Elastic scattering

$$\frac{\Delta\sigma}{\Delta\Omega} = \frac{4\alpha^2(\hbar c)^2 E'^2 cos^2\frac{\theta}{2}}{Q^4}\frac{E'}{E}\left(\frac{G_E^2(Q^2)+\tau G_M^2(Q^2)}{1+\tau} + 2\tau\, tan^2\frac{\theta}{2}\, G_M^2(Q^2)\right)$$

(

where $\tau = \nu^2/Q^2$.

Inelastic Scattering

Note: Q^2 = 2EE'(1-cosθ) => ΔQ^2 = 2EE'sinθΔθ, ΔΩ = 2π sinθΔθ = π/EE' ΔQ^2 => Δσ/ΔQ^2 = π/EE' Δσ/ΔΩ

$$\frac{\Delta\sigma}{\Delta Q^2\Delta\nu} = \frac{4\pi\alpha^2(\hbar c)^2 E' cos^2(\theta/2)}{Q^4 E}(W_2(Q^2,\nu) + 2\,tan^2(\theta/2)\,W_1(Q^2,\nu))$$

$$\frac{\Delta\sigma}{\Delta Q^2\Delta\nu} = \frac{4\pi\alpha^2(\hbar c)^2 E' cos^2(\theta/2)}{Q^4 E}\frac{W_1(Q^2,\nu)}{\epsilon(1+\tau)}(1+\epsilon R(Q^2,\nu))$$

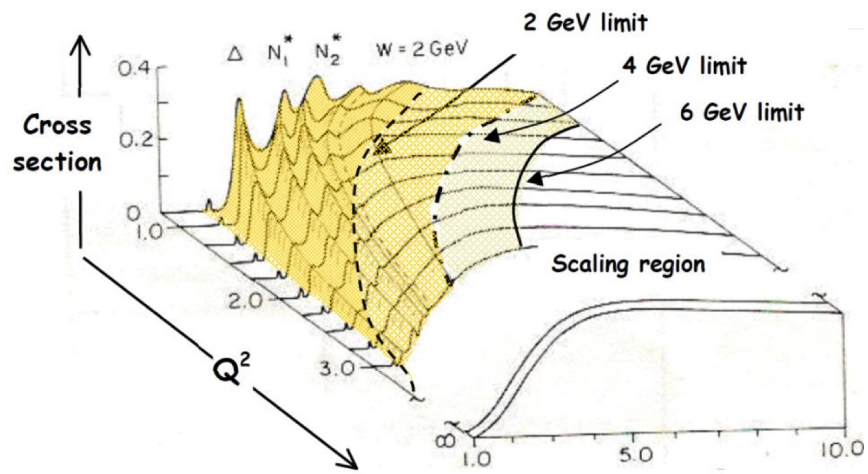
with $\epsilon = (1+2(1+\tau)tan^2(\theta/2))^{-1}$

Deep Inelastic Scattering (large Q^2, large ν)

$$\frac{\Delta\sigma}{\Delta Q^2\Delta\nu} = \frac{4\pi\alpha^2(\hbar c)^2 E' cos^2(\theta/2)}{Q^4 E}(\frac{1}{\nu}F_2(x) + 2\,tan^2(\theta/2)\frac{1}{M}F_1(x))$$

$$F_1(x) = MW_1(Q^2,\nu) \qquad F_2(x) = \nu W_2(Q^2,\nu)$$

Only "mildly" dependent on Q^2



$\omega' = \frac{1}{x} + \frac{m^2}{Q^2}$
= W^2/Q^2 +1

Picture from F. Gross,
«Making the case for Jefferson Lab»
The first decade of Science at Jefferson Lab
JoP, Conf. Series 299 (2011) 012001

Inelastic Cross Section – What's different

Phase space factor $d^3k' = k'^2 d\Omega_{k'} dk'$

(δ-function drops since E' can have any value)

— conversion —

$k'^2 d\Omega dk' = E'^2 2\pi \sin\theta d\theta dE'$

$= -2E'^2 d\cos\theta \, \pi dE'$

$= \frac{\pi E'}{E} \underbrace{(-2EE' d\cos\theta)}_{+dQ^2} \underbrace{dE'}_{|d\nu|}$ since $\nu = E - E'$

Replace $\frac{G_E^2 + \tau G_M^2}{1+\tau}$ with $W_2(Q^2, \underline{\underline{\nu}})$ *)

and τG_m^2 with $W_1(Q^2, \nu) \Rightarrow$

$$\Delta\sigma = \frac{4\pi\alpha^2(\hbar c)^2}{Q^4} \frac{E'}{E} \cos^2\theta/2 \left[W_2 + 2\tan^2\frac{\theta}{2} W_1\right] \Delta Q^2 \Delta\nu$$

*) $\Rightarrow W_2^{el}(Q^2, \nu) = \frac{G_E^2 + \tau G_M^2}{1+\tau} \cdot \delta(\nu - \nu_{el})$

$W_1^{el}(Q^2, \nu) = \tau G_m^2 \cdot \delta(\nu - \nu_{el})$

Elastic Cross section – final form

$$\Delta\sigma = \frac{4z^2\alpha^2(\hbar c)^2}{Q^4} \cos^2\frac{\theta}{2} \left[\underbrace{\frac{G_E^2 + \tau G_M^2}{1+\tau}}_{\text{LONGITUDINAL (charge)}} + \underbrace{2\tau \tan^2\frac{\theta}{2} G_M^2}_{\text{TRANSVERSE (magnetic)}}\right] \cdot E'^2 \Delta\Omega \underbrace{\frac{E'}{E}}_{\text{Recoil}}$$

$\tau = \frac{\nu^2}{Q^2}$, $\underbrace{G_E(Q^2), \; G_M(Q^2)}_{\text{Form Factors}}$:

Dirac Particle : $G_E = G_M = 1$ (const.)

Anomalous magnetic moment : $G_M \approx (1+\kappa) G_E$

Extended Charge distribution :

$G_E(Q^2) \simeq$ Fourier transform of $\rho(r)$

Ex: $\rho(r) \simeq \frac{a^3}{8\pi} e^{-ar} \Rightarrow G_E(Q^2) \simeq \left(\frac{1}{1 + \frac{Q^2}{a^2}}\right)^2$

(Dipole Form). $p : a^2 = 0.71\ \mathrm{GeV}^2$

Inelastic Cross Section – What's different

Phase space factor $d^3k' = k'^2 d\Omega_{k'} dk'$

(δ-function drops since E' can have any value)

— conversion —

$$k'^2 d\Omega dk' = E'^2 2\pi \sin\theta \, d\theta \, dE'$$
$$= -2E'^2 d\cos\theta \, \pi \, dE'$$
$$= \frac{\pi E'}{E} \underbrace{(-2EE' d\cos\theta)}_{+dQ^2} \underbrace{dE'}_{|d\nu|} \quad \text{since } \nu = E - E'$$

Replace $\frac{G_E^2 + \tau G_M^2}{1+\tau}$ with $W_2(Q^2, \underline{\underline{\nu}})$ *)

and τG_m^2 with $W_1(Q^2,\nu) \Rightarrow$

$$\Delta\sigma = \frac{4\pi\alpha^2(\hbar c)^2}{Q^4}\frac{E'}{E}\cos^2\theta/2 \left[W_2 + 2\tan^2\frac{\theta}{2} W_1\right]\Delta Q^2 \Delta\nu$$

*) $\Rightarrow$ $W_2^{el}(Q^2,\nu) = \frac{G_E^2 + \tau G_M^2}{1+\tau} \cdot \delta(\nu - \nu_{el})$

