…back to nuclei

Question

- What keeps nuclei from falling apart? Gluons?
  - Not likely (need at least 2 -> short range)
  - How do water molecules stick together? Residual (van der Waals) forces (e.g., dipole-dipole interaction)
- First need to study how protons and neutrons interact with each other
  - E&M (Coulomb repulsion, magnetic interaction through spins)
  - Strong Force
- HOW do we figure this out?

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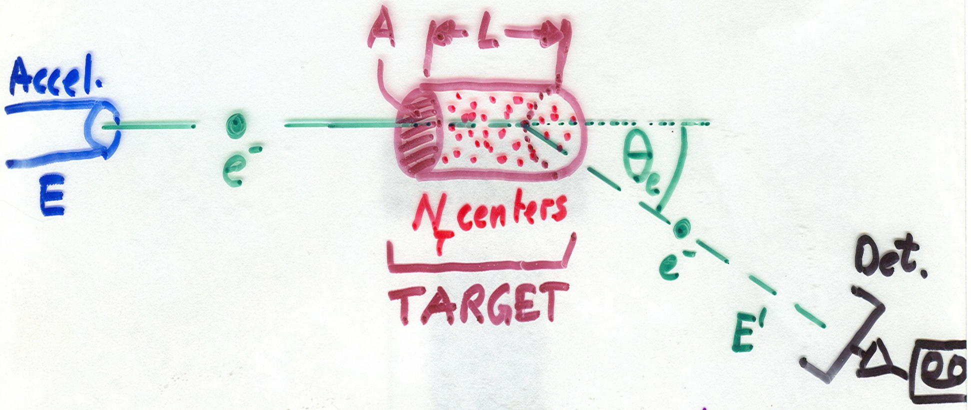
  - Strong Force
- HOW do we figure this out?
- OF COURSE: Through scattering experiments

NN Scattering – a Reminder

what can we measure ?



What is the likelihood to find the electron scattered into the detector?

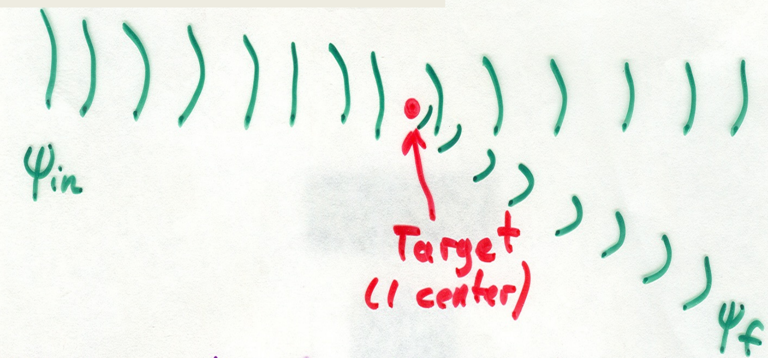
$$P \sim n_T \cdot L = \frac{N_T}{A\,L} \cdot L = \frac{N_T}{A} \;^{*}$$

$$\Rightarrow \text{call} \quad \Delta\sigma = P / \left(\frac{N_T}{A}\right) \quad \text{(cross section)}$$

$\Delta\sigma$ DEPENDS on the kinematics (E, E', θ_e) and is $\approx$ proportional to SIZE of kinematic bin spanned by the detector

*Note: $\frac{N_T}{A} = \rho\left[\frac{g}{cm^3}\right] \cdot L\,[cm] \cdot \frac{\text{Avogadro}}{\text{Atomic Weight [u]}}$

Theorist's View



What is the transition rate $W_{i\to f}$?

$$\dot N_{e,f} = \dot N_{e,in} \cdot P(i\to f) = I_{e,in} \cdot \frac{N_T}{A} \cdot \Delta\sigma$$

$$= \frac{I_{e,in}}{A} \cdot N_T \cdot \Delta\sigma = |\vec{j}_{e,in}|_z \cdot N_T \cdot \Delta\sigma$$

$$\Rightarrow W_{i\to f} = j_{in} \cdot \Delta\sigma$$

Fermi's GOLDEN Rule:

$$W_{i\to f} = \frac{2\pi}{\hbar} |\mathcal{M}_{fi}|^2 \Delta\phi$$

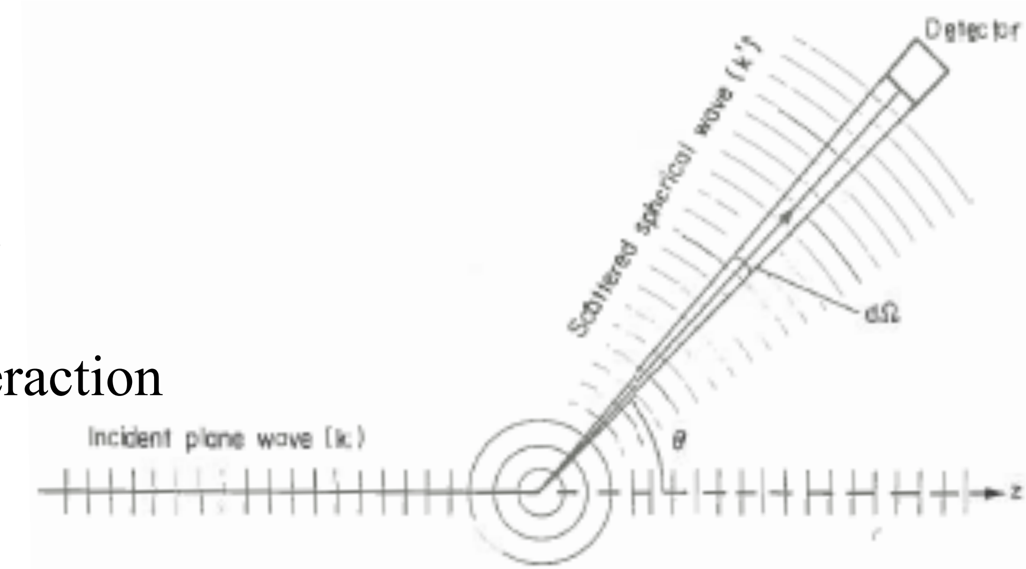
Phase space spanned by detector/kinematic bin

$$\mathcal{M}_{fi} = \langle \psi_f | H_{int} | \psi_{in} \rangle$$

NN Scattering

Short range nuclear force,
potential V(r)

V(r) = 0 except in the interaction region



$$\psi(r,\theta,\phi) \rightarrow e^{ikz} + f(\theta,\phi)\frac{e^{ikr}}{r}$$

Incoming plane wave
+ unscattered

Scattering amplitude

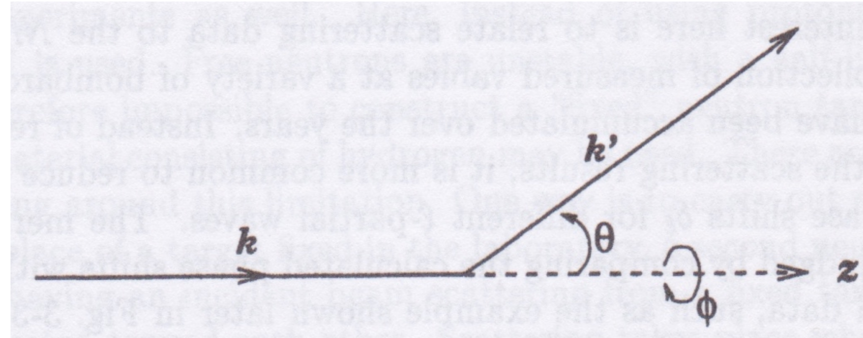
Outgoing spherical wave

$k = \sqrt{2\mu E} / \hbar$

μ is the reduced mass

Slide by Dr. Gail Dodge

NN scattering



- Basic scattering theory
  - Solve Schrödinger Equation: $-\frac{\hbar^2}{2\mu}\nabla^2\psi + (V - E)\psi = 0$
  - Asymptotic free states: Plane wave plus spherical outgoing wave $\psi(r,\theta,\phi) \xrightarrow[r\to\infty]{} e^{ikz} + f(\theta,\phi)\frac{e^{ikr}}{r}$
  - Current densities

$$S(r,t) = \frac{\hbar}{2i\mu}\{\psi^*\nabla\psi - \psi\nabla\psi^*\} = \Re\left\{\psi^*\frac{\hbar}{i\mu}\nabla\psi\right\} = \Re\left\{e^{-ikz}\frac{\hbar}{i\mu}\frac{d}{dz}e^{ikz}\right\} = \frac{\hbar k}{\mu} = v$$

$$S_r = \Re\left\{\left(f(\theta)\frac{e^{ikr}}{r}\right)^*\frac{\hbar}{i\mu}\frac{d}{dr}\left(f(\theta)\frac{e^{ikr}}{r}\right)\right\} = \frac{v}{r^2}|f(\theta)|^2 + O(r^{-3})$$ See HW 8

  - Cross section

$$d\Omega = \frac{da}{r^2} \qquad N_r = S_r\,da = S_r r^2\,d\Omega \qquad \frac{d\sigma}{d\Omega} = \frac{S_r r^2}{S_i} = |f(\theta)|^2$$

Potential Scattering

- Angular momentum decomposition

$$\psi(r,\theta) = \sum_{\ell=0}^{\infty} a_\ell Y_{\ell 0}(\theta)\, R_\ell(k,r)$$

$$R_\ell(k,r) \xrightarrow{\text{free}} j_\ell(kr) \quad \xrightarrow[r\to\infty]{\text{free}} \frac{1}{kr}\sin(kr - \tfrac{1}{2}\ell\pi) \quad \xrightarrow[r\to\infty]{\text{scatt.}} \frac{1}{kr}\sin(kr - \tfrac{1}{2}\ell\pi + \delta_\ell)$$

