Graduate Quantum Mechanics - Problem Set 1 - Solutions

Problem 1)

\[ E = T_{\text{kin}} + V(x) = H(p, x) = \frac{p^2}{2m} + mgx. \]

\[ \frac{\partial H}{\partial p} = \frac{p}{m} = v = \dot{x} \]

\[ -\frac{\partial H}{\partial x} = -mg = F = \dot{p} \]

Problem 2)

\[ \bar{A}(r, \varphi, z) = \frac{r}{2}b \hat{\varphi} \Rightarrow A_{\varphi} = \frac{r}{2}b, A_r = 0, A_z = 0 \Rightarrow \]

\[ \bar{B} = \bar{\nabla} \times \bar{A} = -\frac{\partial A_{\varphi}}{\partial z}\hat{r} + \frac{1}{r}\frac{\partial A_r}{\partial r}\hat{z} = \frac{1}{r}rb\hat{z} = b\hat{z} \]

where we took only the non-zero terms from the curl in cylindrical coordinates in the formula sheet. This leads to motion in a circle of radius

Problem 3)

\[ \vec{F} = q\vec{v} \times \bar{B} = -e(0.1c)(0.1T)(-\hat{y}) = 3 \cdot 10^7 e \text{ N/C} \hat{y} = 4.8 \cdot 10^{-13} \text{ N} \hat{y} \]

The magnitude of this force is constant and it is always perpendicular to the direction of motion.

Therefore, the electron moves on a circle of radius \( R = \frac{mv}{eB} = 0.0017 \text{ m} \) centered at \( x = 0, y = R \). Its angular velocity is \( \omega = \frac{v}{R} = 1.76 \cdot 10^{10} \text{ rad/s} \) which corresponds to a full orbit every 0.36 ns.

Problem 4)

For each of the following statements about Quantum Mechanics, indicate whether you believe them to be correct or wrong. Give a 1-2 sentence explanation for each of your responses:

a. If all possible information on a system is given, Quantum Mechanics can predict the outcome of any future measurement on the system accurately. **WRONG**: in general, only probabilities can be predicted

b. Quantum Mechanics cannot predict anything precisely. **WRONG** – then it wouldn’t be physics!
c. Quantum Mechanics cannot predict with certainty the result of any particular measurement on a single particle. **DEPENDS** – if a particle is in an eigenstate of an observable, I can predict the outcome of a measurement of that observable precisely.
d. The Heisenberg Uncertainty principle means that nothing can be measured precisely. **WRONG** – see above
e. The x- and y-components of any angular momentum cannot simultaneously be measured with arbitrary precision. **CORRECT**
f. The time evolution of a quantum mechanical wave function is described by a unitary operator. **CORRECT**

**Problem 5)**

The most general solution is
\[ y(x) = A \exp(mx) + B \exp(-mx) \]
which can be shown by plugging it in (as a 2\(^{nd}\) order differential equation, there must be two integration constants, \(A\) and \(B\)).
Since \(y(0) = A + B\) and \(y'(0) = mA - mB\), we can solve for \(A\) and \(B\) in terms of the initial conditions at \(x = 0\).

**Problem 6)**

\[
\begin{align*}
z &= \exp(c) = \exp(\text{Re}(c) + i \text{Im}(c)) = \exp(\text{Re}(c))(\cos(\text{Im}(c)) + i\sin(\text{Im}(c))) \\
z' &= \exp(\text{Re}(c))(\cos(\text{Im}(c)) - i\sin(\text{Im}(c))) = \exp(\text{Re}(c))\exp(-i \text{Im}(c)) = \exp(c^*)
\end{align*}
\]

**Problem 7)**

See next recitation