$W_1^{el}(Q^2,\nu) = \tau G_m^2 \cdot \delta(\nu - \nu_{el})$

Interpretation of $W_1(Q^2,\nu)$

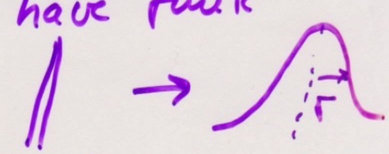
(transverse) electromagnetic transition probability to final state characterized by ν, with resolution $\sim \frac{1}{\sqrt{Q^2}}$

Transition to discrete final states (resonances):

$$W_1(Q^2,\nu) \simeq \tau G_m^2(\text{transition}) \cdot \delta(\nu - \nu_R)$$

ν_R is given by $M^2_{Res} \doteq W^2_{Res} = M^2 + 2M\nu_R - Q^2$

In reality: Resonances have finite width $\Gamma \sim \frac{1}{\tau_{life}} \Rightarrow$

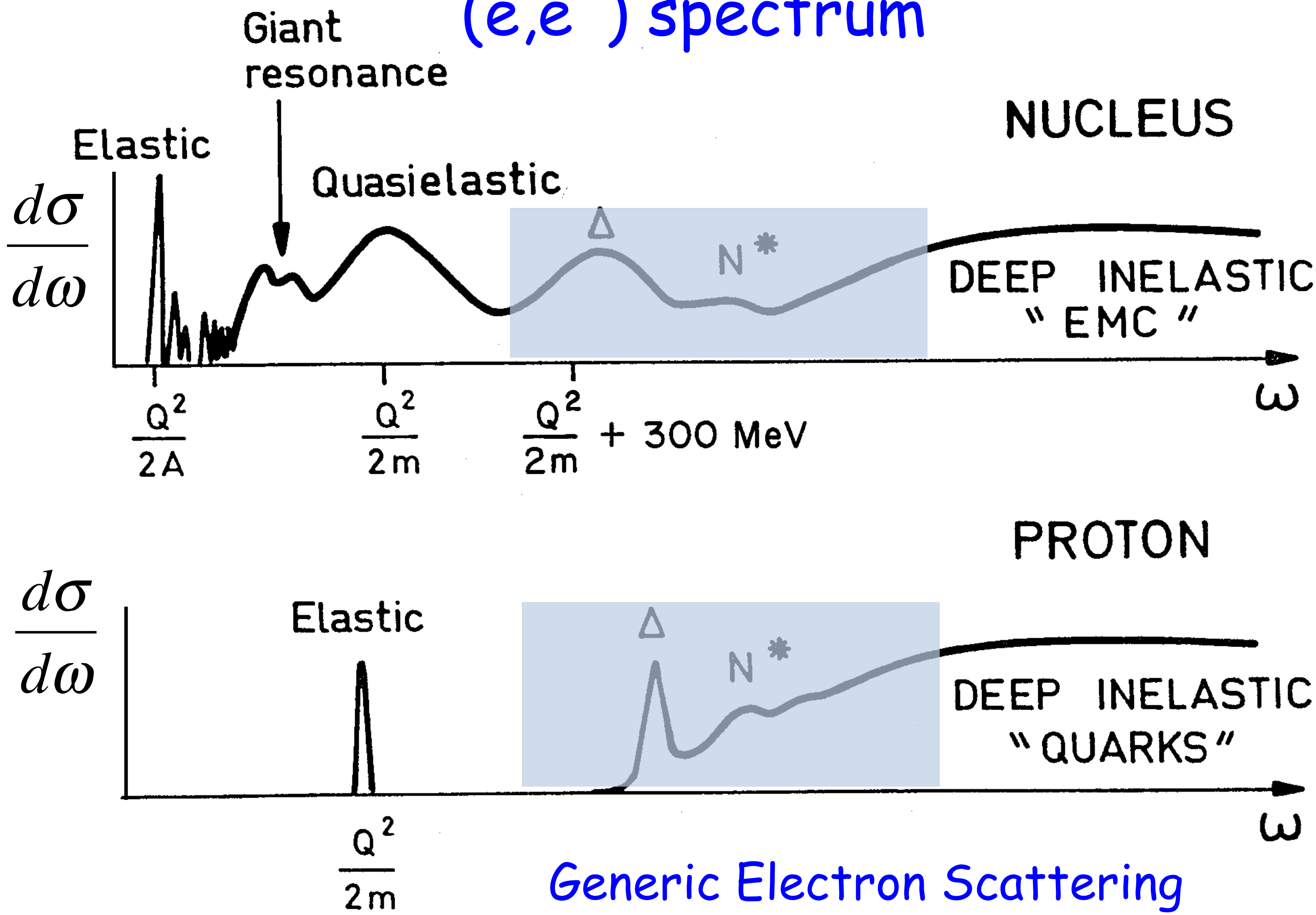


δ-function

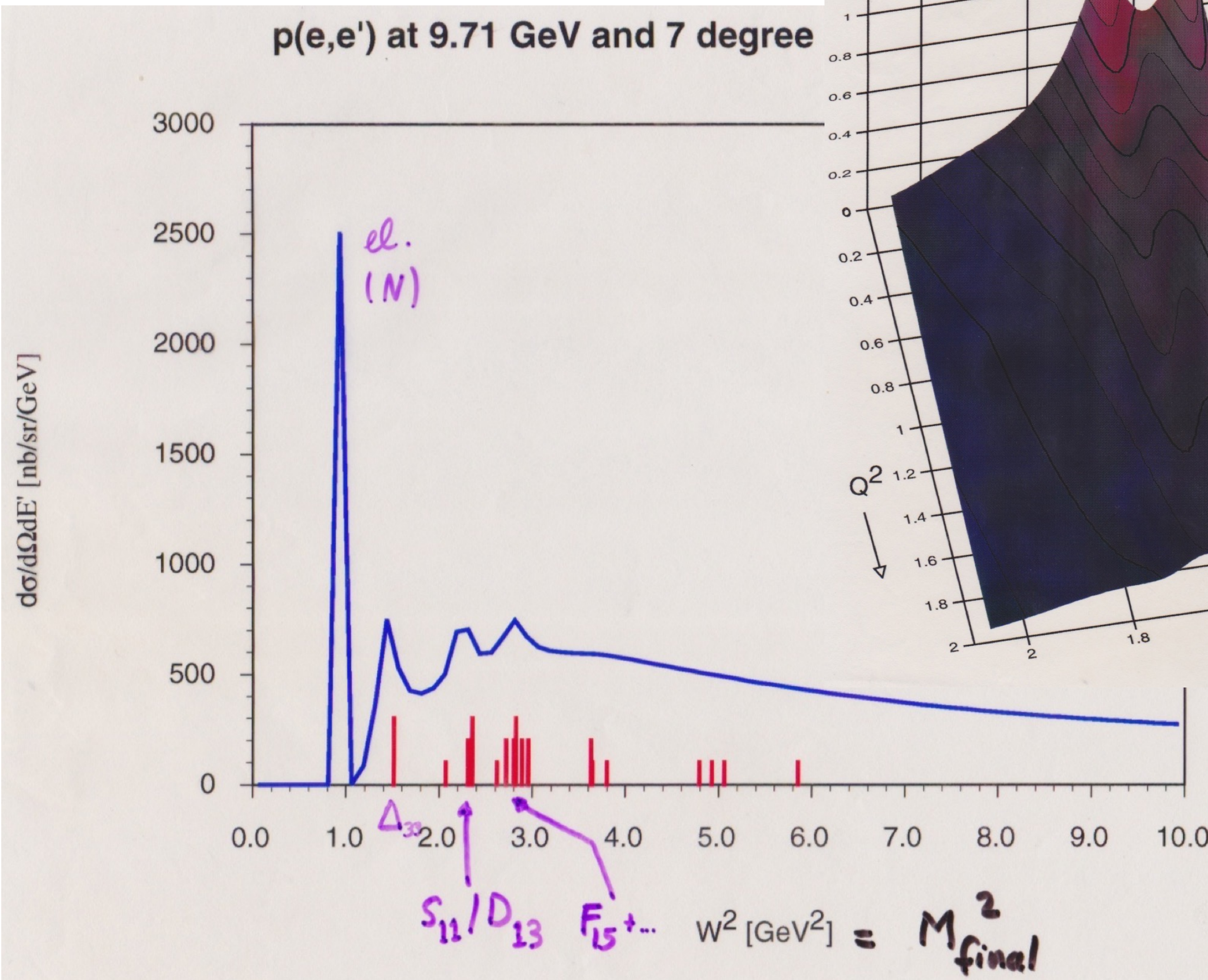
(except elastic transition)