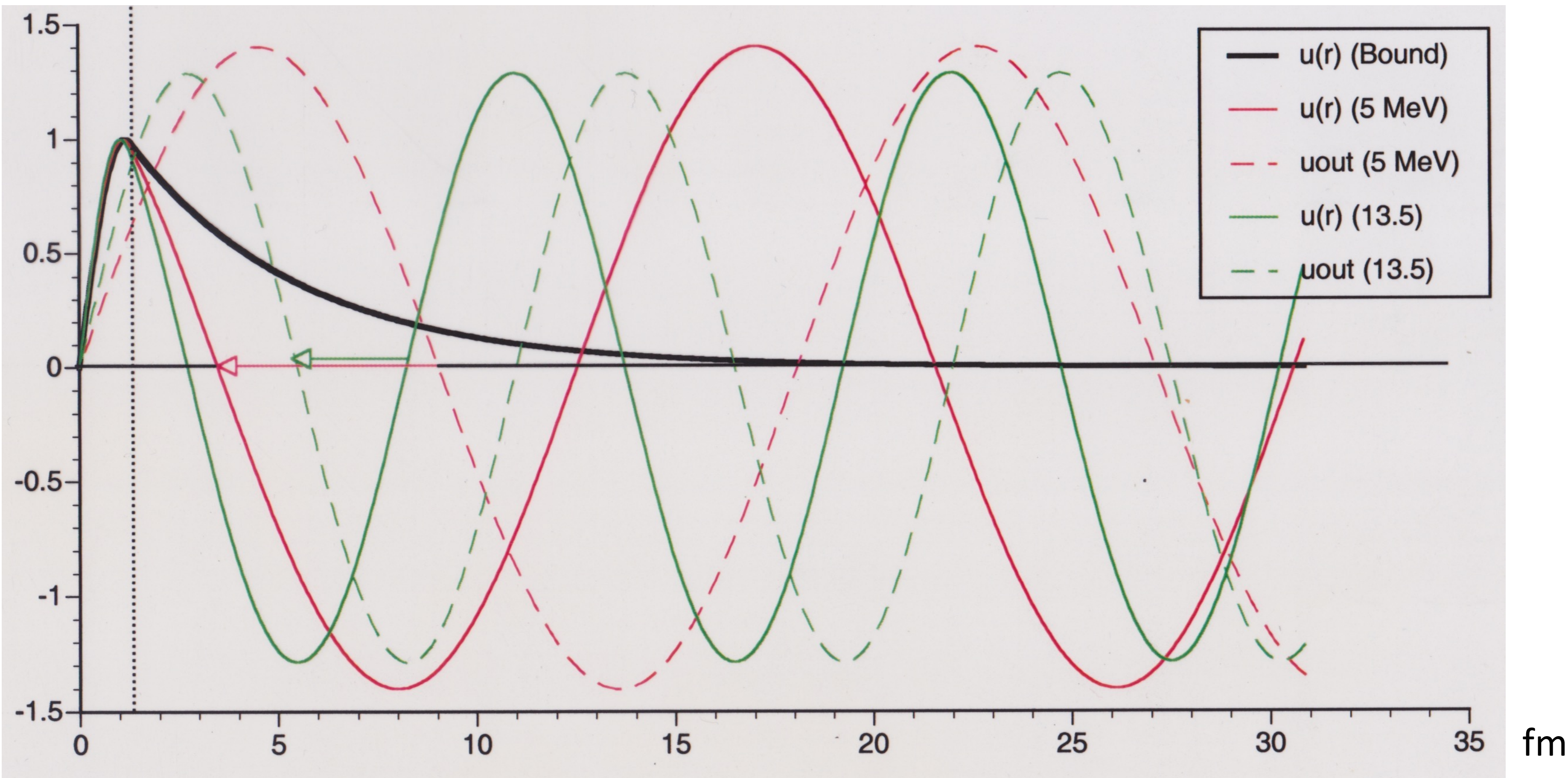
- Phase shifts -> scattering amplitude ->cross section

$$f(\theta) = \frac{\sqrt{4\pi}}{k}\sum_{\ell=o}^{\infty}\sqrt{2\ell+1}\, e^{i\delta_\ell}\sin\delta_\ell Y_{\ell 0}(\theta)$$

Phase Shifts – Example square potential scattering

$$\kappa = \frac{1}{\hbar}\sqrt{2\mu(E+V_0)} \qquad u_0(r) = \sin(kr+\delta_0) \quad \text{for} \quad r > r_0$$

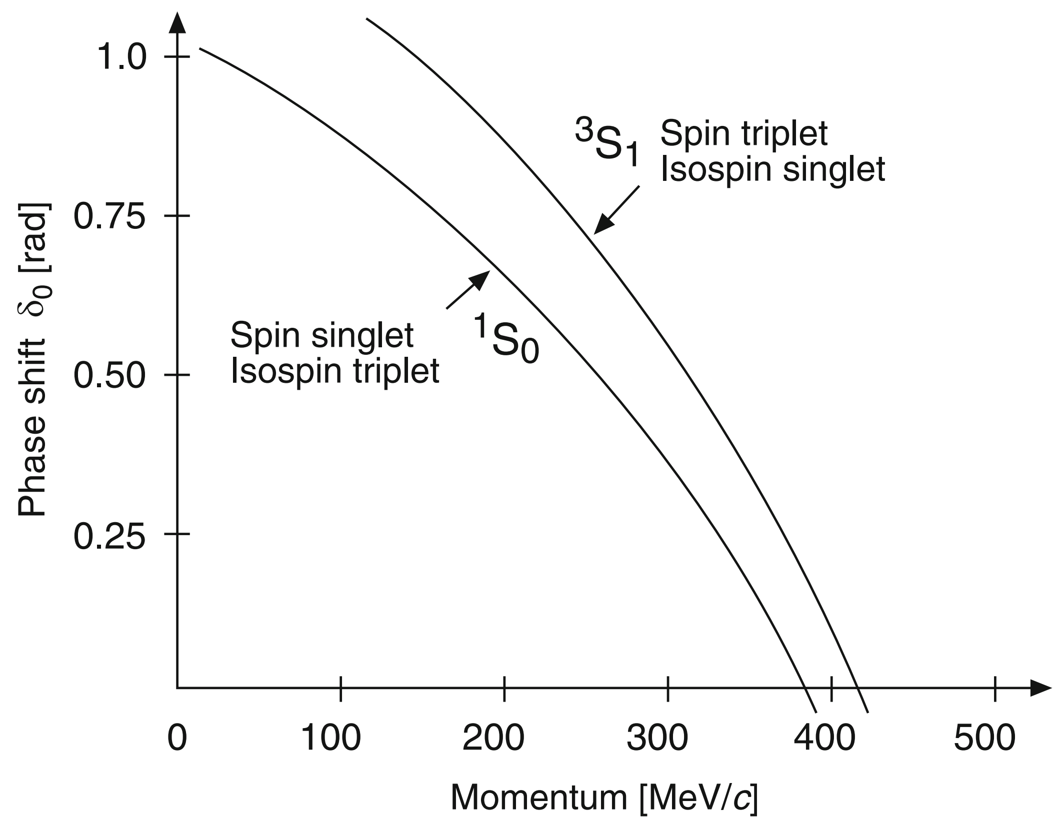
$$\frac{\sin \kappa r_0}{\kappa \cos \kappa r_0} = \frac{\sin(kr_0+\delta_0)}{k\cos(kr_0+\delta_0)}$$

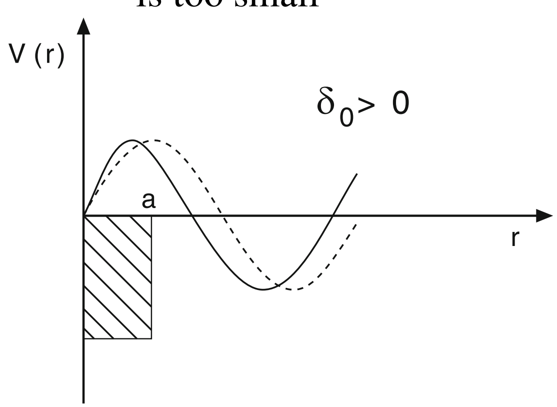


Explain on Whiteboard:

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2 \qquad f(\theta) = \frac{1}{k} \sum_{\ell=0}^{\infty} (2\ell + 1)\, e^{i\delta_\ell} \, \sin\delta_\ell \, P_\ell(\cos\theta)$$

Fig. 17.1 The phase shift δ_0 as determined from experiment both for the spin triplet-isospin singlet $^3S_1$ and for the spin singlet-isospin triplet $^1S_0$ systems plotted against the relative momenta of the nucleons. The rapid variation of the phases at small momenta is not plotted since the scale of the diagram is too small





Selection rules for NN phase shifts

- I = 0 (pn only):
  - Isospin WF antisymmetric ->
  - Spin-orbital WF symmetric ->
  - Either S = 1 and L = 0,2,4,…
  - Or S = 0 and L = 1, 3,…
- I = 1 (pp, pn and nn)
  - Either S = 0 and L = 0,2,4,…
  - Or S = 1 and L = 1, 3,…
- Can also have transition Phase shifts!

