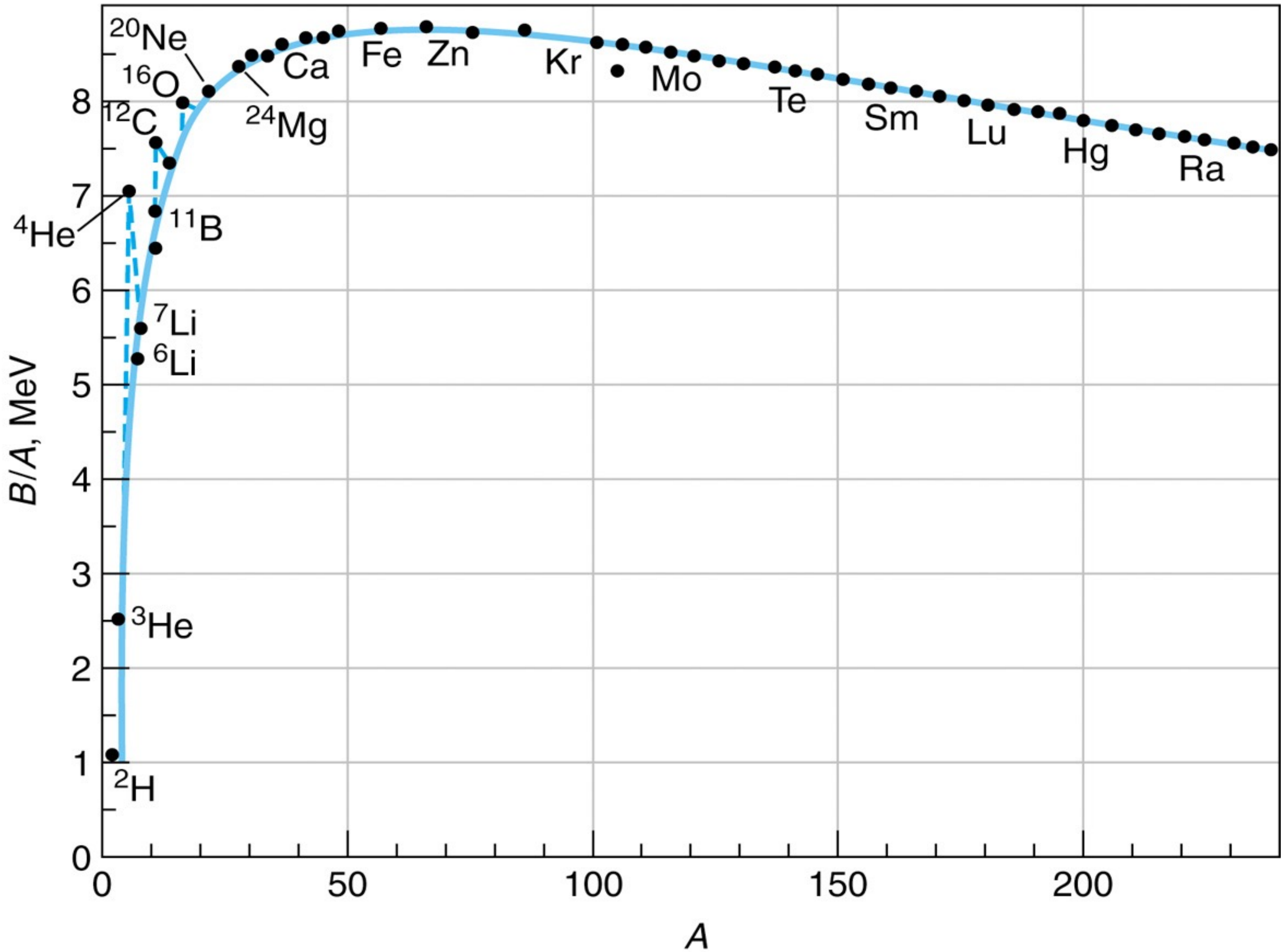
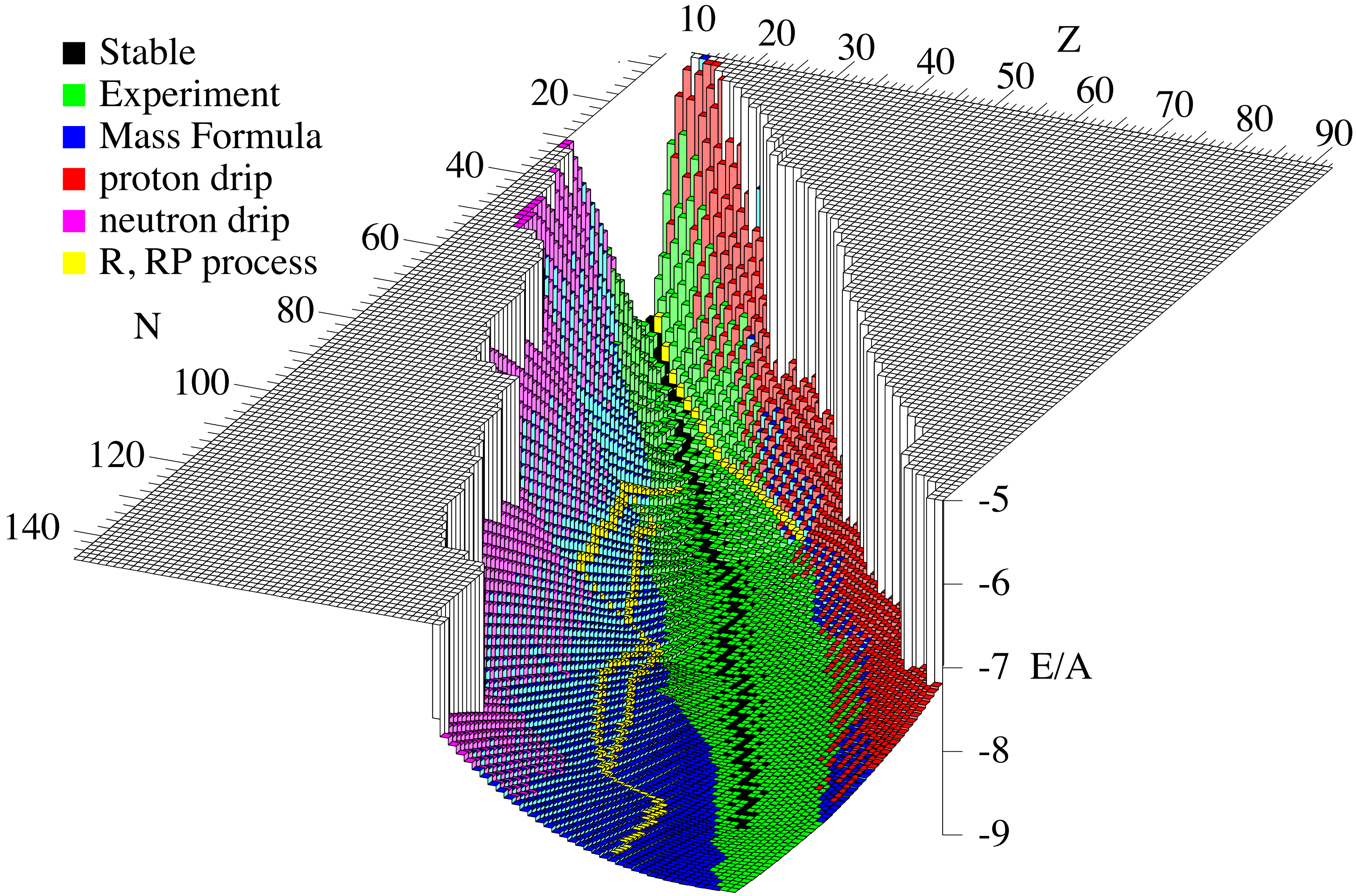
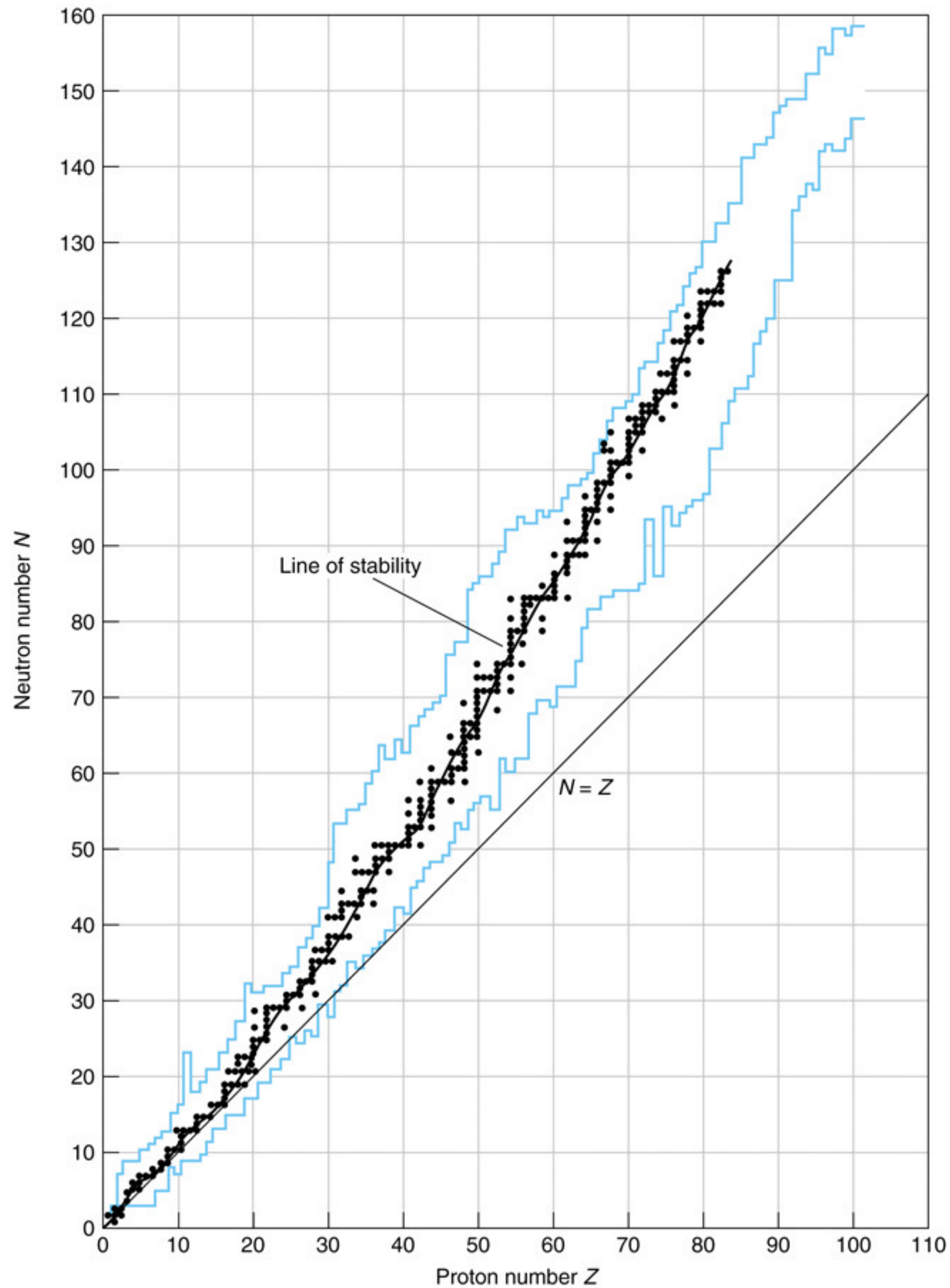
Nuclear Binding energies