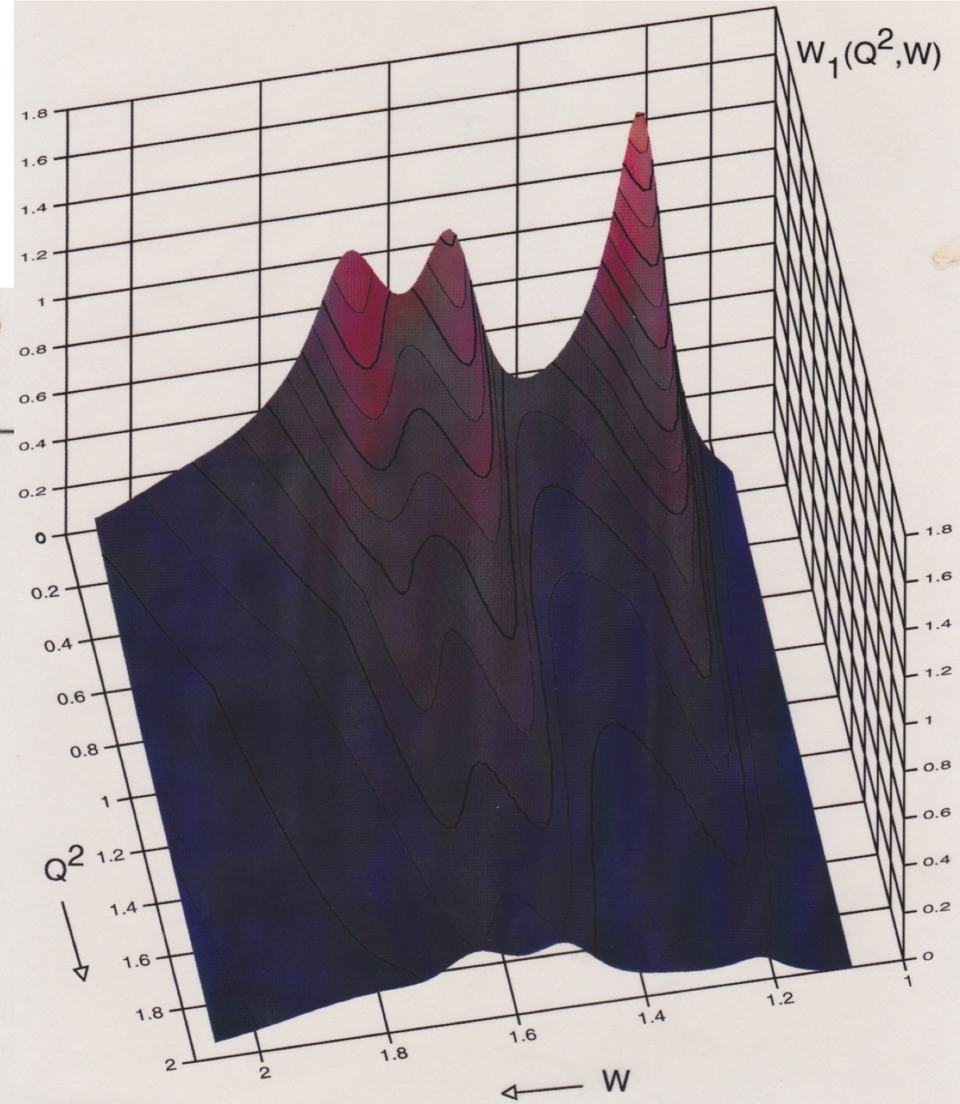
Threshold: $W^2_{min} = (M + m_\pi)^2 = (1.08\ \text{GeV})^2$

(e,e') spectrum

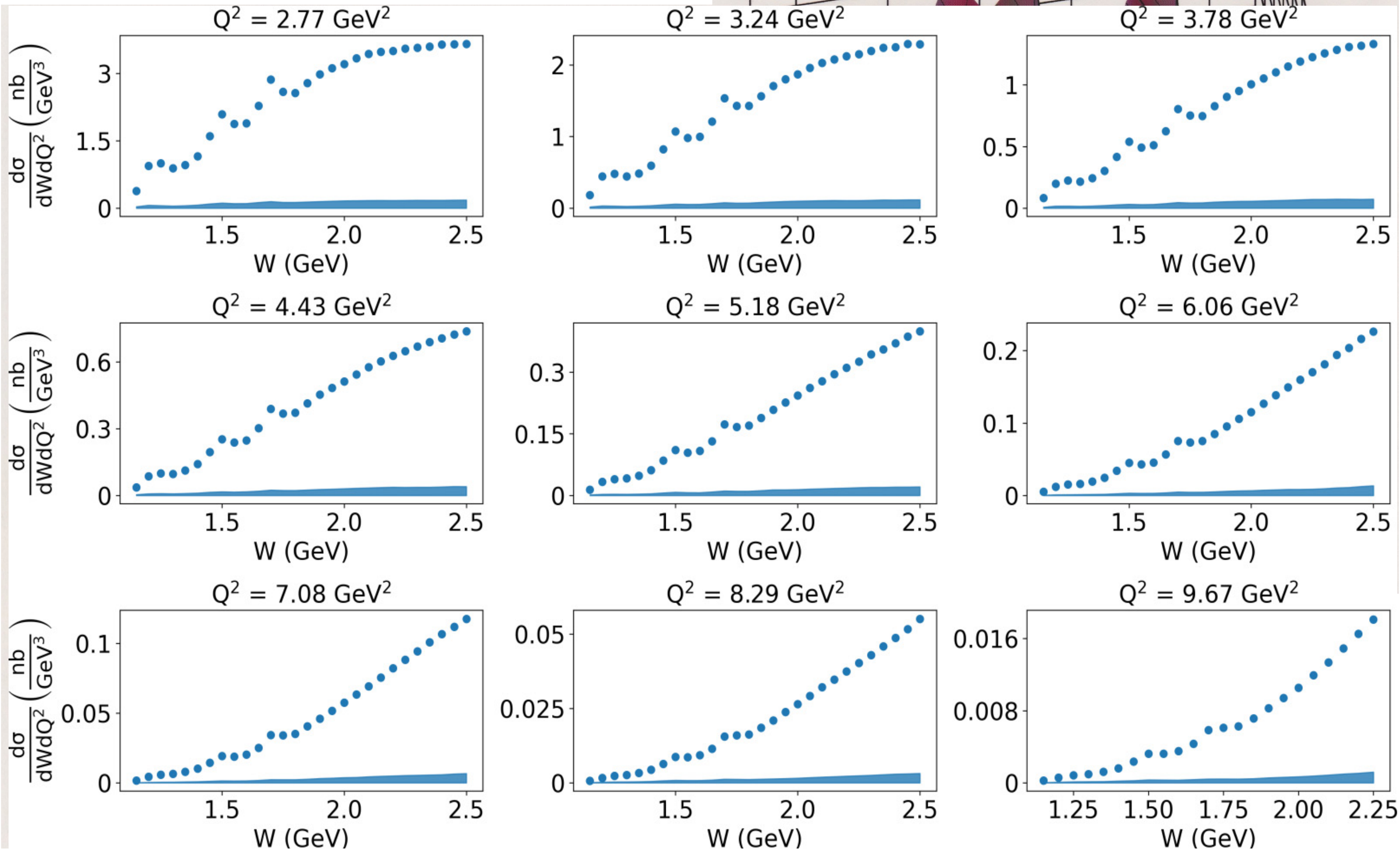


Generic Electron Scattering
at fixed momentum transfer


p(e,e') at 9.71 GeV and 7 degree
dσ/dΩdE' [nb/sr/GeV]
el.
(N)
Δ33
S11 / D13
F15 +...
W2 [GeV2] = M2final