Nomenclature:

$^{2S+1}L_J$; $\mathbf{J} = \mathbf{L} + \mathbf{S}$

Ex.: $\boxed{^3S_1,\ ^3D_1}$, $^3D_2$, $^3D_3$

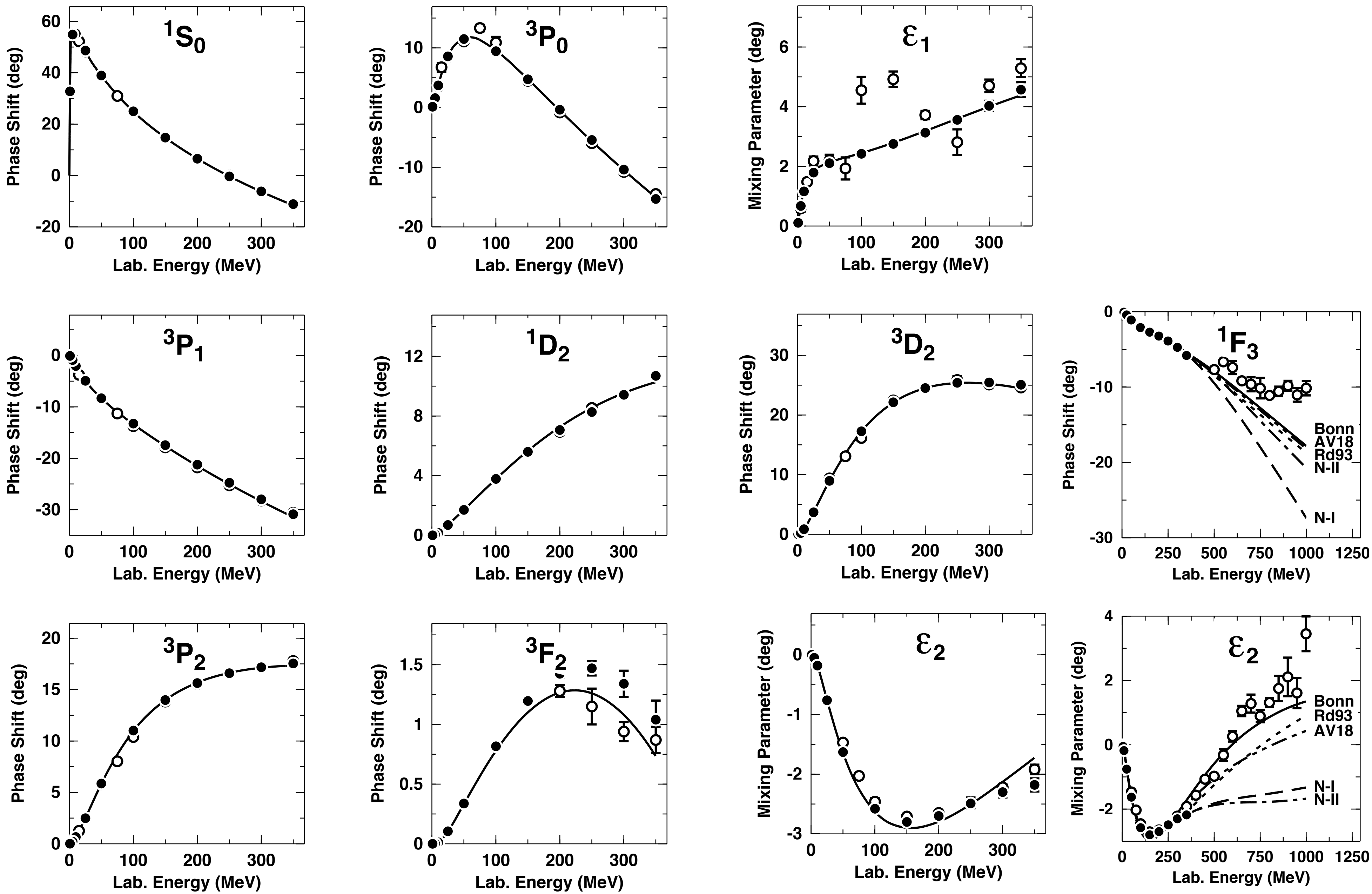
Ex.: $^1P_1$, $^1F_3$,

Ex.: $^1S_0$, $^1D_2$,

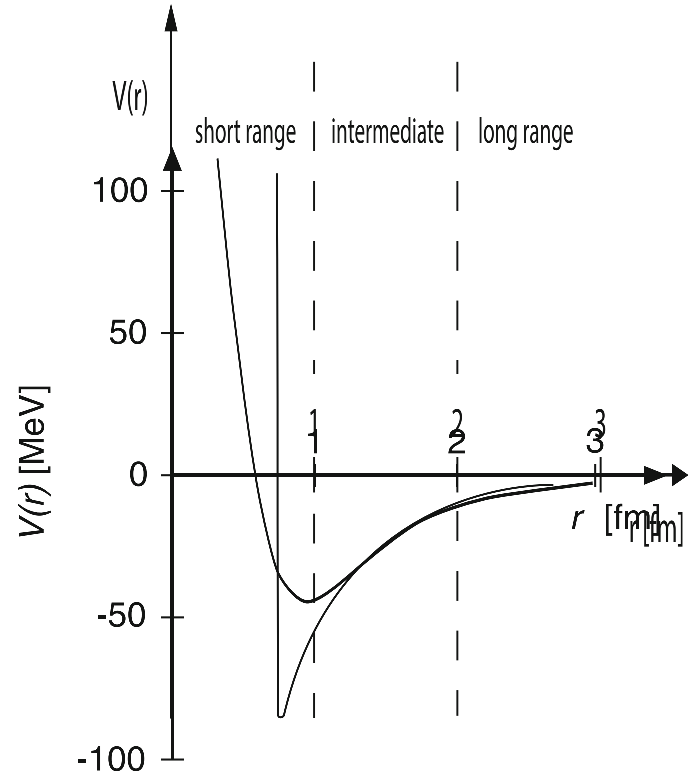
Ex.: $^3P_0$, $^3P_1$, $^3P_2$, $^3F_2$, $^3F_3$, $^3F_4$

Ex.: $^3S_1 \leftrightarrow {}^3D_1$, $^3P_2 \leftrightarrow {}^3F_2$

NN Phase Shifts

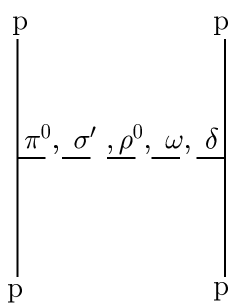


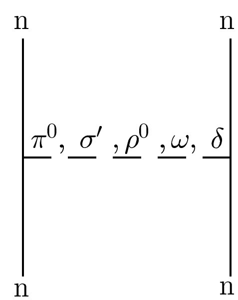
The NN Force

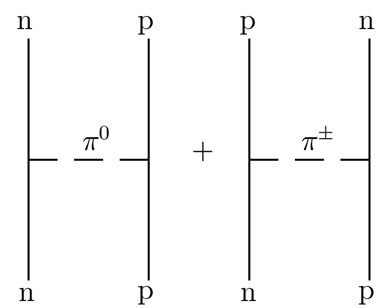


- (L)QCD: The future
- QCD-based effective theories
  - Chiral Perturbation Theory
  - Pion-less effective theory
- Models
  - Quark exchange, gluon van-der Waals force (QCD-"inspired")
  - Meson exchange + hard core
    - Pauli Principle between quarks? Spin-Spin interaction? Vector Mesons?

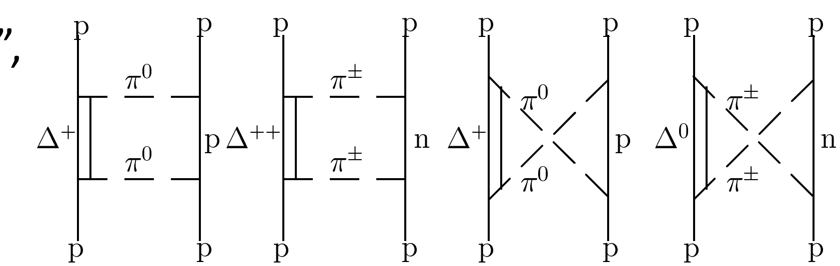
Meson Exchange



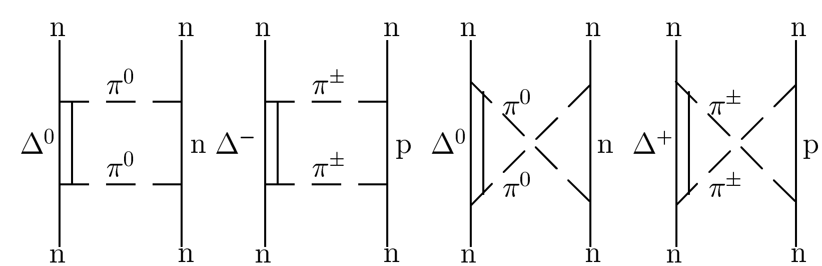




Plus 2-pion ("sigma", rho) exchange plus intermediate nucleon excitations



(a)



One Pion Exchange (OPE)