What do we know about them?



Stable nuclei



Liquid Drop Model

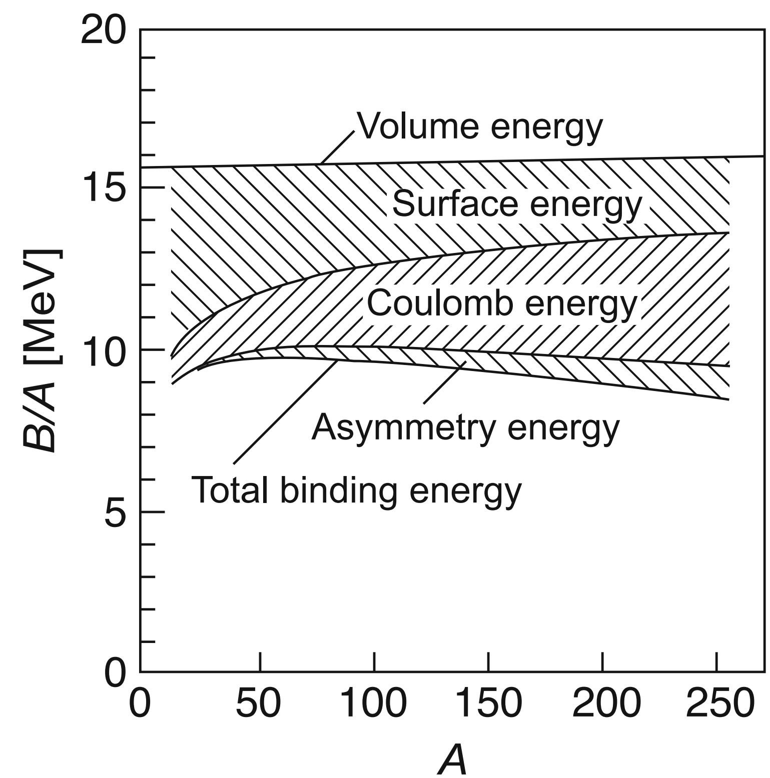
From previous slides, we find that nuclear density is roughly constant, and hence the nuclear radius goes like $A^{1/3}$

$R = 1.22\ \mathrm{fm} \cdot A^{1/3}$
Surface = $19\ \mathrm{fm}^2 \cdot A^{2/3}$

$$M(A,Z) = NM_\mathrm{n} + ZM_\mathrm{p} + Zm_\mathrm{e} - a_\mathrm{v}A + a_\mathrm{s}A^{2/3} + a_\mathrm{c}\frac{Z^2}{A^{1/3}} + a_\mathrm{a}\frac{(N-Z)^2}{4A} + \frac{\delta}{A^{1/2}}$$

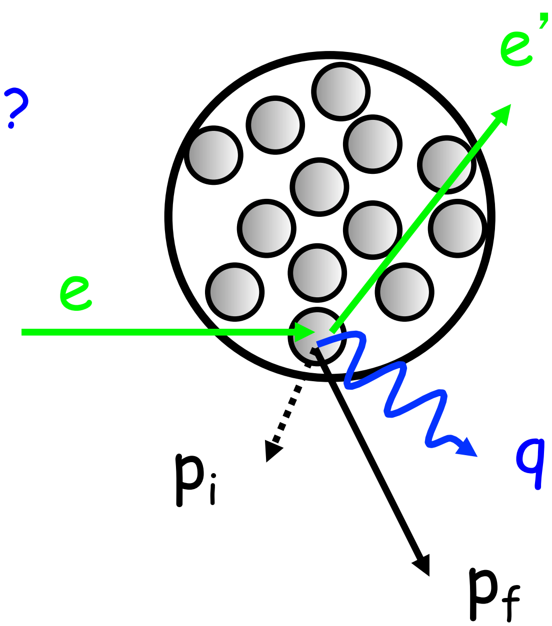
$$a_\mathrm{v} = 15.67\ \mathrm{MeV}/c^2$$
$$a_\mathrm{s} = 17.23\ \mathrm{MeV}/c^2$$
$$a_\mathrm{c} = 0.714\ \mathrm{MeV}/c^2$$
$$a_\mathrm{a} = 93.15\ \mathrm{MeV}/c^2$$
$$\delta = \begin{cases} -11.2\ \mathrm{MeV}/c^2 & \text{for even } Z \text{ and } N \text{ (even-even nuclei)} \\ 0\ \mathrm{MeV}/c^2 & \text{for odd } A \text{ (odd-even nuclei)} \\ +11.2\ \mathrm{MeV}/c^2 & \text{for odd } Z \text{ and } N \text{ (odd-odd nuclei).} \end{cases}$$

Central (saturation) density:
$$\varrho_0 \approx 0.17\ \mathrm{nucleons/fm^3} = 3 \cdot 10^{17}\ \mathrm{kg/m^3}$$
Average density: $0.13\ \mathrm{nucleons/fm^3}$



Fermi gas model:

how simple a model can you make ?