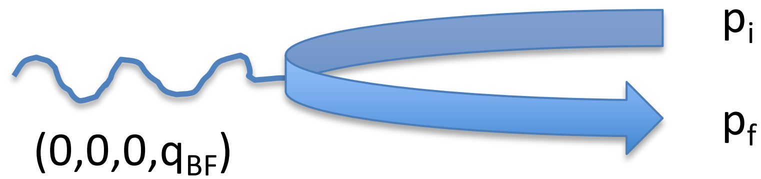

W1(Q2,W)
Q2
W

New Jefferson Lab Data 2025



Deep Inelastic Scattering (DIS)

Breit ("Brickwall") Frame



$$\begin{pmatrix} \Gamma & 0 & 0 & \Gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \Gamma\beta & 0 & 0 & \Gamma \end{pmatrix} \begin{pmatrix} \nu \\ 0 \\ 0 \\ -q \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ q_{BF} \end{pmatrix} \Rightarrow$$

$$p_i = p_f = \frac{q_{BF}}{2} = \frac{\sqrt{Q^2}}{2}$$

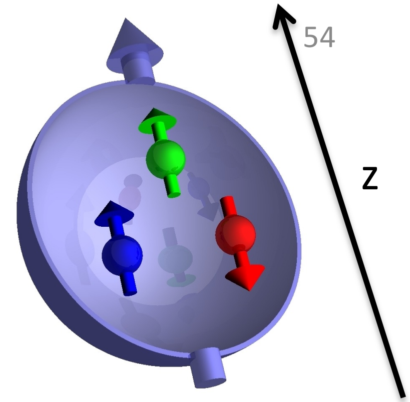
$$\Gamma = \frac{q}{\sqrt{Q^2}}, \Gamma\beta = \frac{\nu}{\sqrt{Q^2}}, q_{BF} = \sqrt{Q^2}$$

$$\begin{pmatrix} \Gamma & 0 & 0 & \Gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \Gamma\beta & 0 & 0 & \Gamma \end{pmatrix} \begin{pmatrix} M \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \Gamma M \\ 0 \\ 0 \\ \Gamma\beta M \end{pmatrix} = \begin{pmatrix} \frac{qM}{\sqrt{Q^2}} \\ 0 \\ 0 \\ \frac{\nu M}{\sqrt{Q^2}} \end{pmatrix} \Rightarrow$$

$$x = \frac{p_i^{BF}}{P_i^{BF}} = \frac{\frac{\sqrt{Q^2}}{2}}{\frac{\nu M}{\sqrt{Q^2}}} = \frac{Q^2}{2M\nu}$$

$$Q^2 \to \infty, \nu \to \infty, x = \frac{Q^2}{2M\nu} \text{ fixed: } q = \nu\sqrt{1 + \frac{Q^2}{\nu^2}} = \nu\sqrt{1 + \frac{4M^2x^2}{Q^2}} \approx \nu$$

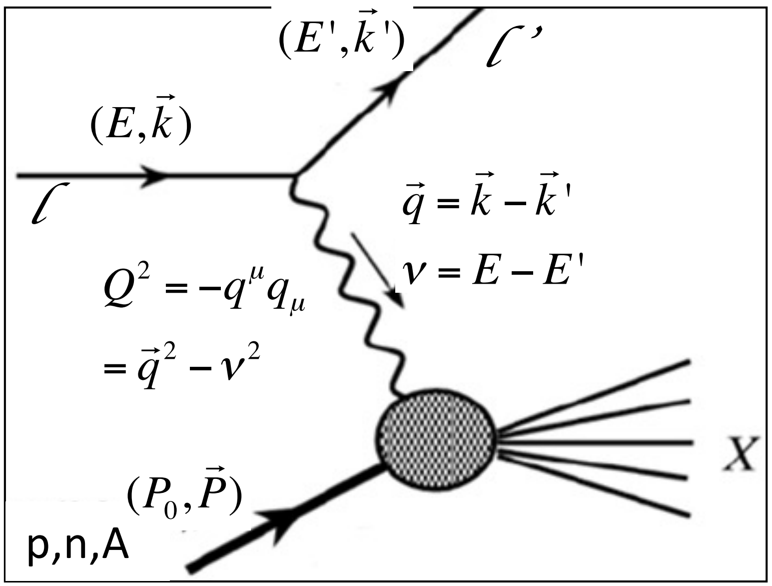
Parton Distribution Functions



- The 1D world of nucleon/nuclear collinear structure:
  - Take a nucleon/nucleus
  - Move it real fast along z
    ⇒ momentum P_z (>>M)
  - Select a “parton” (quark, gluon) inside
  - Measure **its**
    p_z (m≈0)
  - ⇒ Momentum Fraction x = p_z/P_z
  - In DIS *): p_z/P_z
    ≈ $x_{Bj} = Q^2/2M\nu$
  - Probability:

$$f_1^i(x), i = u, d, s, \ldots, G$$

In the following, will often write “$q_i(x)$” for $f_1^i(x)$



*) DIS = “Deep Inelastic (Lepton) Scattering” -> very large Q^2, ν

Deep Inelastic Scattering (DIS)

Reminder: Elastic scattering

$$\frac{\Delta\sigma}{\Delta \cancel{\Omega}\, Q^2} = \frac{4\alpha^2(\hbar c)^2 \overset{\pi}{\cancel{E'^2}} cos^2\frac{\theta}{2}}{Q^4}\,\frac{E'}{E}\left(\frac{G_E^2(Q^2)+\tau G_M^2(Q^2)}{1+\tau} + 2\tau\, tan^2\frac{\theta}{2}\, G_M^2(Q^2)\right)$$

Elastic scattering from quarks:

$$\Delta\sigma = \frac{4\pi z_q^2 \alpha^2(\hbar c)^2 E' cos^2(\theta/2)}{Q^4 E}(q(x)\Delta x + 2\nu^2/Q^2 tan^2(\theta/2) q(x)\Delta x)\Delta Q^2. \tag{12}$$

We can use the relation $\Delta x = -Q^2/(2M\nu^2)\Delta\nu = -x\Delta\nu/\nu$ to rewrite this as

$$\frac{\Delta\sigma}{\Delta Q^2 \Delta\nu} = \frac{4\pi\alpha^2(\hbar c)^2 E' cos^2(\theta/2)}{Q^4 E}(\frac{x}{\nu} z_q^2 q(x) + \frac{1}{M} tan^2(\theta/2) z_q^2 q(x)). \tag{13}$$

Reminder: IN-Elastic scattering

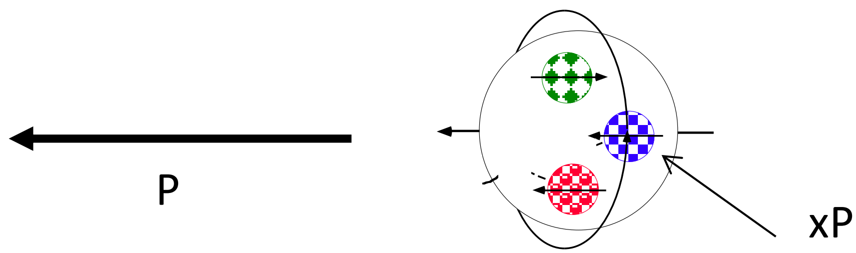
$$\frac{\Delta\sigma}{\Delta Q^2 \Delta\nu} = \frac{4\pi\alpha^2(\hbar c)^2 E' cos^2(\theta/2)}{Q^4 E}(\frac{1}{\nu} F_2(x) + 2\, tan^2(\theta/2)\frac{1}{M} F_1(x))$$

NOTE: $\Rightarrow$
$F_2 = 2xF_1$
Callan-Gross

$$\Rightarrow \quad F_1(x) = \frac{1}{2}\left(\frac{4}{9}\left[u(x)+\bar{u}(x)\right] + \frac{1}{9}\left[d(x)+\bar{d}(x)+s(x)+\bar{s}(x)\right] + \ldots\right)$$

No Q^2!