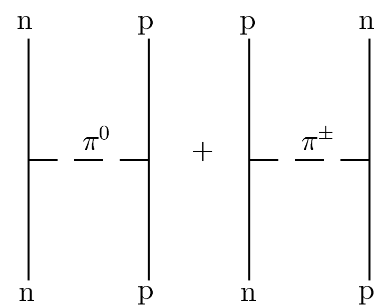
(again borrowing from Rocco: Rev. Mod. Phys., Vol. 70, No. 3, July 1998)

$$v_{ij}^{OPE} = \frac{f_{\pi NN}^2}{4\pi} \frac{m_\pi}{3} [Y_\pi(r_{ij}) \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j + T_\pi(r_{ij}) S_{ij}] \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j , \tag{2.2}$$

$$Y_\pi(r_{ij}) = \frac{e^{-\mu r_{ij}}}{\mu r_{ij}}, \tag{2.3}$$

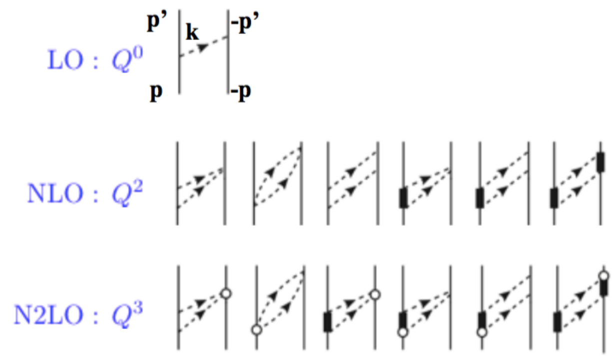
$$T_\pi(r_{ij}) = \left[1 + \frac{3}{\mu r_{ij}} + \frac{3}{(\mu r_{ij})^2} \right] \frac{e^{-\mu r_{ij}}}{\mu r_{ij}}, \tag{2.4}$$

$$S_{ij} \equiv 3 \boldsymbol{\sigma}_i \cdot \hat{\mathbf{r}}_{ij} \boldsymbol{\sigma}_j \cdot \hat{\mathbf{r}}_{ij} - \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \tag{2.5}$$



(see HW assignments 8 and 9!)

χPT effective potential
(Slide by Rocco Schiavilla)



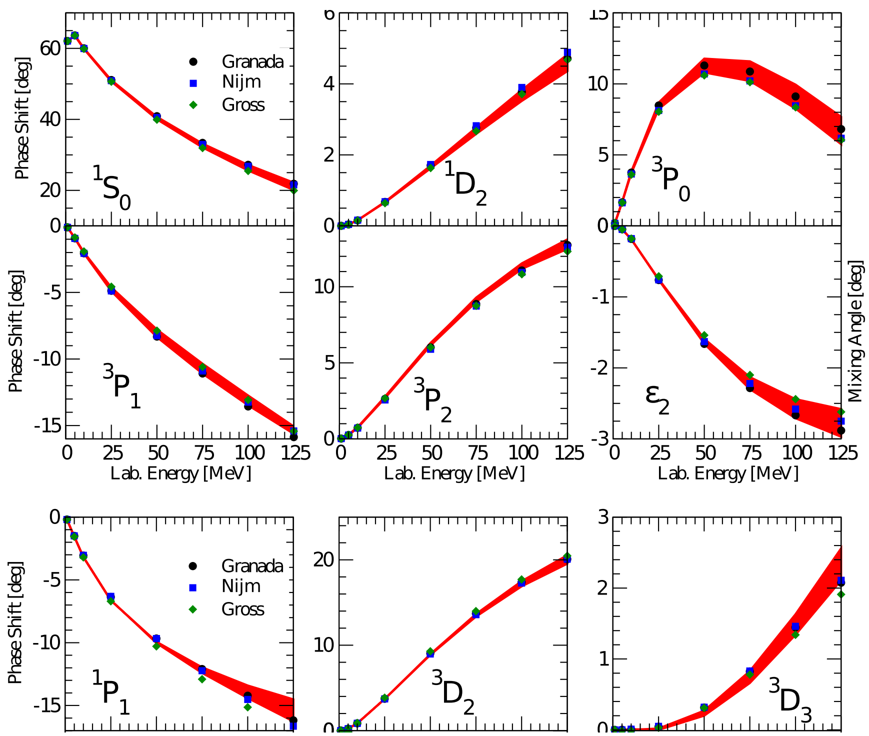
From PHYSICAL REVIEW C **94**, 054007 (2016):

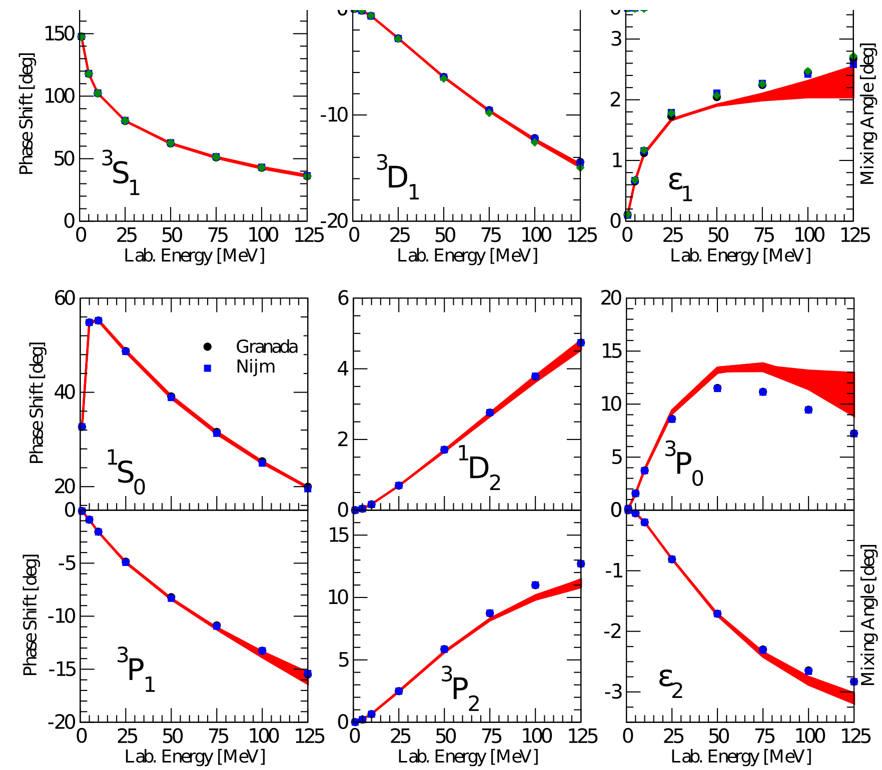
The v_L part includes the one-pion-exchange (OPE) and two-pion-exchange (TPE) (including Deltas) contributions up to N2LO. Short range part "ad hoc".

$$v_{12}^{\mathrm{L}} = \left[\sum_{l=1}^{6} v_{\mathrm{L}}^{l}(r)\, O_{12}^{l}\right] + v_{\mathrm{L}}^{\sigma T}(r)\, O_{12}^{\sigma T} + v_{\mathrm{L}}^{tT}(r)\, O_{12}^{tT},$$

where

$$O_{12}^{l=1,\ldots,6} = [\mathbf{1}\,,\, \boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\,,\, S_{12}] \otimes [\mathbf{1}\,,\, \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2],$$



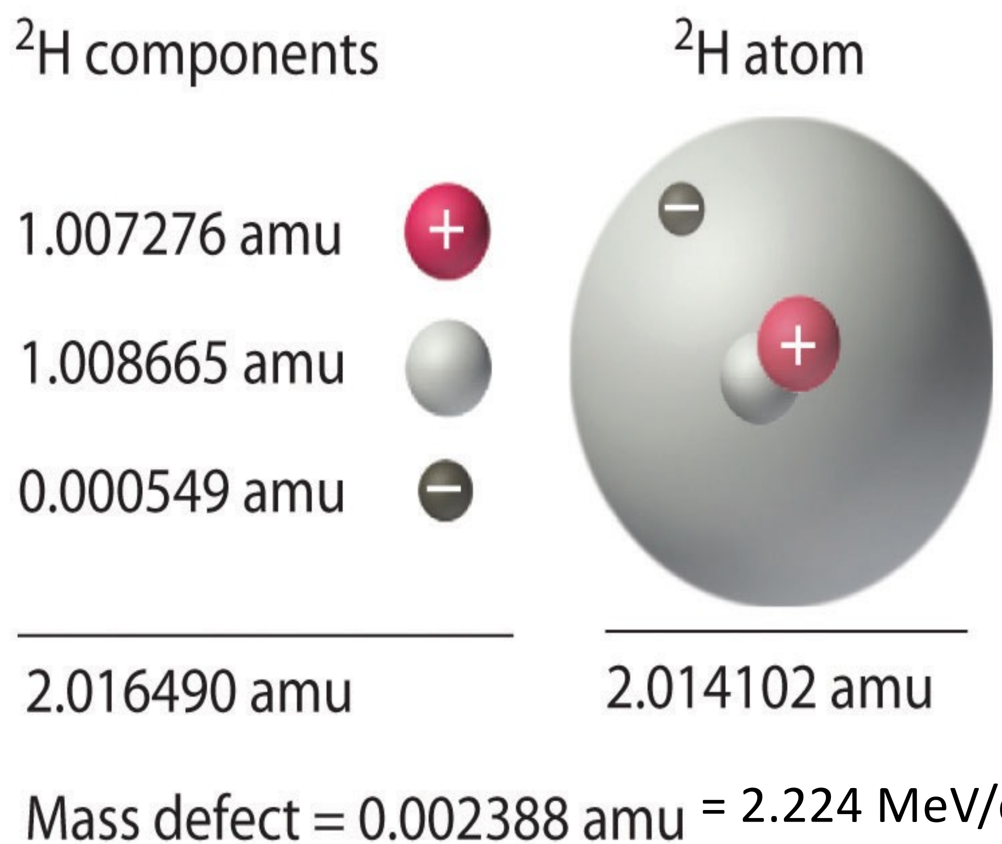


Plus information from the Deuteron!