Initial nucleon energy: $KE_i = p_i^2 / 2m_p$

Final nucleon energy: $KE_f = p_f^2 / 2m_p = (\vec{q} + \vec{p}_i)^2 / 2m_p$

Energy transfer: $\nu = KE_f - KE_i = \dfrac{\vec{q}^{\,2}}{2m_p} + \dfrac{\vec{q} \cdot \vec{p}_i}{m_p}$

Expect:

- Peak centroid at $\nu = q^2/2m_\mathrm{p} + \varepsilon$
- Peak width $2qp_\mathrm{fermi}/m_\mathrm{p}$
- Total peak cross section = $Z\sigma_\mathrm{ep} + N\sigma_\mathrm{en}$

Fermi Gas

- Pauli exclusion principle: No two fermions (spin 1/2 particles) can be in the same quantum state

$$h = 2\pi\hbar$$

- Heisenberg uncertainty principle: $\Delta p \cdot \Delta x \approx \hbar$ => two states are indistinguishable if they occupy the same "cell" $dV \cdot d^3p = h^3$ in "phase space" (except for factor 2 because of spin degree of freedom) => for volume V and "momentum volume" $d^3p = 4\pi p^2 dp$ we find for the Number of states between $p \ldots p+dp$:

$$dN = 2\frac{V}{h^3}4\pi p^2 dp = \frac{V}{\pi^2\hbar^3}p^2 dp \quad \Rightarrow \quad N_{tot} = \frac{V}{\pi^2\hbar^3}\frac{p_f^3}{3} \quad \Rightarrow \quad p_f = \hbar\left(3\pi^2\right)^{1/3} n^{1/3}; \quad n = \frac{N_{tot}}{V}$$

- $R = 1.22\ \text{fm} \cdot A^{1/3}$ => $V = 7.6\ \text{fm}^3 \cdot A$ =>
  if we count p and n separately but assume equal number, n = ½ /7.6 fm^3 =>
  $n^{1/3} = 0.404/\text{fm}$
  => p_f is approximately 250 MeV/c

Nuclear Fermi Momenta of ^{2}H, ^{27}Al and ^{56}Fe from an Analysis of CLAS data

Hui Liu,[1,2,*] Na-Na Ma,[3,†] and Rong Wang[1,4,‡]

TABLE III. Fermi Momenta of some nuclei determined in this work. $k_{\mathrm{F,\ exp.}}$ denotes the Fermi momentum given by our analysis of the CLAS data. The errors are the statistical errors only. $k_{\mathrm{F,\ theo.}}$ denotes the Fermi momentum given by the calculation from the Fermi gas model for the nucleus (see Eq. (9)).

Nucleus	$k_{\mathrm{F,\ exp.}}$ (MeV/c)	$k_{\mathrm{F,theo.}}$ (MeV/c)
^{2}H	116 ± 7	140
^{27}Al	232 ± 27	226
^{56}Fe	244 ± 28	231

Fermi Gas

- Pauli exclusion principle: No two fermions (spin 1/2 particles) can be in the same quantum state
- Heisenberg uncertainty principle: $\Delta p \cdot \Delta x \approx \hbar$ => two states are indistinguishable if they occupy the same "cell" $dV \cdot d^3p = h^3$ in "phase space" (except for factor 2 because of spin degree of freedom) => for volume V and "momentum volume" $d^3p = 4\pi p^2 dp$ we find for the Number of states between $p \ldots p+dp$:

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$$h = 2\pi\hbar$$

- Sirius B: p_f = 670 keV/c for electrons (semi-relativistic - m_e = 511 keV/c^2!)

Sun: $6 \cdot 10^{56}$ e^-

– total kinetic energy:

$$E_{tot}^{kin} = \int_0^{p_f} E(p)\frac{V}{\pi^2\hbar^3}p^2\,dp = \begin{cases} \displaystyle\int_0^{p_f}\frac{p^2}{2m}\frac{V}{\pi^2\hbar^3}p^2\,dp = \frac{1}{2m}\frac{V}{\pi^2\hbar^3}\frac{p_f^5}{5} = \frac{3}{5}N_{tot}\frac{p_f^2}{2m} = \frac{3\hbar^2}{10m}N_{tot}\left(3\pi^2\right)^{2/3}\left(\frac{N_{tot}}{V}\right)^{2/3} = \frac{3\hbar^2\left(\frac{9\pi}{4}\right)^{2/3}}{10m}\frac{N_{tot}^{5/3}}{R^2}\text{; non-rel.} \\ \displaystyle\int_0^{p_f} pc\frac{V}{\pi^2\hbar^3}p^2\,dp = \frac{Vc}{\pi^2\hbar^3}\frac{p_f^4}{4} = \frac{3}{4}N_{tot}cp_f = \frac{3}{4}\hbar c N_{tot}\left(3\pi^2\right)^{1/3}\left(\frac{N_{tot}}{V}\right)^{1/3} = \frac{3\hbar c\left(\frac{9\pi}{4}\right)^{1/3}}{4}\frac{N_{tot}^{4/3}}{R}\text{; ultra-relativistic} \end{cases}$$

White Dwarf Stability

- If R **de**creases, gravitational energy more negative:

$$\frac{dV_{pot}^{grav}}{d(-R)} = -\frac{d}{dR}\left(-\frac{3GM^2}{5R}\right) = -\frac{3GM^2}{5R^2}$$

- …while kinetic energy goes up:

$$\frac{dE_{tot}^{kin}}{d(-R)} = -\frac{d}{dR}\left(\frac{3\hbar^2}{10m}\left(\frac{9\pi}{4}\right)^{2/3}\frac{N_{tot}^{5/3}}{R^2}\right) = \frac{3\hbar^2}{5m}\left(\frac{9\pi}{4}\right)^{2/3}\frac{N_{tot}^{5/3}}{R^3}\,;\text{non-rel.}$$

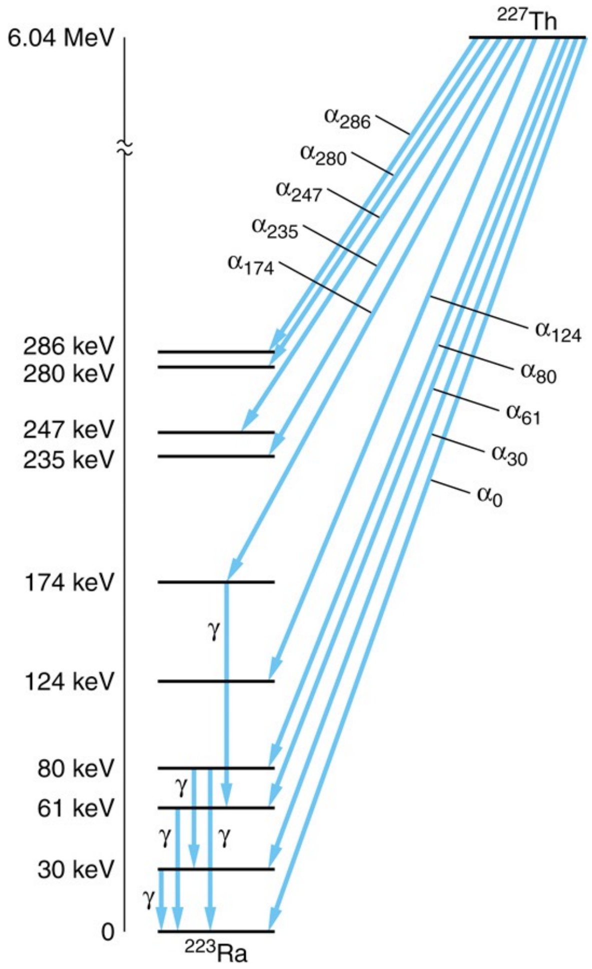
- Compare: Equilibrium if sum of derivatives = 0

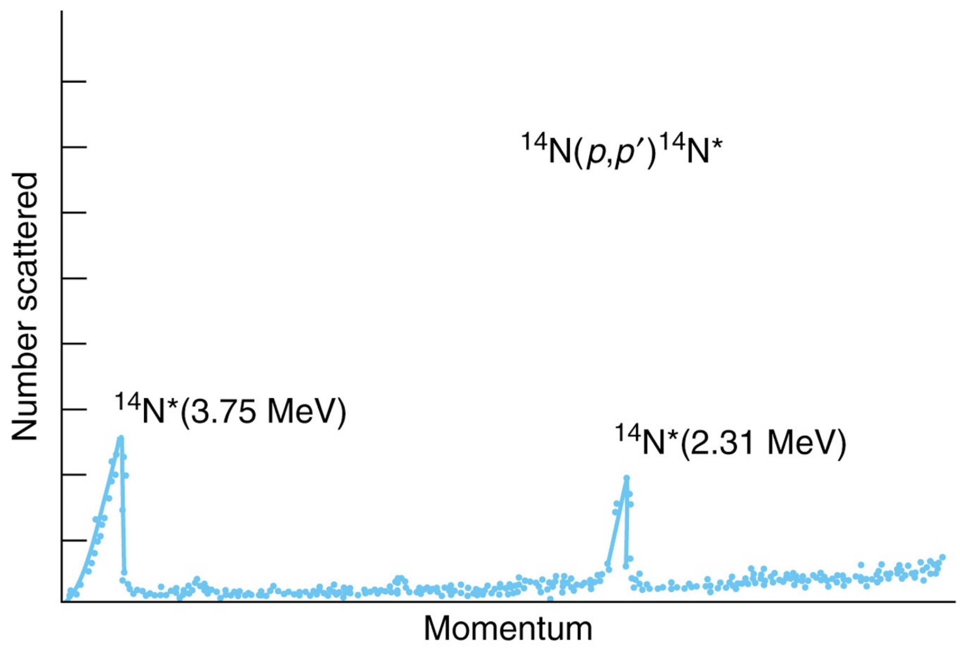
$$-\frac{3GM^2}{5R^2} + \frac{3\hbar^2}{5m}\left(\frac{9\pi}{4}\right)^{2/3}\frac{N_{tot}^{5/3}}{R^3} = 0 \Rightarrow \quad R = \frac{\hbar^2 N_{tot}^{5/3}}{m_e G M^2}\left(\frac{9\pi}{4}\right)^{2/3} \propto \frac{M^{5/3}}{M^{6/3}}$$

