\text{s} \] [Lorentz Transf.; \( \gamma = \frac{1}{\sqrt{1 - 0.9^2}} = 2.294 \)]

3b) Using best measurement practices, what appears to be the length of the spaceship as measured from Earth? \( L = L'/\gamma = 500\text{m}/2.294 = 218 \text{ m} \) [“moving sticks are contracted”]

Problem 4
In 2047, NASA sends the first spaceship to Alpha Centauri, the closest star outside our solar system (at 4 light-years distance). Assume the spacecraft can go at 80% of the speed of light (relative to Earth). Ignoring the (short) periods of acceleration and deceleration at each end of the trip, explain why the astronauts age by less than what an Earth-based observer would expect for the trip. (How many years would the trip take, according to Earth? How many years according to the astronauts?) How can the astronauts explain the fact that they covered a distance of 4 light-years in less than 4 years? I.e., how would THEY describe the trip, from their own reference frame?

Answ.: From the numbers given, \( v/c = 0.8 \) and \( \gamma = 1.667 \). As seen from Earth, the travel time is 5 years (4 light-years divided by \( v = 0.8c \)). As we saw in lecture, “moving clocks run slow” which means that during that same 5 years, the “internal clocks” of the spacecraft and the astronauts
themselves tick off only 5 years / \gamma = 3 \text{ years}. So they age considerably less (by only 3 years) than expected from the trip duration (5 years). But how can they explain to themselves that it took them only 3 years to cover a distance of 4 light-years? Of course, their own reference frame (that of the moving spaceship) is perfectly equivalent to Earth, so there must be a logical and consistent explanation. It comes from the fact that in their own reference frame, the whole (local) universe, including Sun, Earth and Alpha Centauri, are moving with velocity -0.8c. But moving lengths contract – by a factor 1/\gamma as well. That means that the distance they have to travel LOOKS to be only 4 light years / 1.667 = 2.4 light years. This distance can indeed be traveled in only 3 years when going with velocity 0.8 \text{ c}, so there is no contradiction.

**Problem 5**

\[
x = x' + \frac{v}{c} \frac{ct'}{\sqrt{1 - \frac{v^2}{c^2}}} \implies x' = x \sqrt{1 - \frac{v^2}{c^2}} - \frac{v}{c} ct'; ct = \frac{ct' + \frac{v}{c} x'}{\sqrt{1 - \frac{v^2}{c^2}}}; x' = \frac{(1 - \frac{v^2}{c^2})ct'}{\sqrt{1 - \frac{v^2}{c^2}}} + \frac{v}{c} x
\]

\[
\implies ct' = \left(ct - \frac{v}{c} x\right) \sqrt{\frac{1 - \frac{v^2}{c^2}}{1 - \frac{v^2}{c^2}}} = \frac{ct - \frac{v}{c} x}{\sqrt{1 - \frac{v^2}{c^2}}}; x' = x \frac{1 - \frac{v^2}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}; \frac{ct - \frac{v}{c} x}{\sqrt{1 - \frac{v^2}{c^2}}}
\]

\[
= \frac{x - \frac{v}{c} ct}{\sqrt{1 - \frac{v^2}{c^2}}}
\]