Proton : 938.27 MeV =
Neutron : 939.57 MeV =

M_A = “A_{nom}”*u
“A_{nom}” ≈ A := Z + N
“A_{nom}”: Atomic mass #
A = Baryon Number

Binding energy	$B = 2.225$ MeV
Spin and parity	$J^P = 1^+$
Isospin	$I = 0$
Magnetic moment	$\mu = 0.857\ \mu_N$
Elec. quadrupole moment	$Q = 0.282\ e{\cdot}\mathrm{fm}^2$



= 2.224 MeV/c^2

NN forces – the deuteron

Deuteron Properties: Mass = 1865.613 MeV

Binding energy 2.225 MeV

Only bound NN system

$J^P = 1^+$, hence L = 0,2 S = 1, I = 0

RMS radius 1.97 fm (1/2 RMS distance between p and n).

$\mu_D = 0.8574\ \mu_N = \mu_p + \mu_n - 0.0224\ \mu_N$ *)

Electric Quadrupole Moment $Q_D = 0.2859\ e\ \text{fm}^2$ -> some L = 2 admixture!

$P_D = 0.04 - 0.06$ – not an observable! (However, the asymptotic D/S ratio = η is)

TABLE I. Experimental deuteron properties compared to recent *NN* interaction models; meson-exchange effects in μ_d and Q_d are not included.

	Experiment	Argonne v_{18}	Nijm II	Reid 93	CD Bonn	Units
A_S	0.8846(8)[a]	0.8850	0.8845	0.8853	0.8845	$\text{fm}^{1/2}$
η	0.0256(4)[b]	0.0250	0.0252	0.0251	0.0255	
r_d	1.971(5)[c]	1.967	1.9675	1.9686	1.966	fm
μ_d	0.857406(1)[d]	0.847				μ_0
Q_d	0.2859(3)[e]	0.270	0.271	0.270	0.270	fm^2
P_d		5.76	5.64	5.70	4.83	

[a]Ericson and Rosa-Clot, 1983.
[b]Rodning and Knutson, 1990.
[c]Martorell, Sprung, and Zheng, 1995.
[d]Lindgren, 1965.
[e]Bishop and Cheung, 1979.

*) Simple model: $\mu_d = \mu_s - \frac{3}{2}\left(\mu_s - \frac{1}{2}\right) P_D$

NN forces – the deuteron

S- and D-state WF:

$$\psi_M(\mathbf{x}) = \frac{u(r)}{r}\mathcal{Y}^M_{101}(\theta,\phi) + \frac{w(r)}{r}\mathcal{Y}^M_{121}(\theta,\phi)\,, \tag{2}$$

where

$$\mathcal{Y}^M_{JLS}(\theta,\phi) = \sum_{m_L,m_S} \langle J, M\,|L, m_L; S, m_S\rangle\, Y_{LM}(\theta,\phi)\,|S, m_s\rangle \tag{3}$$

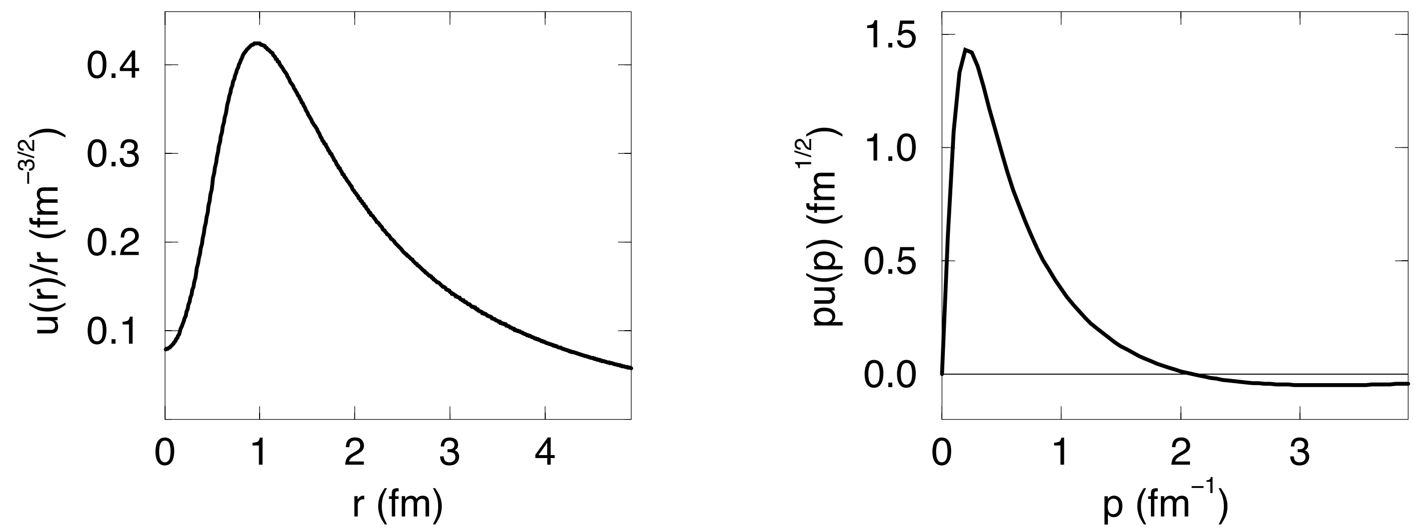


Figure 2: The deuteron S wave function in configuration space and in momentum space: $u(r)/r$ and $pu(p)$ (calculated from the Argonne v_{18} potential).

NN forces – the deuteron

S- and D-state WF:

$$\psi_M(\mathbf{x}) = \frac{u(r)}{r}\mathcal{Y}^M_{101}(\theta,\phi) + \frac{w(r)}{r}\mathcal{Y}^M_{121}(\theta,\phi)\,, \tag{2}$$

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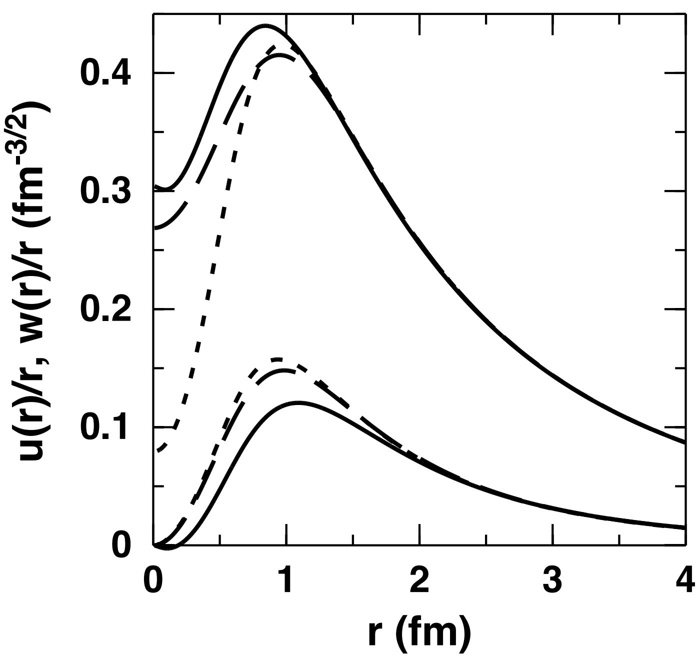
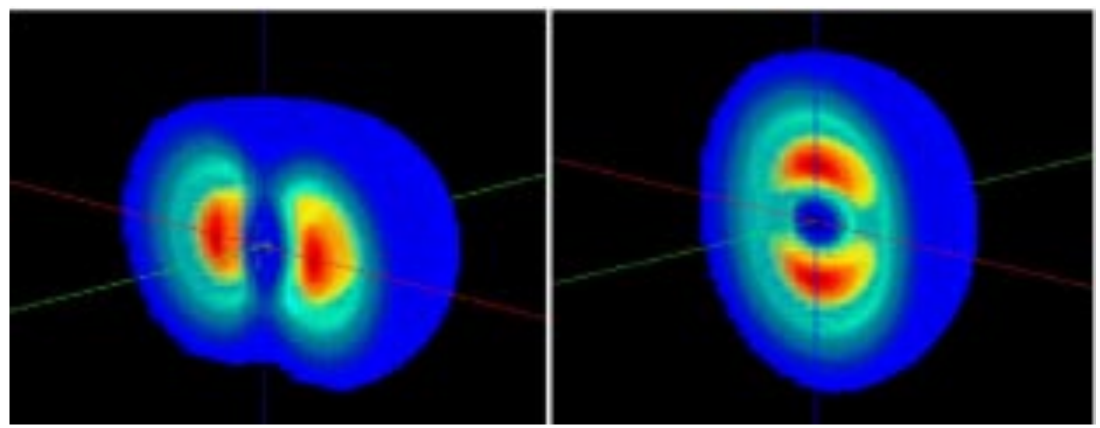


The deuteron wave functions. The family of large curves are u(r)/r and the family of small curves are w(r)/r.