= 7280 (really: 5500) km / $(M/M_{sun})^{1/3}$

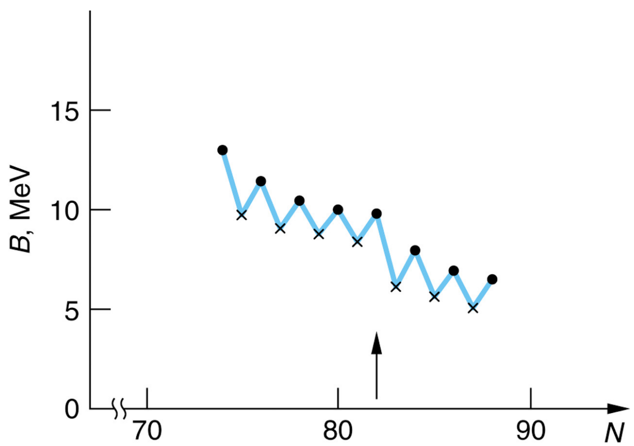
Shell Model
- slides by Dr. Weinstein

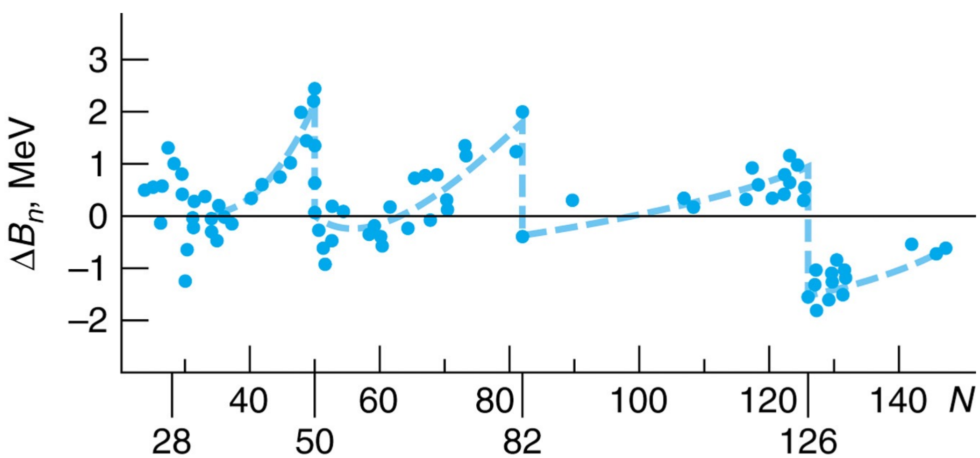
Evidence for excited states

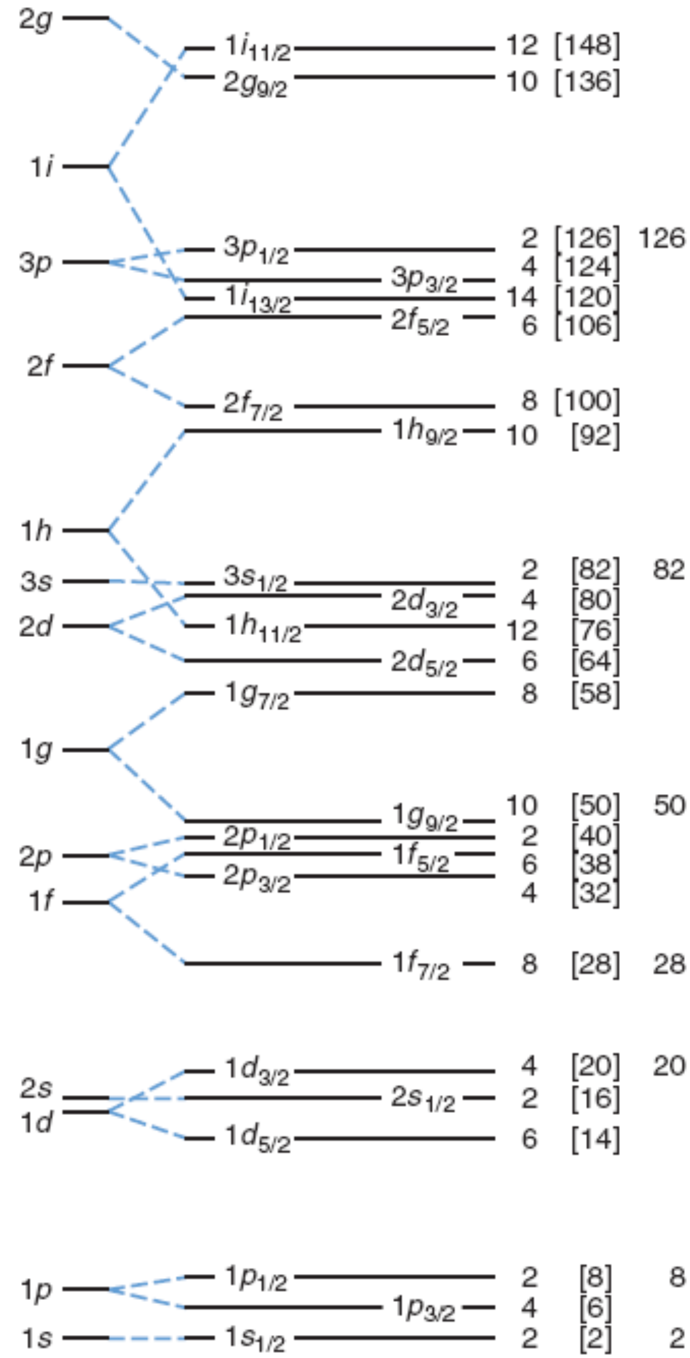




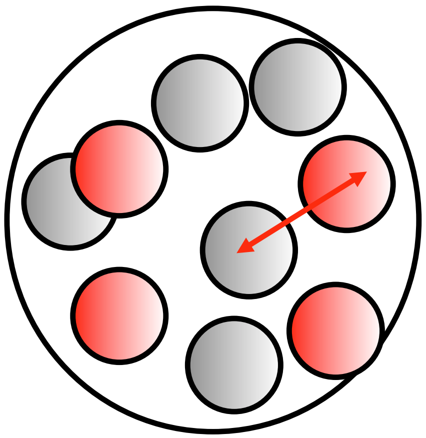
Nuclear Level Scheme





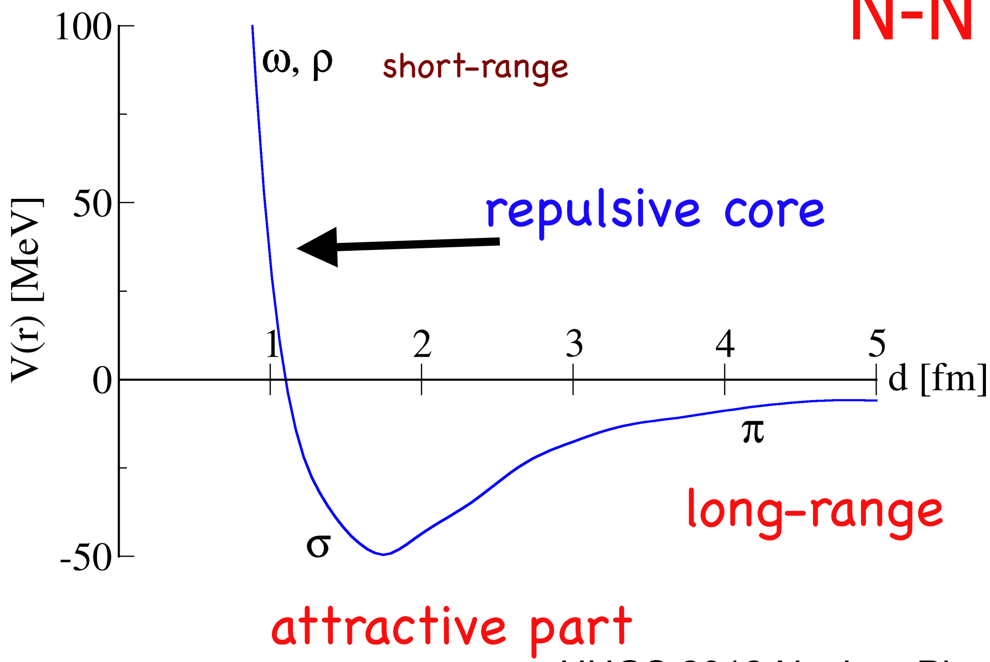


Structure of the nucleus

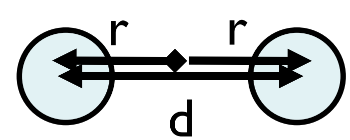


- nucleons are bound
  - •energy (*E*) distribution
  - •shell structure
- nucleons are not static
  - •momentum (*k*) distribution