Quark-Parton Structure of the Proton – with spin

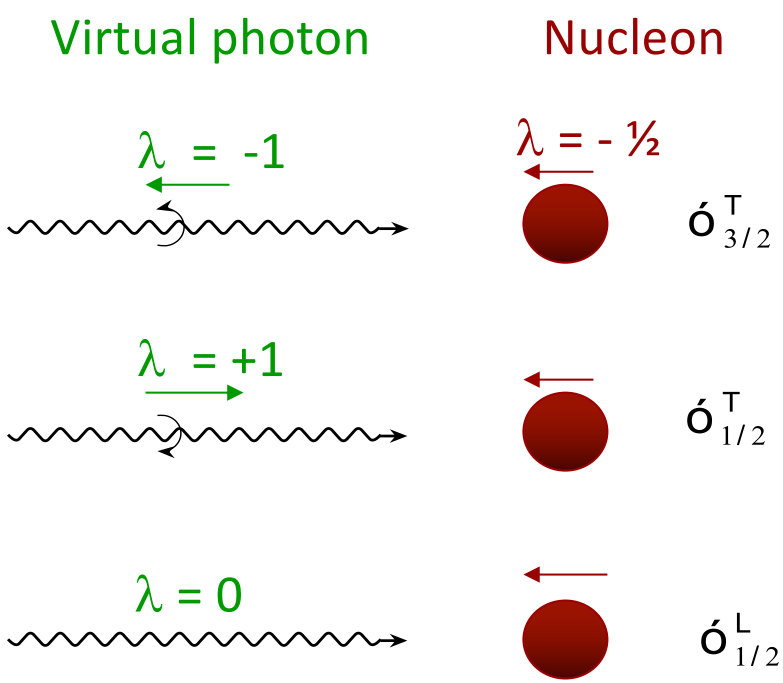


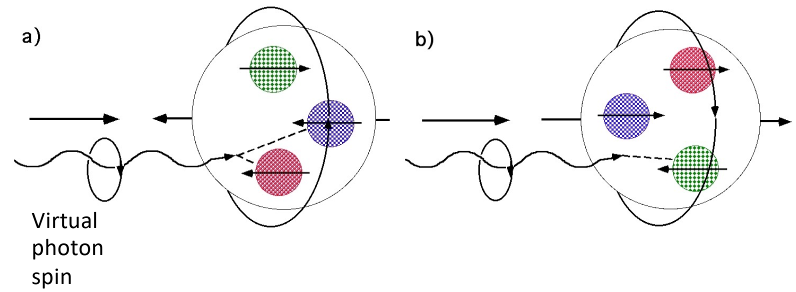
$$q(x) \sim \left\langle P,s \middle| \bar{q}\gamma^{\mu} q \middle| P,s \right\rangle = q \Uparrow\uparrow(x) + q \Uparrow\downarrow(x) + \bar{q} \Uparrow\uparrow(x) + \bar{q} \Uparrow\downarrow(x)$$

Nucleon Spin Quark Spin Quarks Anti-Quarks

Similarly G(x) for gluons

Virtual Photon Asymmetries





$$\mathbf{A_1} = \frac{\sigma_{1/2} - \sigma_{3/2}}{\sigma_T} \qquad \mathbf{A_2} = \frac{\sigma_{LT'}}{\sigma_T}$$

related to quark polarizations $\Delta q/q$

$$\Delta q(x) = q \Uparrow\uparrow(x) - q \Uparrow\downarrow(x) + \bar{q} \Uparrow\uparrow(x) - \bar{q} \Uparrow\downarrow(x) \sim \left\langle P,s \middle| \bar{q}\gamma^{\mu}\gamma^{5} q \middle| P,s \right\rangle$$

Structure Functions

$$\frac{\Delta\sigma}{\Delta Q^2 \Delta\nu} = \frac{4\pi\alpha^2(\hbar c)^2 E' cos^2(\theta/2)}{Q^4 E}\left(\frac{1}{\nu}F_2(x) + 2\,tan^2(\theta/2)\frac{1}{M}F_1(x)\right)$$

$$\frac{\Delta\sigma}{\Delta Q^2\Delta\nu}\downarrow\Uparrow - \frac{\Delta\sigma}{\Delta Q^2\Delta\nu}\uparrow\Uparrow = \frac{4\pi\alpha^2}{M\nu Q^2 E^2}\left[\left(E + E'\cos\theta\right)\mathbf{g_1} - 2xM\mathbf{g_2}\right]$$

Electron Spin   Nucleon Spin

Unpolarized: $F_1(x,Q^2)$ and $F_2(x, Q^2)$

Polarized: $g_1(x,Q^2)$ and $g_2(x, Q^2)$

Parton model:

$$F_1(x) = \frac{1}{2}\sum_i e_i^2 q_i(x) \text{ and } F_2(x) = 2xF_1(x)$$

$$g_1(x) = \frac{1}{2}\sum_i e_i^2 \Delta q_i(x) \text{ and } g_2(x) = 0$$

$i =$ quark flavor

$e_i =$ quark charge

the structure functions $\mathbf{g_1}$ and $\mathbf{g_2}$ are linear combinations of $\mathbf{A_1}$ and $\mathbf{A_2}$

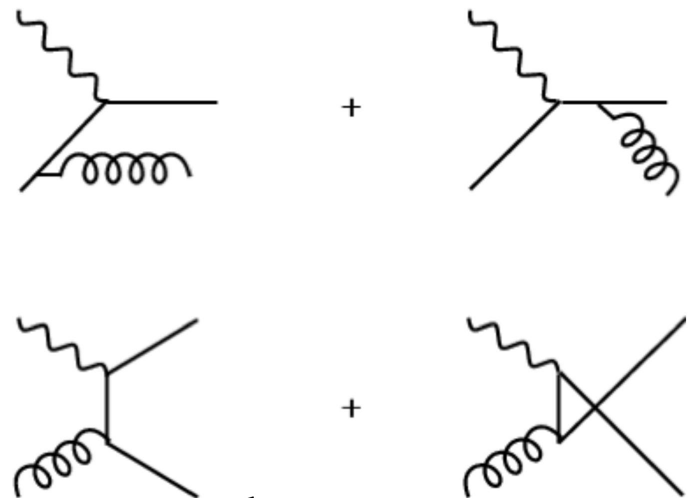
$$g_1(x,Q^2) = \frac{\tau}{1+\tau}\left(A_1 + \frac{1}{\sqrt{\tau}}A_2\right)F_1$$

$$g_2(x,Q^2) = \frac{\tau}{1+\tau}\left(\sqrt{\tau}A_2 - A_1\right)F_1$$

$$\tau = \frac{\nu^2}{Q^2}$$

Parton Distribution Functions and NLO pQCD

Two effects modify simple parton picture:



