Modern Physics - Problem Set 4 – Sol

Problem 1

Please answer the following questions with “Y” or “N”:

1a) Quantum Mechanics tells us that nothing can be predicted with certainty. True? N

[Counterexamples are the possible values of binding energy in an atom, or probabilities for certain measurement results.]

1b) In many cases, Quantum Mechanics can make only statements about probabilities for outcomes of measurements. True? Y

1c) From the Modern Physics point of view, are photons as “real” as electrons? Y

[Photons are the quanta of the electromagnetic field. They have specific energy, momentum, and can be emitted, absorbed or scattered, like any elementary particles including electrons. Ultimately, all elementary particles are only accessible to indirect observation – we cannot see them. Or one could argue that photons are the only elementary particles that we can actually see...]

Problem 2

The following is a set of multiple choice questions. Answer each with a single digit number.

2a) Which of the following statements about the “state vector” (a.k.a. “wave function”) in Quantum Mechanics is not true? 1

1 – The state vector is directly proportional to the probability of a measurement outcome.
[No, if anything it’s the absolute magnitude squared of its projection that is proportional to some probability]

2 – The state vector contains all “knowable” information about a system. [YES!]

3 – The state vector can change over time.
[Yes, and the Schrödinger Equation shows how it changes with time.]

4 – In Quantum Mechanics, the state vector plays the same fundamental role momentum and position play in Classical Mechanics. [Yes]

2b) Assume Anton buys a lottery ticket on Wednesday night, and at the drawing on Thursday he finds out that his are the winning numbers. Which of the following statements is NOT true? 4

1 – On Wednesday, the probability for Anton to win the lottery was extremely small. Y

2 – In a world where Anton knows the position of every atom in the lottery drawing machine right before the drawing, he could have predicted his win. Y

3 – On Thursday evening, the probability that Anton has won the lottery is 1. Y

4 – Because the probability of winning the lottery is so small, nobody should ever win. [No, even if the probability for a specific person to win is tiny, if you “repeat the experiment many times” – i.e., if lots and lots (millions!) of people play -, the probability that someone wins will become sizable.]

2c) Which of the following statements about the relationship between uncertainty and quantization is correct? 3

1 – Even an observable with a continuous (not quantized) spectrum of values can be known with perfect precision in principle.

2 – Any observable that can be predicted with certainty must be quantized.

3 – Any observable that is quantized can always be predicted with certainty.

4 – Both position and momentum are quantized and can therefore be predicted with certainty.
2d) An elevator in a 5-floor building has a 50% chance to be at the ground floor (level 1) and a 12.5% chance to be at any of the other 4 floors at any given time. Which statement is wrong? 2
1 – The mode of the probability distribution for the position of the elevator is the 1st floor.
2 – The median of the same probability distribution is the 3rd floor.
3 – The mean of the same probability distribution is the 2.25th floor.
4 – The standard deviation of the probability distribution is 1.48 floors.
[1 – Correct, because it’s the most probable position
2 – Wrong. ½ of the time it’s on the first floor, and ½ of the time any other floor, so the median is somewhere between 1st and 2nd floor, e.g. 1.5
3 – Correct: The correct answer is 0.5*1+0.125*2+0.125*3+0.125*4+0.125*5 = 2.25 for the expectation value.
4 – Also correct. Use the equation $\sigma = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$]

**Problem 3**

Assume that the position of a particle located within an interval of 0…1 m is given by the following probability density: $p(x) = 7.5(x^2 - x^4)$. Answer the following questions with a simple numerical results:

3a) What is the probability to find the particle between 0.59 m and 0.61 m? (approximate answer is sufficient)
   $\Delta P(0.59...0.61) = p(0.6) \cdot \Delta x = 1.728 \cdot 0.02 = 0.03456$

3b) What is the most probable position? (Mode of the distribution) $x = 1/\sqrt{2} \approx 0.707$
   [Setting the first derivative to zero yields $2x = 4x^3$ or $x^2 = \frac{1}{2}$]

3c) What is the median of the distribution?
   $\int_0^x p(x) \, dx = 7.5 \int_0^x (x^2 - x^4) \, dx = 7.5 \left( \frac{X^3}{3} - \frac{X^5}{5} \right) = \frac{1}{2}$ if $X \approx 0.643$
   [You can just get this by trial and error – an approximate result is good enough]

3d) What is the mean of the distribution?
   $\langle x \rangle = \int_0^1 xp(x) \, dx = 7.5 \int_0^1 (x^3 - x^5) \, dx = 7.5 \left( \frac{X^4}{4} - \frac{X^6}{6} \right) \Bigg|_0^1 = \frac{5}{8} = 0.625$

3e) What is the standard deviation of the distribution?
   $\langle x^2 \rangle = \int_0^1 x^2 p(x) \, dx = 7.5 \int_0^1 (x^4 - x^6) \, dx = 7.5 \left( \frac{X^5}{5} - \frac{X^7}{7} \right) \Bigg|_0^1 = \frac{3}{7}$
   $\sigma^2 = \langle x^2 \rangle - \langle x \rangle^2 = \frac{3}{7} - \frac{25}{64} = \frac{17}{448} = 0.0379 \Rightarrow \sigma = \sqrt{0.0379} = 0.195$
Problem 4

Explain, in your own words, why the light emitted from Na vapor lamps (typical yellow street lights) contains a few sharp wavelengths (frequencies) and is not a continuous distribution over all frequencies like the light coming from the sun. Make the connection to photons and energy!

Answ.: The light emitted by Na vapor lengths comes directly from specific transitions between different bound state energy levels in sodium atoms. The atoms are excited from their ground states to higher energy states by an electric discharge, and then decay back to the ground state through the emission of a photon. The energy difference between the two states must equal the energy carried away by the photon since energy is conserved in the process. Because the energy of each atomic state is quantized (can only have a specific, discrete value), only photons with very specific energies can be emitted. Given that the frequency of the light we observe is directly related to the energy of each photon, via $E = hf$, this means that we can only observe discrete frequencies, not a continuous band.