determined by the N-N potential

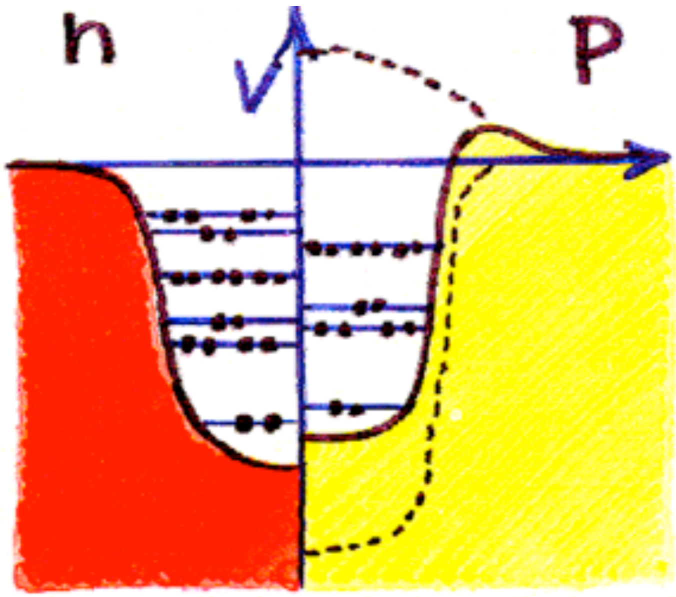


on average:
Net binding energy: ≈ 8 MeV
distance: ≈ 2 fm



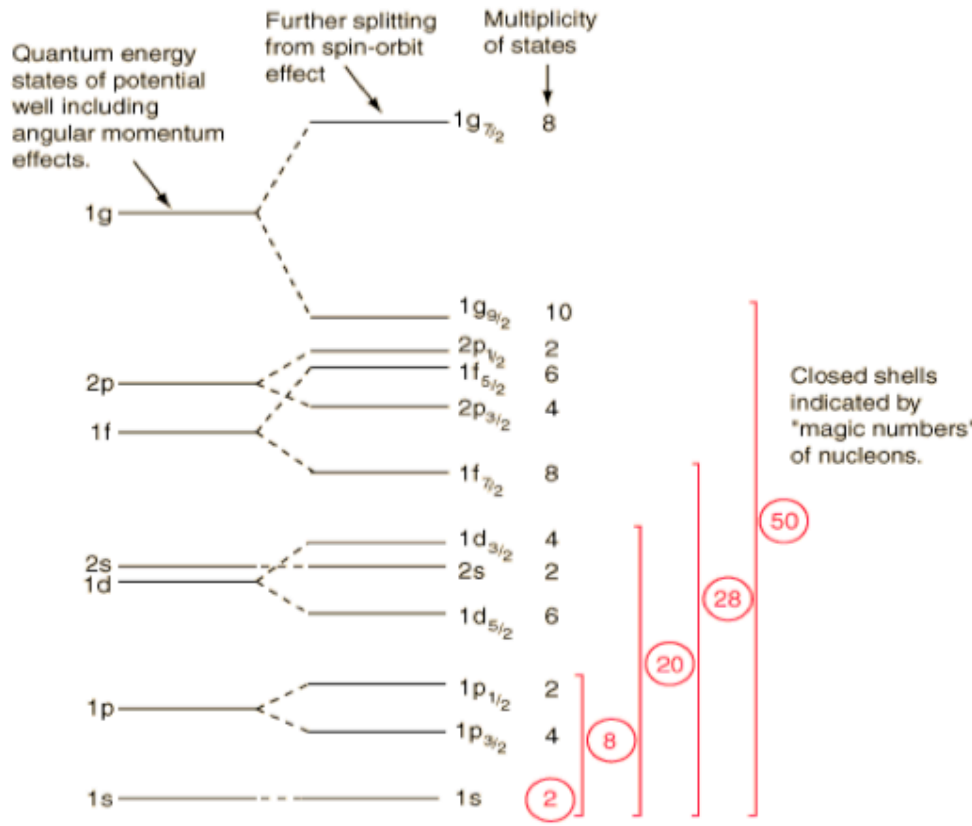
Strong repulsion ⟹ NN correlations

Shell Structure (Maria Goeppert-Mayer, Jensen, 1949, Nobel Prize 1963)



nuclear density 10^{18} kg/m^3

With the enormous strong force acting between them and with so many nucleons to collide with, how can nucleons possibly complete whole orbits without interacting?



But: there is experimental evidence for shell structure

Pauli Exclusion Principle: $\longrightarrow$ nucleons can not scatter into occupied levels: Suppression of collisions between nucleons

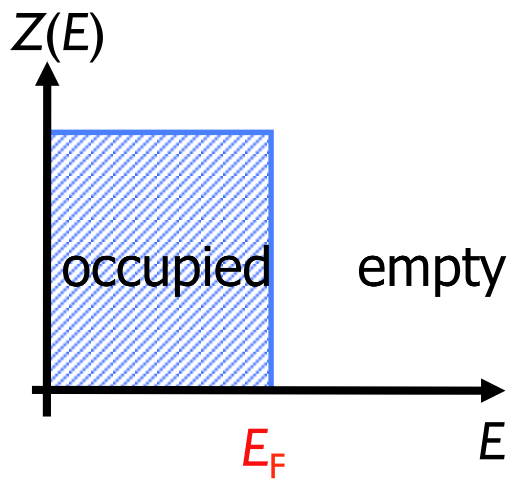
Magnetic moments

- "Natural" unit: 1 nuclear magneton $\mu_N = \dfrac{e\hbar}{2m_p}$
- Classical prediction: $\boldsymbol{\mu} = \mu_N J$
- Generally: $\boldsymbol{\mu} = \mu_N\, g_J\, J$
- Dirac/Relativity: for J = S, $g_S = 2$ (pretty good for electrons)
- For protons, $g_S = 5.58 \rightarrow$ anomalous moment $\kappa = (g-2)/2 = 1.79$; for neutrons $g_S = -3.83 \rightarrow \kappa = -1.91$ (huh? n is neutral!!!)
- Orbital motion only: $g_L = 1$ (p), 0 (n)
- For nucleon w/ S,L,J: $\dfrac{\mu}{\mu_N} = \left(g_l \pm \dfrac{g_s - g_l}{2l+1} \right) j \quad \text{where} \quad j = l \pm \dfrac{1}{2}$

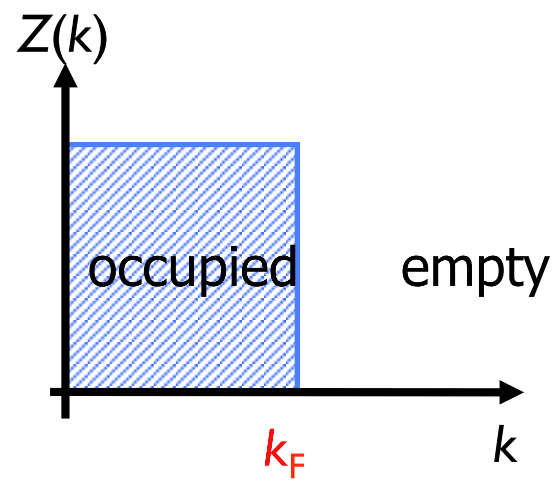
Independent Particle Shell model (IPSM)