1) (Gluon) radiative corrections change elementary cross section

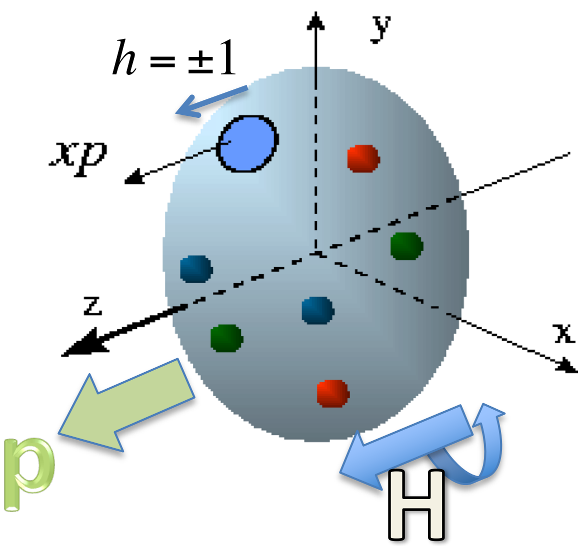
$$g_1(x,Q^2)_{pQCD} = \frac{1}{2}\sum_q^{N_f} e_q^2\left[(\Delta q + \Delta\bar{q}) \otimes \left(1 + \frac{\alpha_s(Q^2)}{2\pi}\delta C_q\right) + \frac{\alpha_s(Q^2)}{2\pi}\Delta G \otimes \frac{\delta C_G}{N_f}\right]$$

$\delta C_q, \delta C_G - Wilson$ coefficient functions

2) pQCD evolution makes PDFs Q^2-dependent

Inclusive lepton scattering

Parton model: DIS can access



$$q(x;Q^2), \langle h \cdot H \rangle q(x;Q^2)$$

Traditional "1-D" Parton Distributions (PDFs)
(integrated over many variables)

Callan-Gross

Wandzura-Wilczek

$$F_1(x) = \tfrac{1}{2}\sum_i e_i^2 q_i(x) \ \left(\text{and } \boxed{F_2(x) \approx 2xF_1(x)}\right)$$

$$g_1(x) = \tfrac{1}{2}\sum_i e_i^2 \Delta q_i(x) \ \left(\text{and } g_2(x) \approx \boxed{-g_1(x) + \int_x^1 \frac{g_1(y)}{y} dy}\right)$$

At finite Q^2: pQCD evolution ($q(x,Q^2)$, $\Delta q(x,Q^2)$ $\Rightarrow$ DGLAP equations), and gluon radiation

$$g_1(x,Q^2)_{pQCD} = \frac{1}{2}\sum_q^{N_f} e_q^2 [(\Delta q + \Delta\bar{q}) \otimes (1 + \frac{\alpha_s(Q^2)}{2\pi}\delta C_q) + \frac{\alpha_s(Q^2)}{2\pi}\Delta G \otimes \frac{\delta C_G}{N_f}]$$

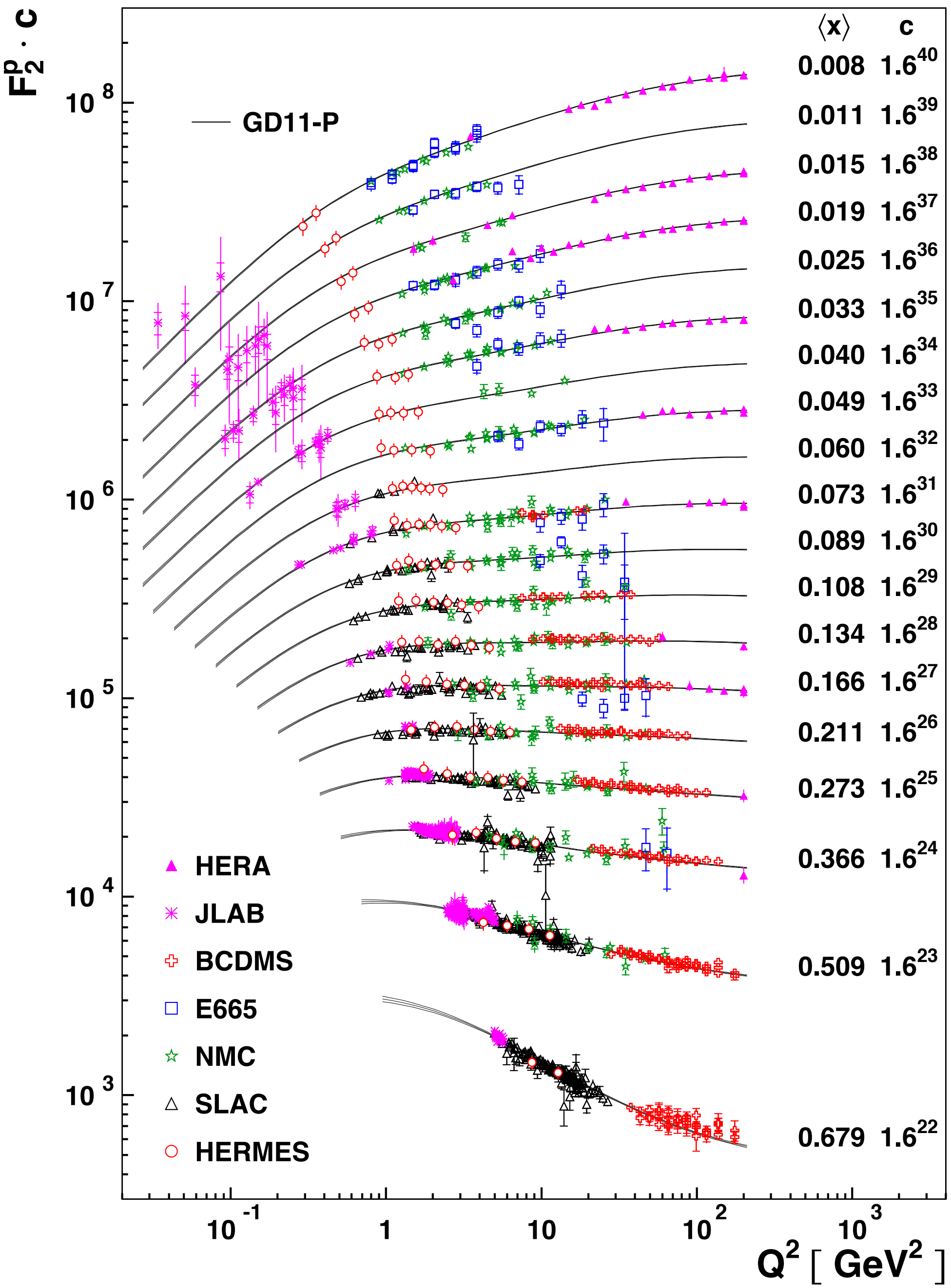
$\Rightarrow$ access to gluons. $\delta C_q, \delta C_G$ – *Wilson* coefficient functions

SIDIS: Tag the flavor of the struck quark with the leading FS hadron $\Rightarrow$ separate $q_i(x,Q^2)$, $\Delta q_i(x,Q^2)$