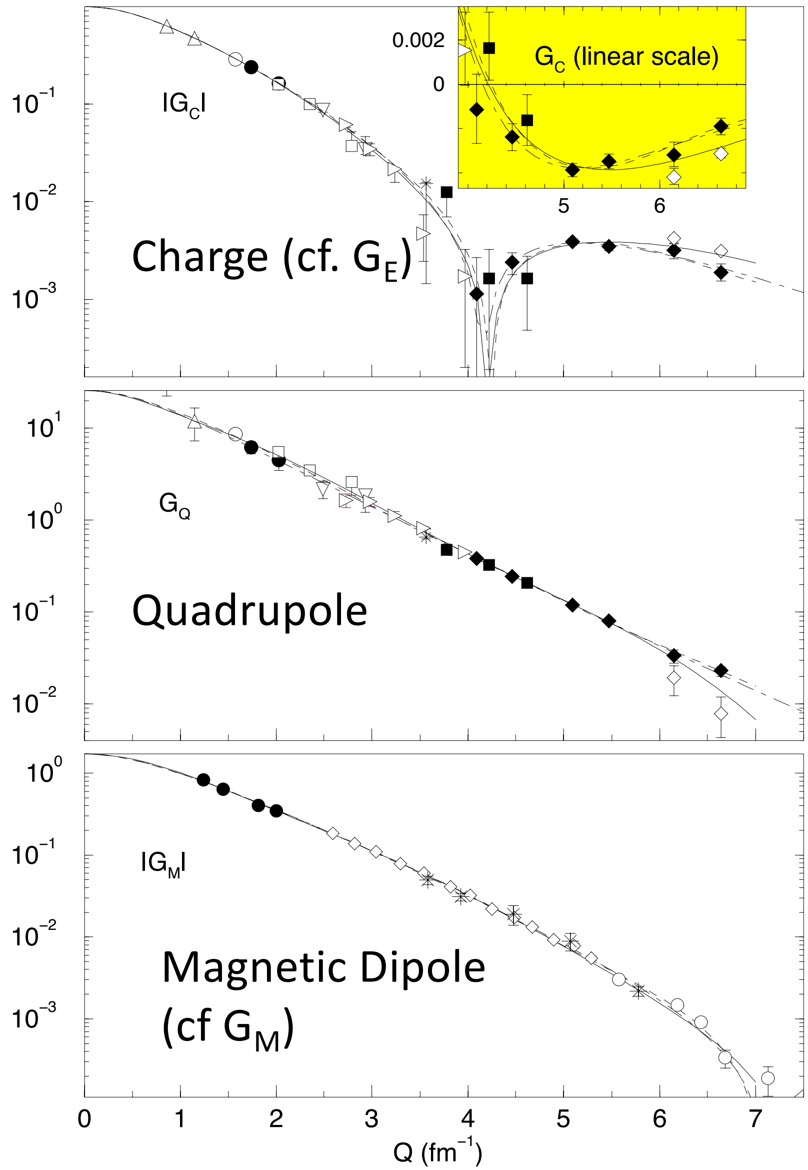
Spatial density contours of the deuteron due to S-D state interference for S_z = ±1 (left) and S_z = 0 (right). z points up.

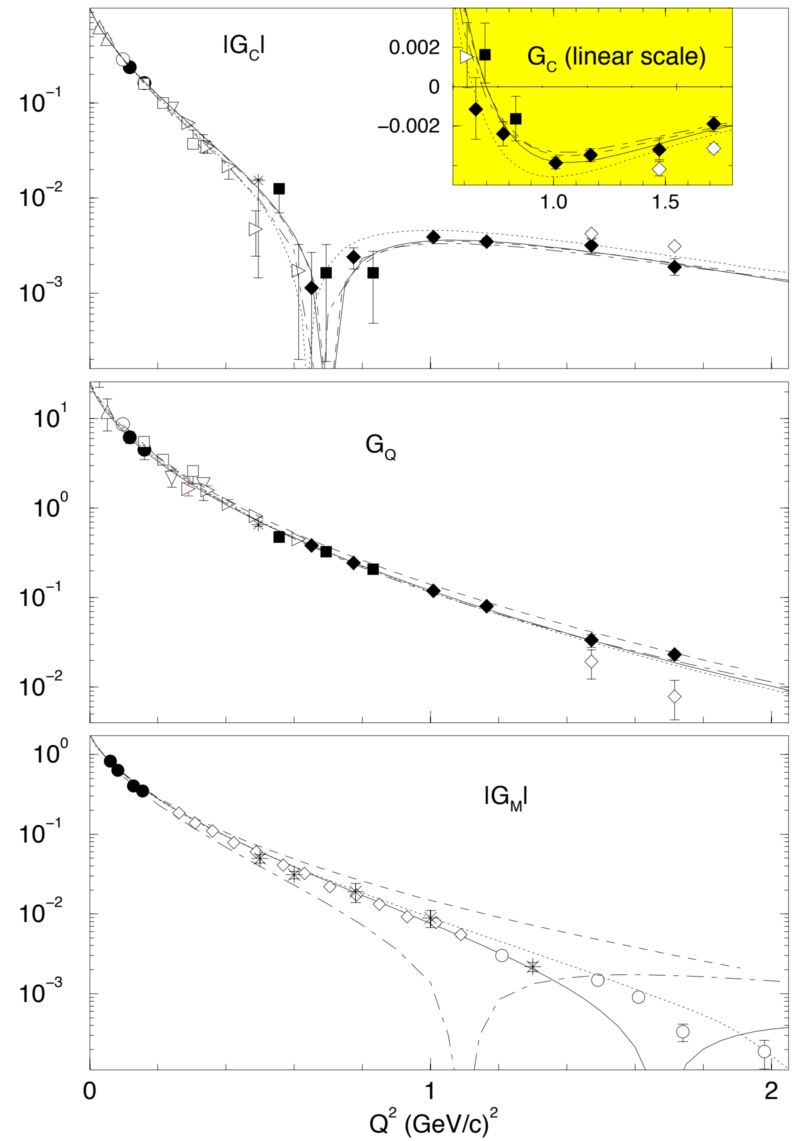
Deuteron Form Factors

Left: non-relativistic potential model

Right: fully relativistic calculation

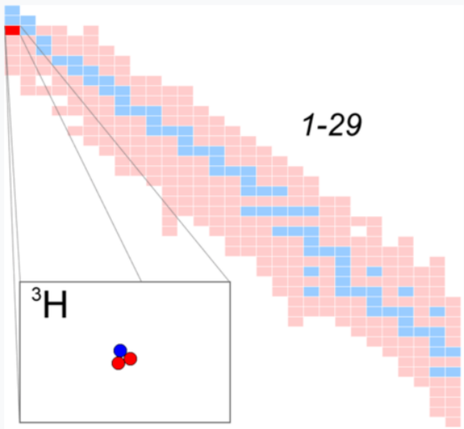
Note different horizontal scale (Q vs Q^2, although same maximum)





Light Nuclei – from Wikipedia

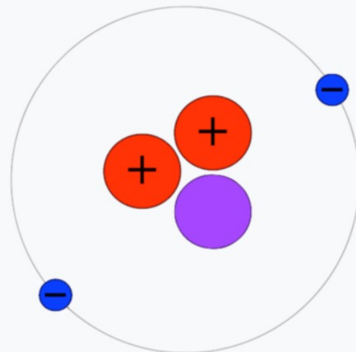
Tritium, ^3H



General	
Name, symbol	tritium,^{3}H
Neutrons	2
Protons	1
Nuclide data	
Natural abundance	trace
Half-life	12.32 years
Decay products	^{3}He
Isotope mass	3.0160492 u
Spin	$^1/_2$
Excess energy	14,949.794± 0.001 keV
Binding energy	8,481.821± 0.004 keV
Decay modes	
Decay mode	**Decay energy (MeV)**
Beta emission	0.018590

Complete table of nuclides

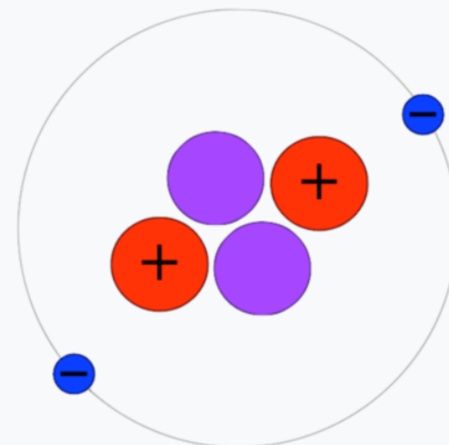
Helium-3, ^3He



General	
Name, symbol	Helium-3, He-3,^{3}He
Neutrons	1
Protons	2
Nuclide data	
Natural abundance	0.000137% (% He on Earth)
Half-life	stable
Parent isotopes	^{3}H (beta decay of tritium)
Isotope mass	3.0160293 u
Spin	$^1/_2$

Note: Isospin doublet
Just like p and n

Helium-4, ^4He



General	
Name, symbol	Helium-4, He-4,^{4}He
Neutrons	2
Protons	2
Nuclide data	
Natural abundance	99.999863%
Half-life	stable
Isotope mass	4.002602 u
Spin	0
Binding energy	28300.7 keV