- single particle approximation:
  nucleons move independently from each other
  in an average potential created by the other nucleons (mean field)
  - spectral function $S(E,k)$:
    probability of finding a proton with initial momentum k and energy E in the nucleus
- factorizes into energy & momentum part

nuclear matter:



$$E_F = \frac{k_F^2}{2m_p}$$



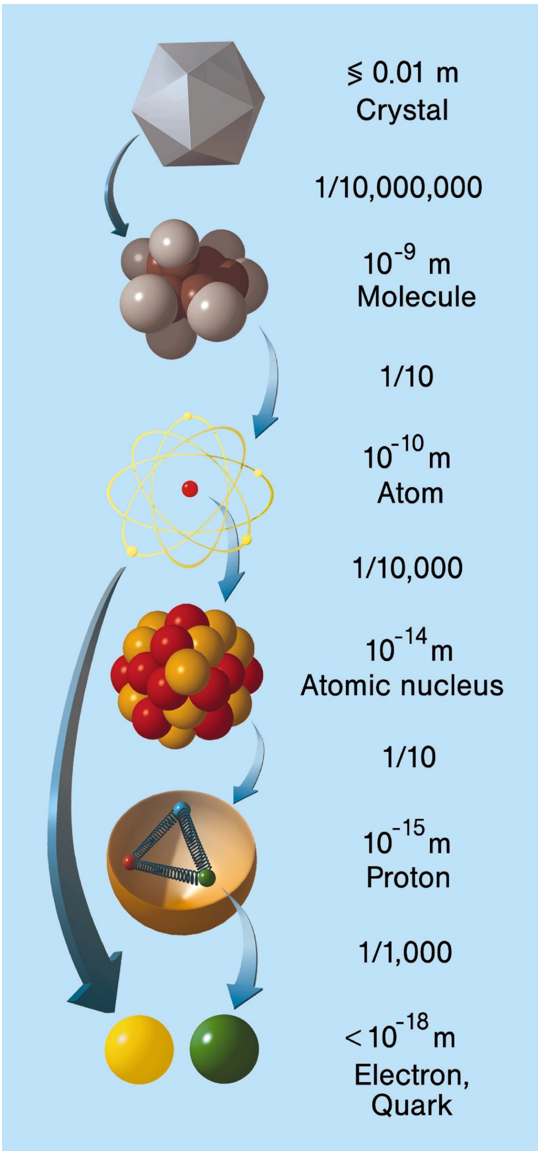
nuclei: $S(\vec{p},E) = \sum_i |\Phi_a(p)|^2 \delta(E+\epsilon_a)$

Not 100% accurate, but a good starting point

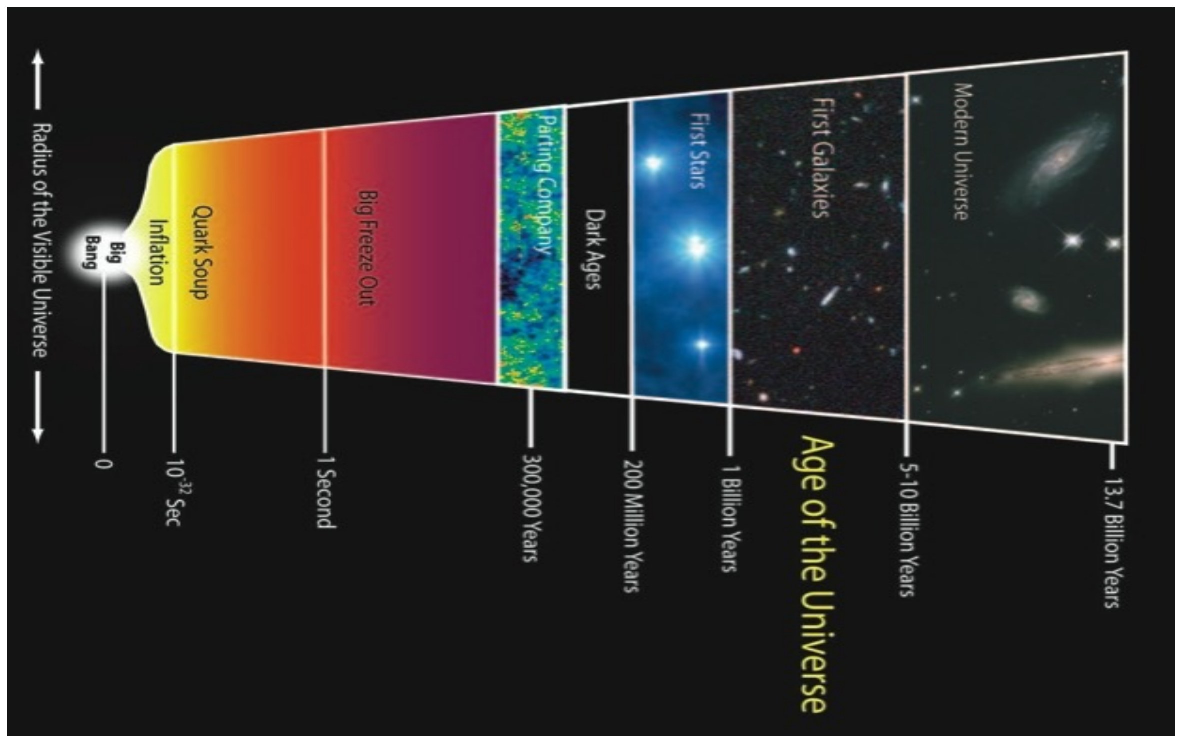
Nuclei in the Cosmos

- When and where were all the known nuclei existing naturally on Earth produced?
- What kind of nuclear reactions are involved?
- What kind of stellar or galactic or Big Bang environments provide these reactions?
- How can we learn more about this with experiments on Earth?

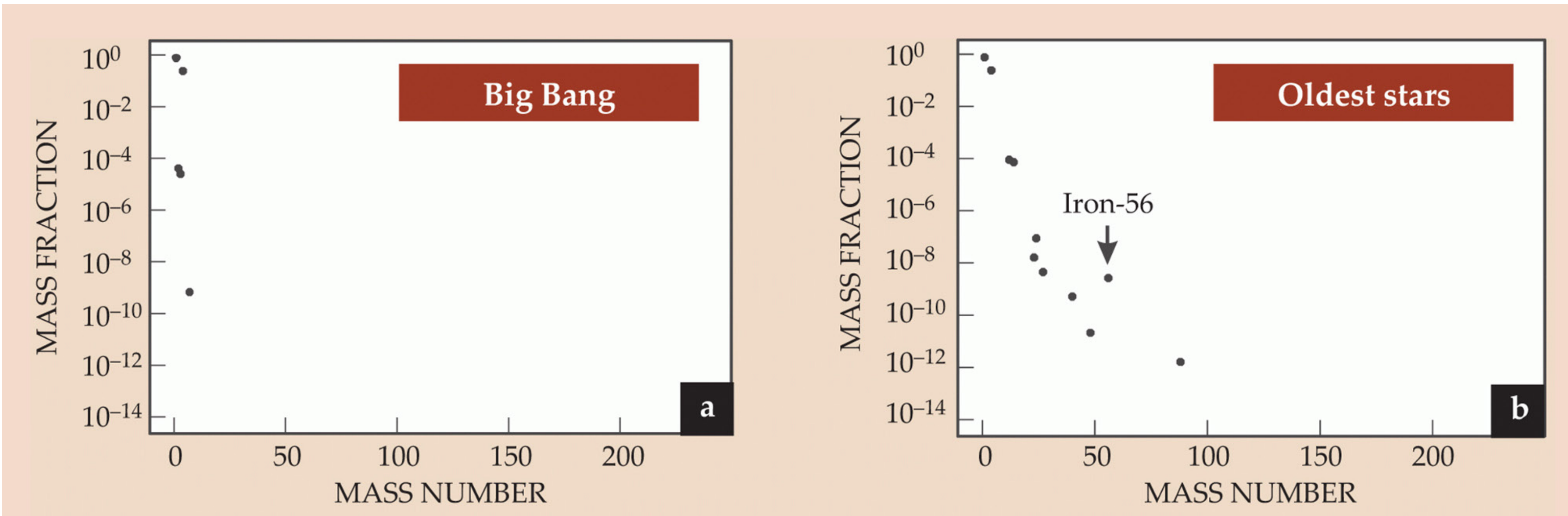
The Structure of Matter



- What **nuclei** is the Universe made off?
- What nuclei where there in the beginning (right after the big bang)?
- When and how did nuclei important for life form?
- Where do heavy nuclei come from?



Where does ^{4}He come from ?



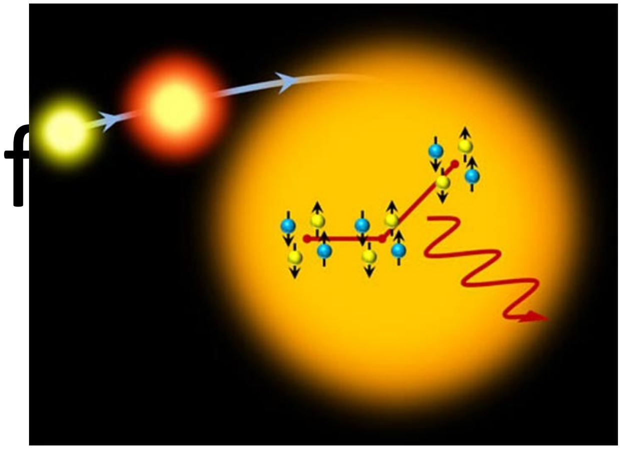
First "3" minutes:

- quarks fuse to p, n (everything else decays)
- p+n = d, d+p=^{3}He, ^{3}He+n = ^{4}He
- Competes with n decay (15 min) => observed abundance = test of Big Bang models
- Smattering of Li,…

"Ordinary" nucleosynthesis in stars (like the sun):

p+p = d + e^+ + ν -> ^{4}He, …

C, N, O: Elements for Lif



- How do you form C?