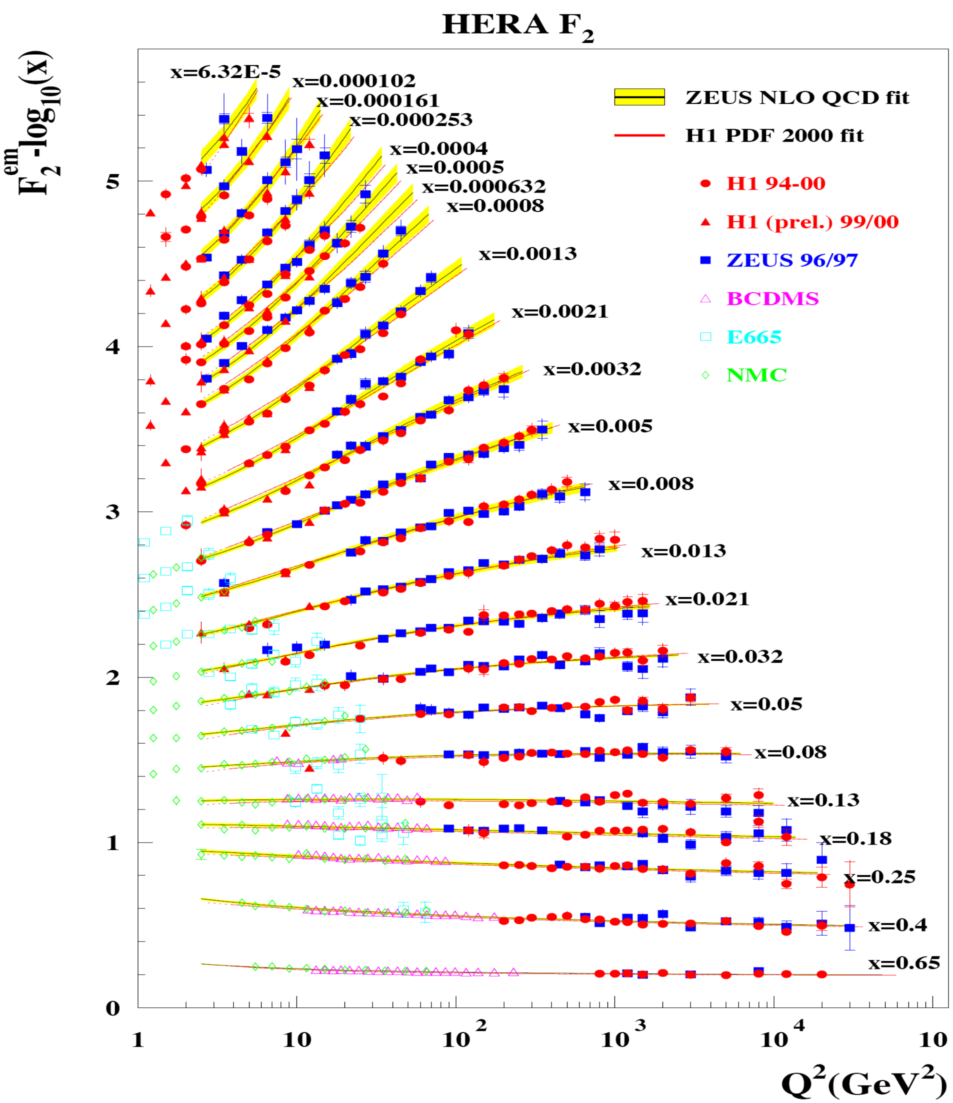
Jefferson Lab kinematics: $Q^2 \approx M^2 \Rightarrow$ target mass effects, higher twist contributions and resonance excitations

- Non-zero $R = \frac{F_2}{2xF_1}\left(\frac{4M^2x^2}{Q^2}+1\right)-1$, $g_2^{HT}(x) = g_2(x) - g_2^{WW}(x)$
- Further Q^2-dependence (power series in $\frac{1}{Q^n}$)

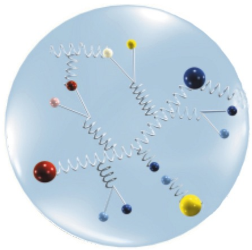
Results



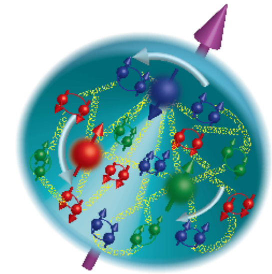
$F_2^p \cdot c$
GD11-P
⟨x⟩ c
0.008 1.6^{40}
0.011 1.6^{39}
0.015 1.6^{38}
0.019 1.6^{37}
0.025 1.6^{36}
0.033 1.6^{35}
0.040 1.6^{34}
0.049 1.6^{33}
0.060 1.6^{32}
0.073 1.6^{31}
0.089 1.6^{30}
0.108 1.6^{29}
0.134 1.6^{28}
0.166 1.6^{27}
0.211 1.6^{26}
0.273 1.6^{25}
0.366 1.6^{24}
0.509 1.6^{23}
0.679 1.6^{22}
HERA
JLAB
BCDMS
E665
NMC
SLAC
HERMES
Q^2 [GeV^2]



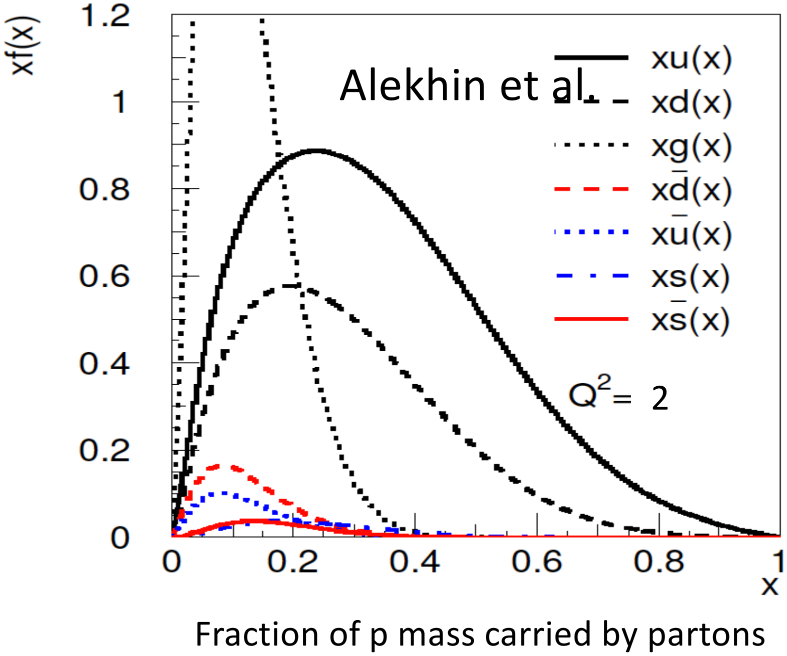
HERA F_2
F_2^{em}-$\log_{10}(x)$
ZEUS NLO QCD fit
H1 PDF 2000 fit
H1 94-00
H1 (prel.) 99/00
ZEUS 96/97
BCDMS
E665
NMC
x=6.32E-5
x=0.000102
x=0.000161
x=0.000253
x=0.0004
x=0.0005
x=0.000632
x=0.0008
x=0.0013
x=0.0021
x=0.0032
x=0.005
x=0.008
x=0.013
x=0.021
x=0.032
x=0.05
x=0.08
x=0.13
x=0.18
x=0.25
x=0.4
x=0.65
$Q^2(GeV^2)$



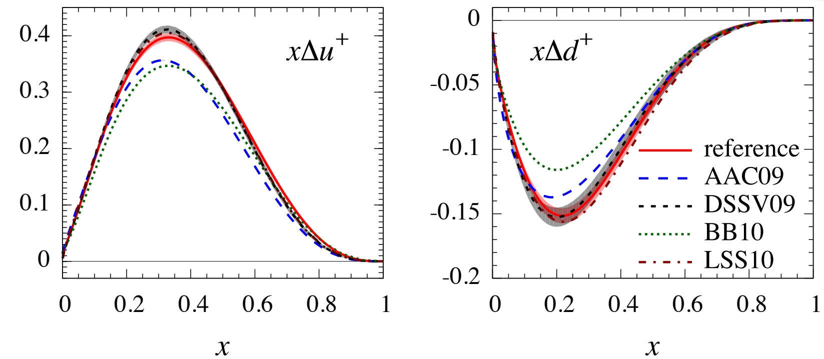
…and what have we learned?