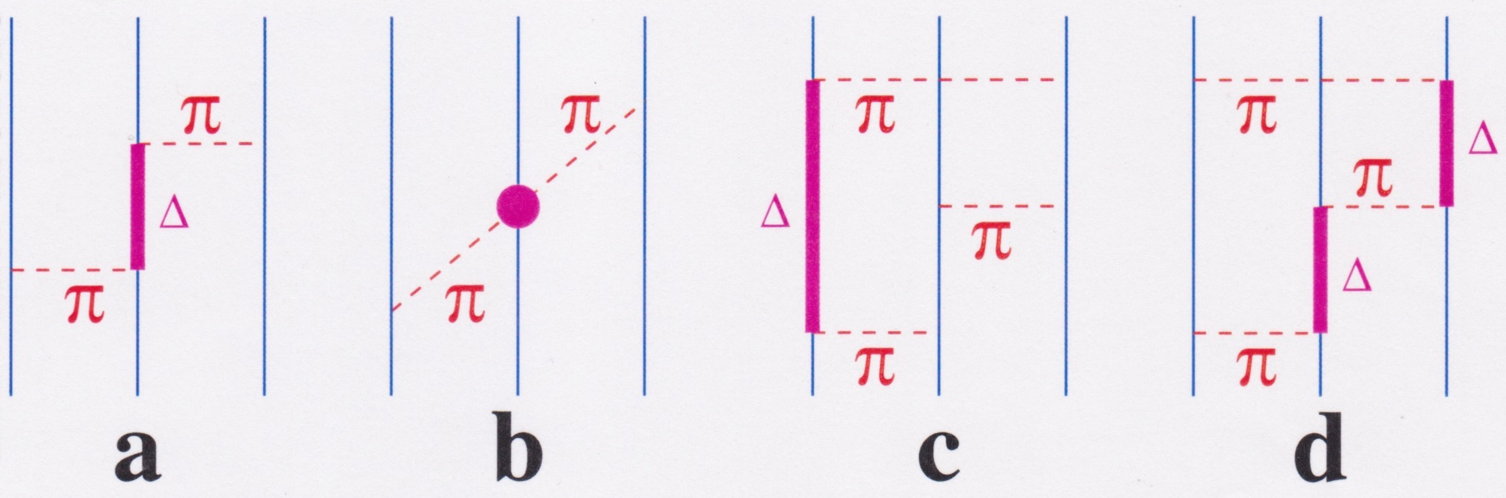
Light Nuclei – How to calculate?
(More from Rev. Mod. Phys. 70, 1998)

N-body Hamiltonian $$H=\sum_i \sqrt{\mathbf{p}_i^2+m^2}+\sum_{i<j} v_{ij}(\mathbf{r}_{ij};\mathbf{P}_{ij})$$ NN potential

May have to add 3-body force $$+\sum_{i<j<k} V_{ijk}(\mathbf{r}_{ij},\mathbf{r}_{ik};\mathbf{P}_{ijk}), \tag{2.12}$$

$$V_{ijk}=V_{ijk}^{2\pi}+V_{ijk}^{R}, \quad V_{ijk}^{R}=U_0\sum_{\text{cyc}} T_\pi^2(r_{ij})T_\pi^2(r_{ik}).$$

Fig. 2 (Pieper, et al.)



Light Nuclei – how to solve Schrödinger Eq.?

- 3-body -> Fadeev approach: applies to both bound states and scattering. Decomposes 3-body wave function in 3 2-body ones.
- 3-4 body: Hyperspherical harmonics
- ≥3: Monte Carlo methods
  - Variational Monte Carlo (uses variational principle with test functions to find minimum energy = G.S.)
  - Green's-function Monte Carlo (Path integral, imaginary time,

Light Nuclei

- Some results:

TABLE VI. ^{4}He binding energies with and without three-nucleon interaction; comparison of different methods: correlated hyperspherical harmonics (CHH), Faddeev-Yakubovsky (FY), variational Monte Carlo (VMC), and Green's-function Monte Carlo (GFMC). Error bars in CHH calculations are estimates of the effects of channel truncation.

Hamiltonian	AV14	AV14+TNI 8
CHH	24.17(5)[a]	27.48[b]
FY	24.01[b]	
VMC		27.6(1)[c]
GFMC	24.23(3)[e]	28.3(2)[f]

[a]Viviani, 1997.
[b]Viviani, Kievksy, and Rosati, 1995.
[c]Glöckle *et al.*, 1995.
[d]Arriaga, Pandharipande, and Wiringa, 1995.
[e]Pudliner *et al.*, 1977.
[f]Carlson and Schiavilla, 1994a.

TABLE VII. Experimental and quantum Monte Carlo energies of $A=3$–7 nuclei in MeV (Pudliner *et al.*, 1997), for variational Monte Carlo (VMC), Green's-function Monte Carlo (GFMC), and experiment.

$^AZ(J^\pi;T)$	VMC	GFMC	Expt.
^{2}H$(1^+;0)$	$-2.2248(5)$		-2.2246
^{3}H$(\frac{1}{2}^+;\frac{1}{2})$	$-8.32(1)$	$-8.47(1)$	-8.48
^{4}He$(0^+;0)$	$-27.76(3)$	$-28.30(2)$	-28.30
^{6}He$(0^+;1)$	$-24.87(7)$	$-27.64(14)$	-29.27
^{6}He$(2^+;1)$	$-23.01(7)$	$-25.84(11)$	-27.47
^{6}Li$(1^+;0)$	$-28.09(7)$	$-31.25(11)$	-31.99
^{6}Li$(3^+;0)$	$-25.16(7)$	$-28.53(32)$	-29.80
^{6}Li$(0^+;1)$	$-24.25(7)$	$-27.31(15)$	-28.43
^{6}Li$(2^+;0)$	$-23.86(8)$	$-26.82(35)$	-27.68
^{6}Be$(0^+;1)$	$-22.79(7)$	$-25.52(11)$	-26.92
^{7}He$(\frac{3}{2}^-;\frac{3}{2})$	$-20.43(12)$	$-25.16(16)$	-28.82
^{7}Li$(\frac{3}{2}^-;\frac{1}{2})$	$-32.78(11)$	$-37.44(28)$	-39.24
^{7}Li$(\frac{1}{2}^-;\frac{1}{2})$	$-32.45(11)$	$-36.68(30)$	-38.76
^{7}Li$(\frac{7}{2}^-;\frac{1}{2})$	$-27.30(11)$	$-31.72(30)$	-34.61
^{7}Li$(\frac{5}{2}^-;\frac{1}{2})$	$-26.14(11)$	$-30.88(35)$	-32.56
^{7}Li$(\frac{3}{2}^-;\frac{3}{2})$	$-19.73(12)$	$-24.79(18)$	-28.00

Light Nuclei – more results

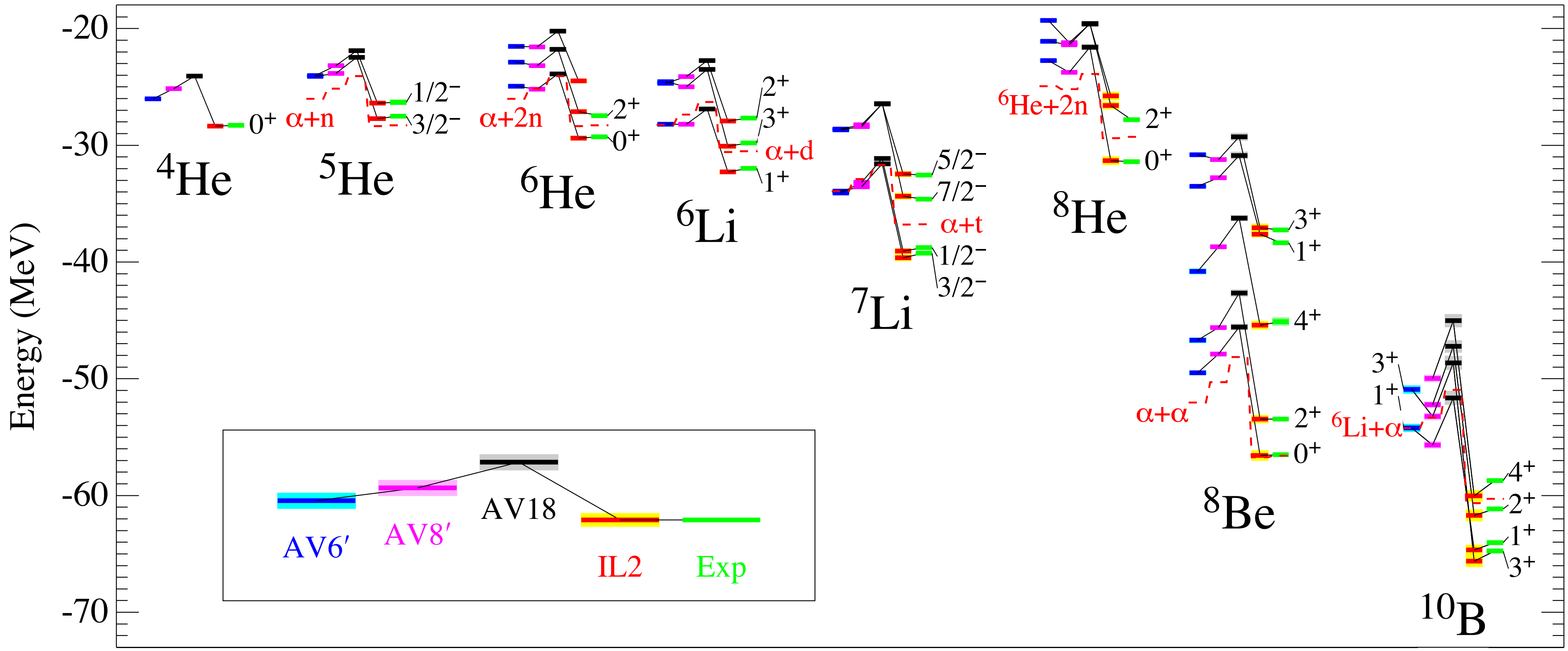


FIG. 2: Nuclear energy levels for the more realistic potential models; shading denotes Monte Carlo statistical errors.