The **Hoyle state** is
the triple-alpha p
of this 7.7 MeV re
burning stars and
The existence of t
properties are stil
The Hoyle state is
a high-temperatu
a consequence of
followed within ~
lived resonance w
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In 2011, an ab init
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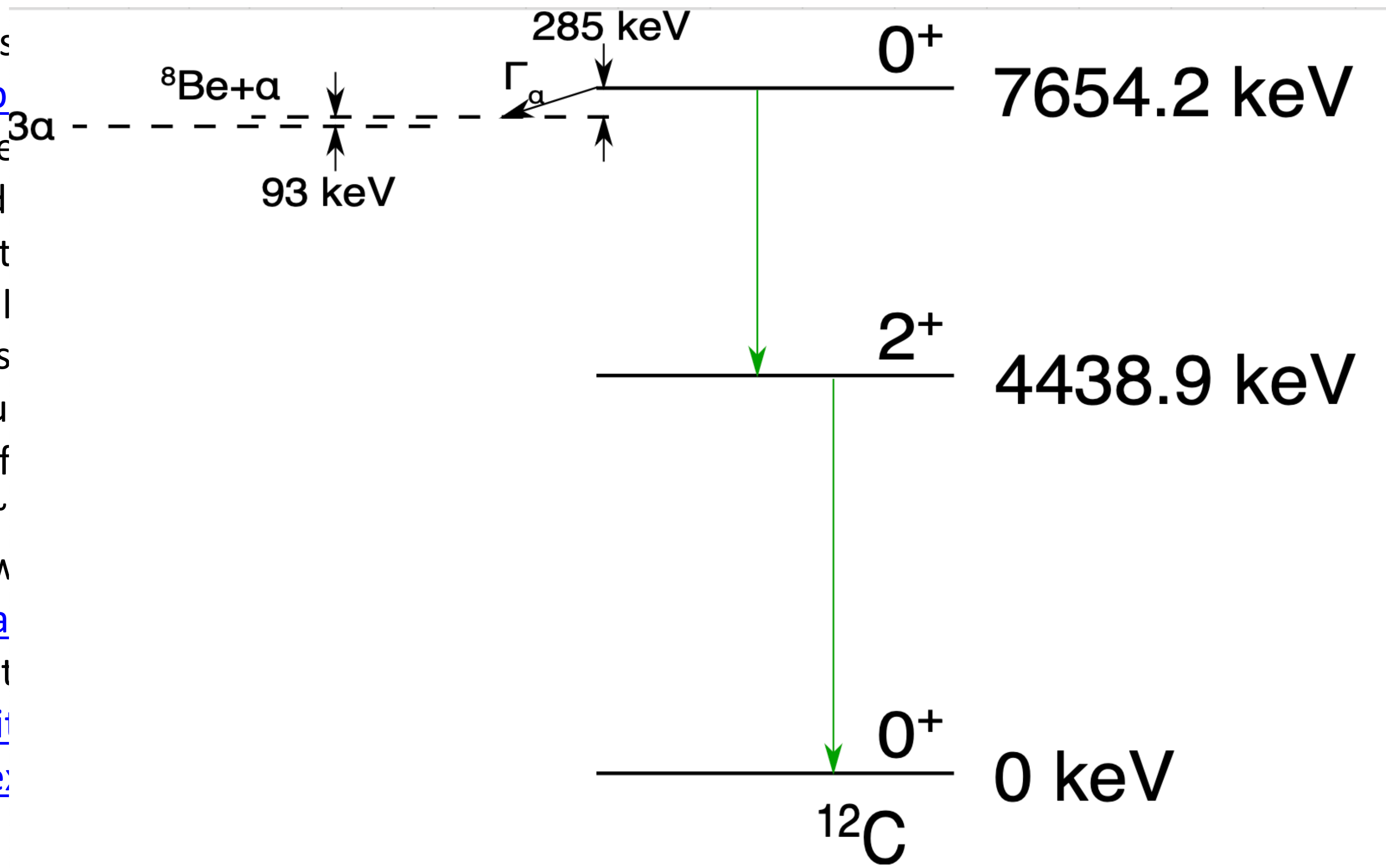
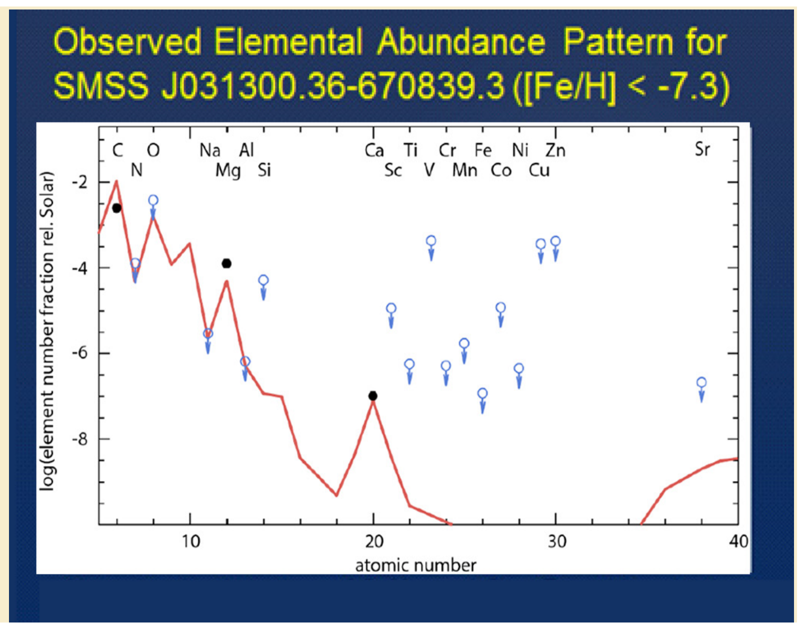


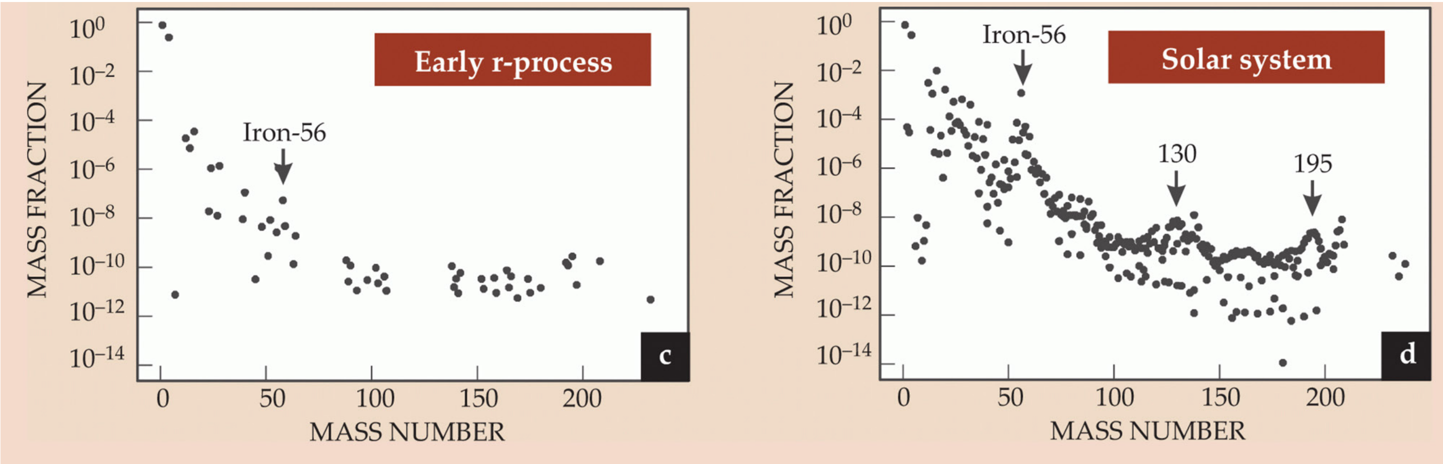
C, N, O: Elements for Life

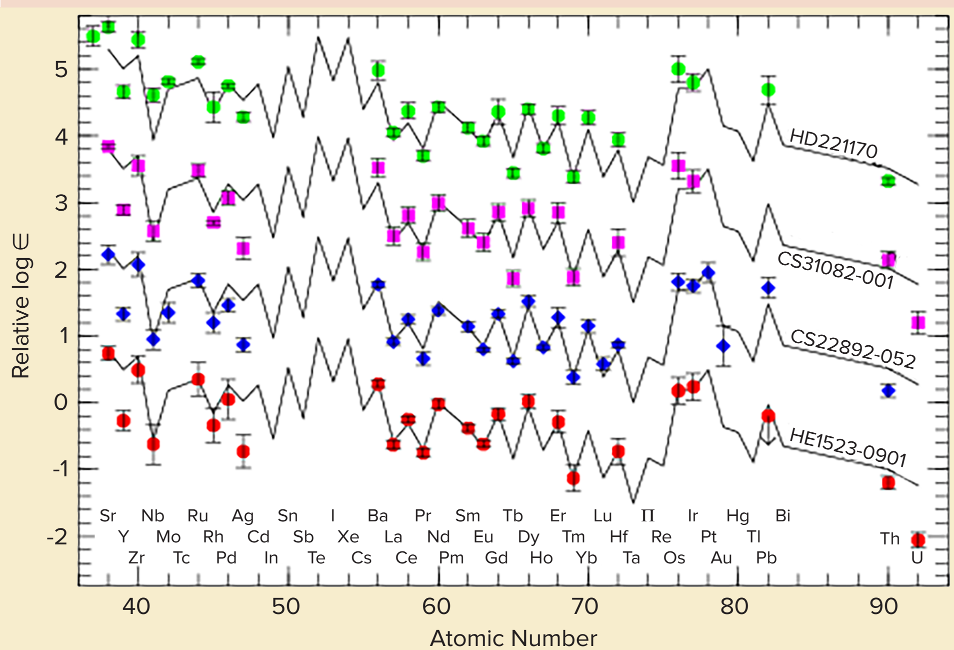
- Carbon/Oxygen ratio in our universe?
- What reaction do we need to study?
- What is the problem?
- What do we need to study it?

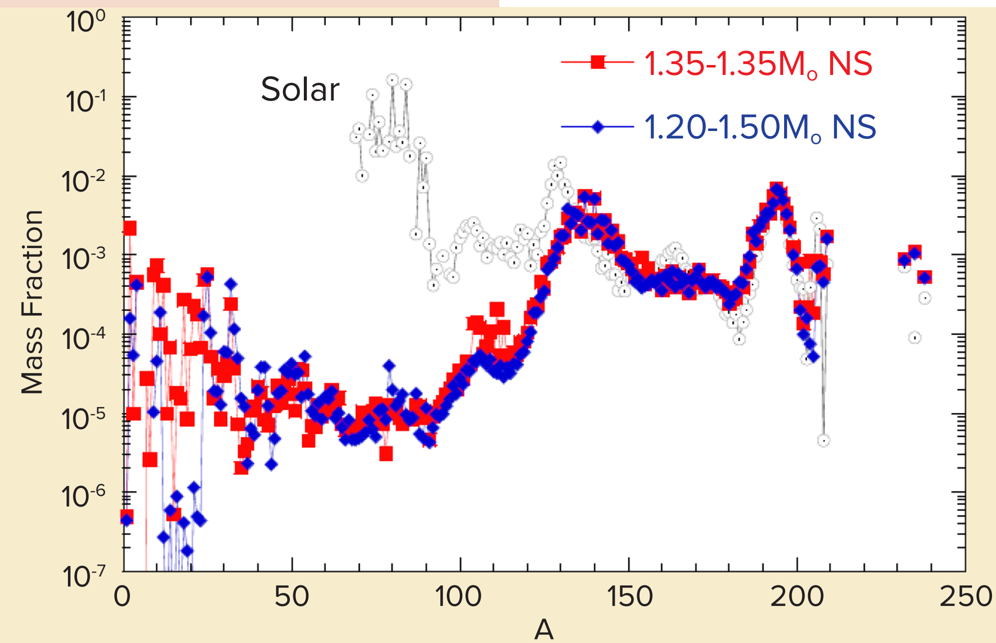


Heavier elements – the r process

- What is the r-process?
- What kind of nuclei do we need to study to understand it?
- What are possible sites for it?
- How can LIGO help?

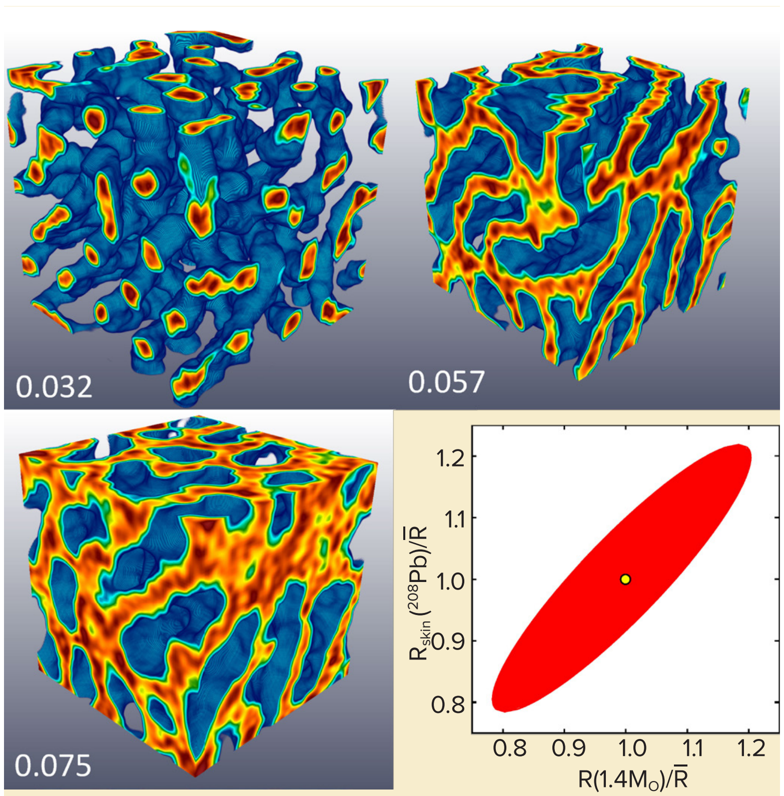


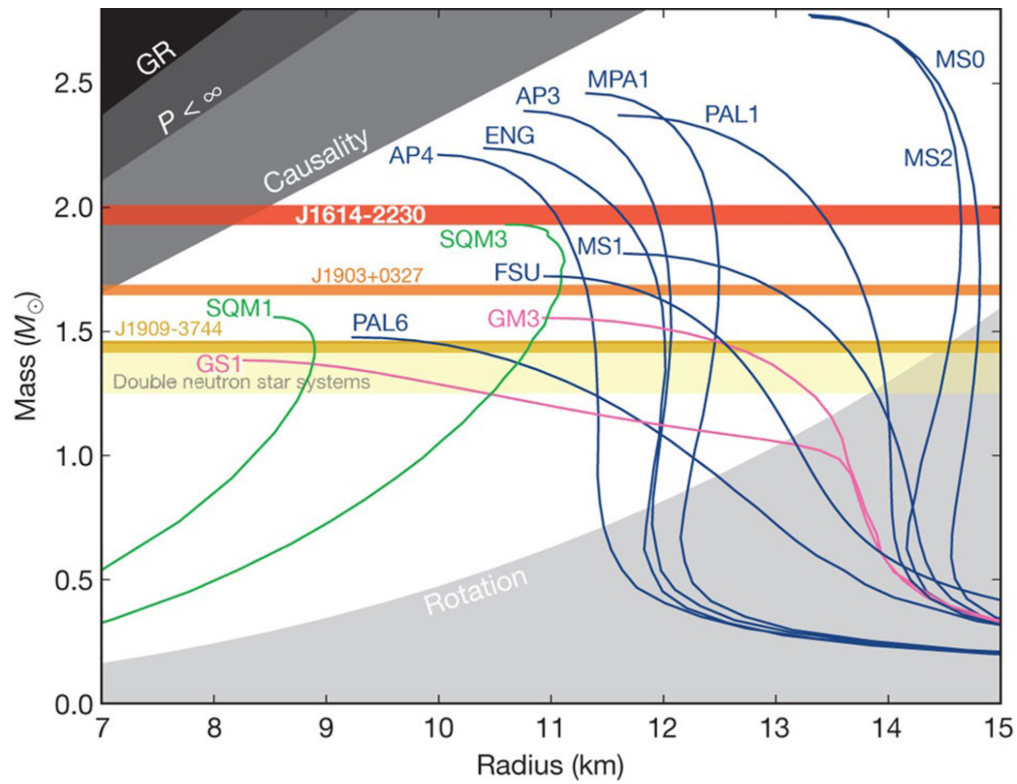




Neutron Stars and Nuclear Pasta

n stars = End states of star collapse for stars > several solar masses (supernovae)
Gigantic nuclei: A = 10^{57} (but superdense core due to gravity >> nuclear force!)





models of neutron stars + observed masses

Nuclear Pasta
(crust of n star)

Measuring the n radius of lead to predict the radius of a n star

Summary

- Much already known about nuclear processes in the universe
- Still more information needed: cross sections of very rare processes, properties for very exotic nuclei, equation of state of nuclear matter, r-process sites,…
- Tools: low energy accelerators (future: underground!), rare isotope facilities (FRIB!), parity violating electron scattering (JLab), LIGO