Form Factors of light nuclei

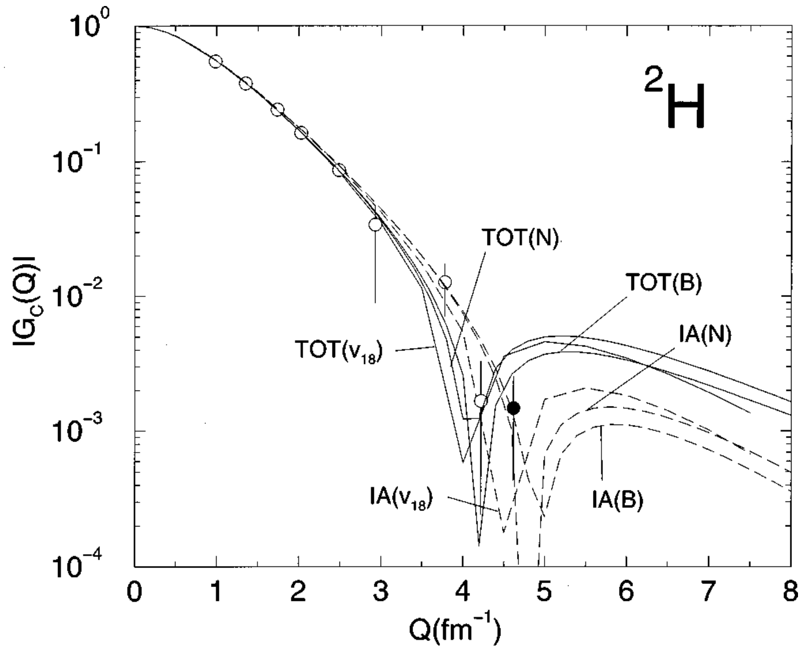


FIG. 18. The charge form factor of the deuteron, obtained in the impulse approximation (IA) and with inclusion of two-body charge contributions and relativistic corrections (TOT), compared with data from Schulze *et al.* (1984), The *et al.* (1991), Dmitriev *et al.* (1985), and Gilman *et al.* (1990) [empty and filled circles denote, respectively, positive and negative experimental values for $G_C(Q)$]. Theoretical results corresponding to the Argonne v_{18} (v_{18}; Wiringa, Stoks, and Schiavilla, 1995), Bonn B (B; Plessas, Christian, and Wagenbrunn, 1995), and Nijmegen (N; Plessas, Christian, and Wagenbrunn, 1995) interactions are displayed. The Höhler parametrization is used for the nucleon electromagnetic form factors.

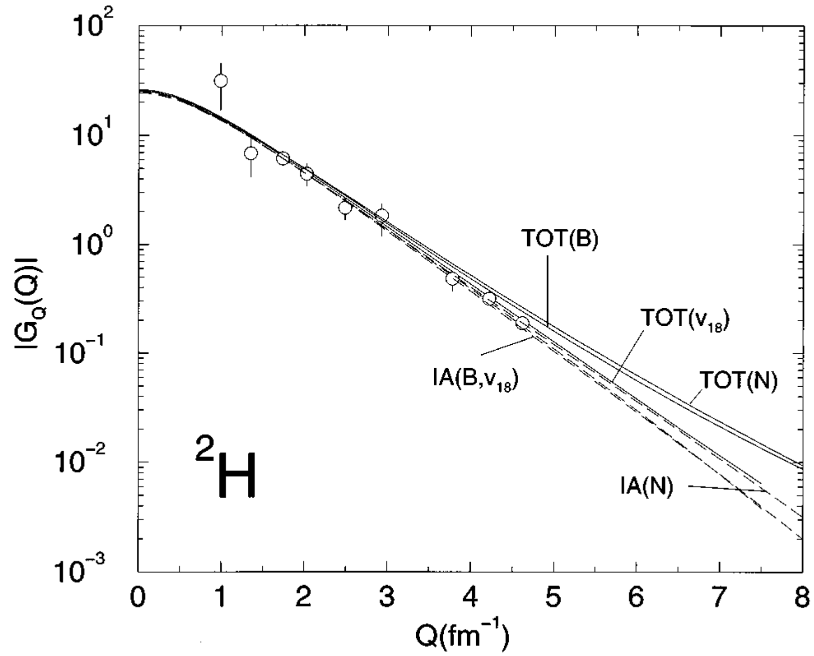


FIG. 19. Same as in Fig. 18, but for the quadrupole form factor of the deuteron.

More light nuclei…

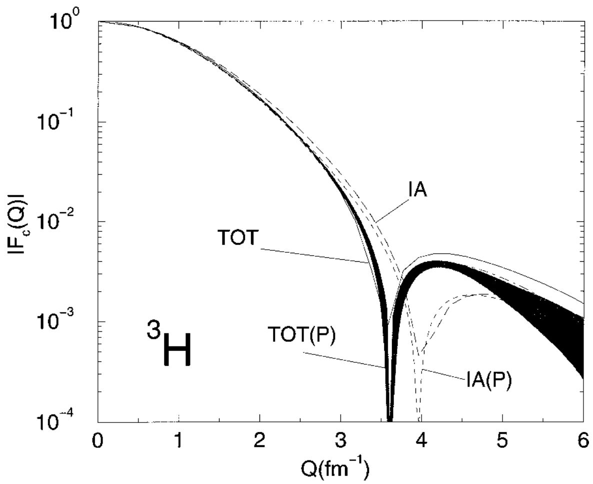


FIG. 27. The charge form factors of ^{3}H, obtained in the impulse approximation (IA) and with inclusion of two-body charge contributions and relativistic corrections (TOT), compared with data (shaded area) from Amround *et al.* (1994).

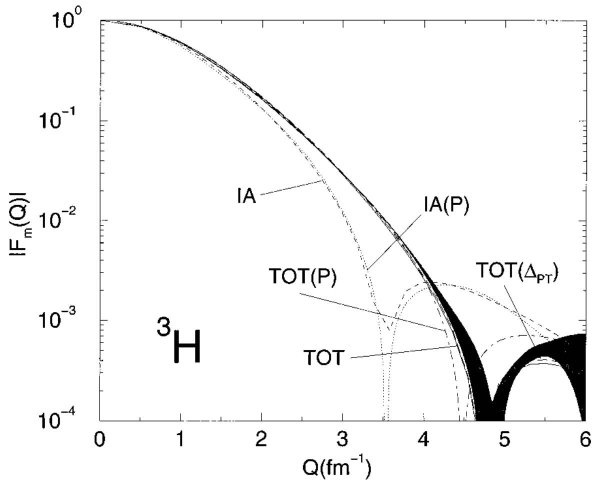


FIG. 25. The magnetic form factors of ^{3}H, obtained in the impulse approximation (IA) and with inclusion of two-body current contributions and Δ admixtures in the bound-state

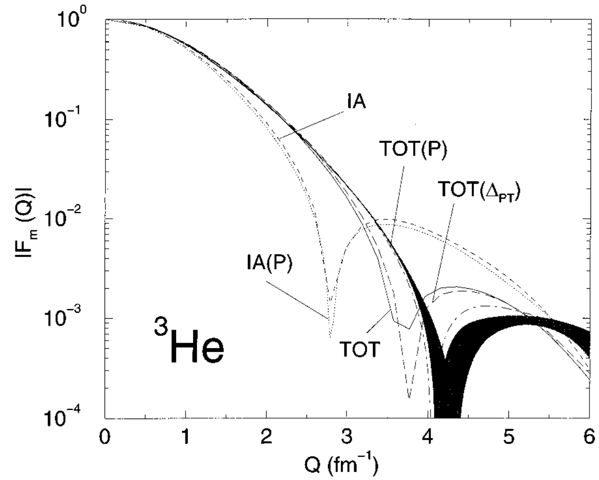


FIG. 26. Same as in Fig. 25, but for ^{3}He.

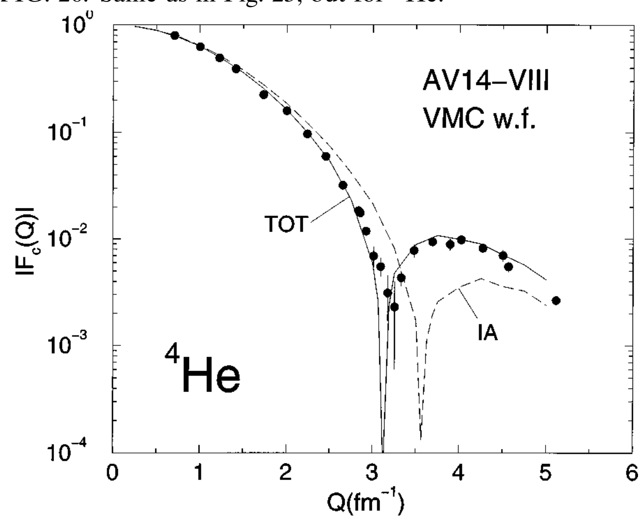


FIG. 29. The charge form factors of ^{4}He, obtained in the impulse approximation (IA) and with inclusion of two-body charge contributions and relativistic corrections (TOT), compared with data from Frosch *et al.* (1968) and Arnold *et al.*

Mixing GFMC wave functions with chiral Effective Field Theory currents:

Magnetic moments in $A \leq 10$ nuclei

Pastore *et al.* (2013)

- GFMC calculations use AV18/IL7 (rather than chiral) potentials with χEFT EM currents
- Predictions for $A > 3$; about 40% of $\mu(^9\mathrm{C})$ due to corrections beyond LO