How many of those partons are there?



…times spin



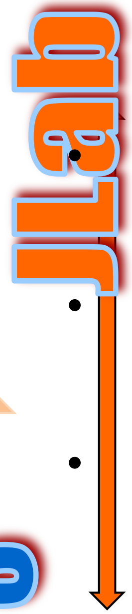
- Begun to map the 1D and even 3D motion of quarks and gluons inside nucleons
- Developed an approximate solution of QCD (Lattice QCD) that can predict masses, excited states etc.
- “Sorta” understand the size, magnetic moment and other properties of nucleons
- Begun to get a QCD-based picture of nuclei
- BUT: Much left to do – will you join us?

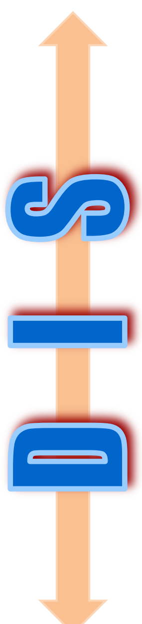
⇒ Our 1D View of the Nucleon

(depends on energy ν and wave length of the virtual photon ~ $1/Q^2$)

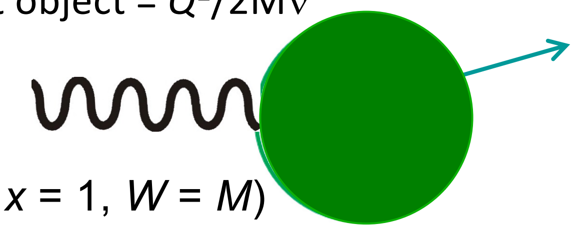
W = final state invariant mass = $\sqrt{M^2 + 2M\nu - Q^2}$

x = energy fraction of hit object = $Q^2/2M\nu$

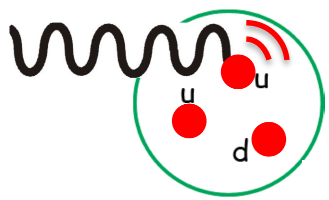




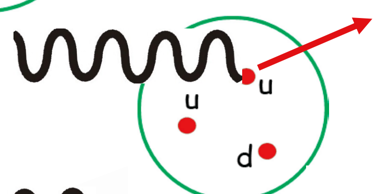
- Elastic scattering
  (Whole system recoils, $x = 1$, $W = M$)



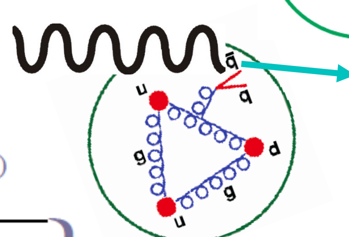
- Resonances
  ($x < 1$, $W < 2$ GeV)



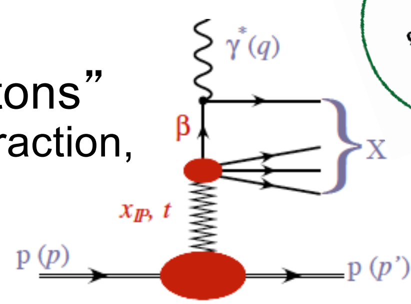
- Valence quarks
  ($x \geq 0.3$, $W > 2$ GeV)

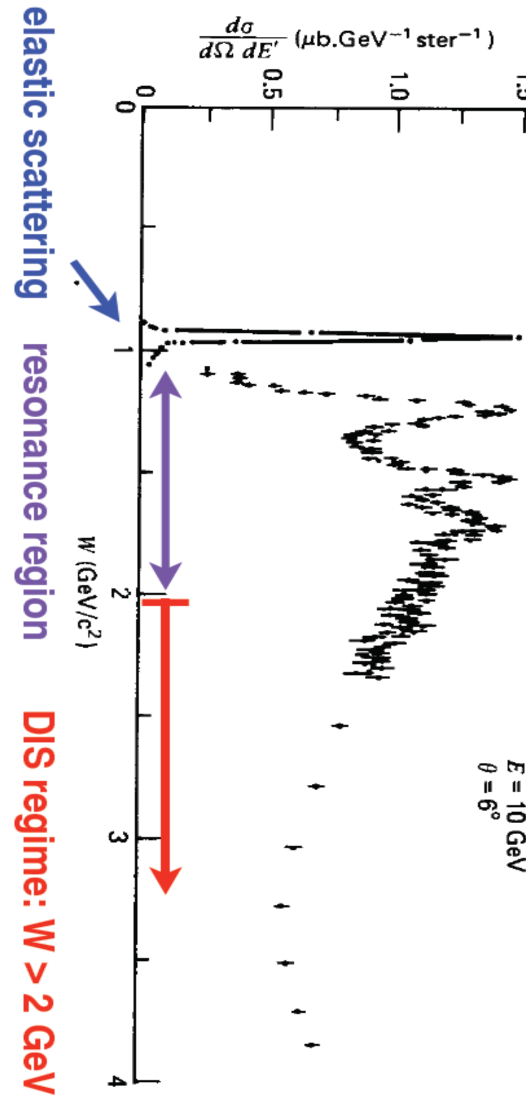


- Sea quarks, gluons
  ($x < 0.3$)



- "Wee Partons"
  ($x \to 0$, Diffraction, Pomerons)





Electron Scattering

Kinematic Variables

$$\nu = E - E' = |\vec{k}| - |\vec{k}'|$$

$$\vec{q} = \vec{k} - \vec{k}'; \quad q^\mu = (\nu, \vec{q}) = k^\mu - k'^\mu$$

$$Q^2 = -(k - k')^2 = \vec{q}^2 - \nu^2 \approx 4EE'\sin^2\frac{\theta_e}{2}$$

$$y = \frac{q^\mu P_\mu}{k^\mu P_\mu} = \frac{\nu}{E}; \quad x = \frac{Q^2}{2q^\mu P_\mu} = \frac{Q^2}{2M\nu}$$

$$W = \sqrt{(P_\mu + q^\mu)^2} = \sqrt{M^2 + (1/x - 1)Q^2}$$

Lepton variables

Inclusive

Semi-Inclusive

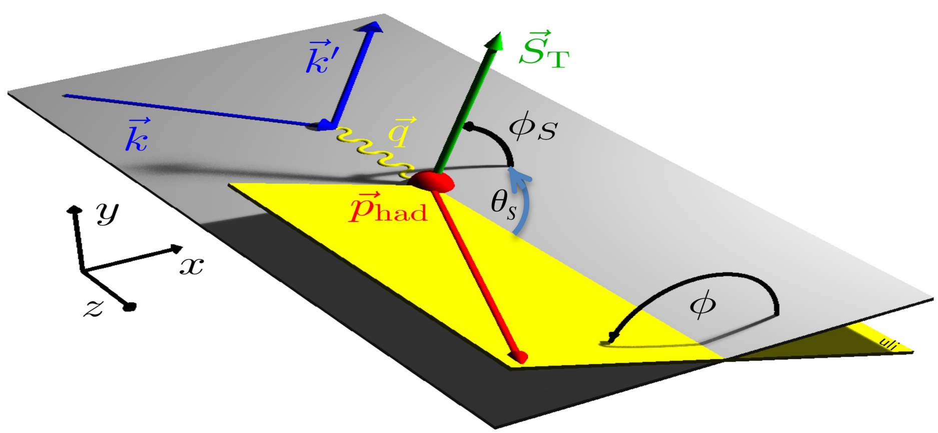
Hadron variables

$$P^\mu_{had} = \left(z\nu, \sqrt{z^2\nu^2 - m_h^2 - P_{hT}^2}, \vec{P}_{hT}\right)$$

$$z = E_h / \nu$$

$$\vec{P}_{hT} = |\vec{P}_{hT}|(\sin\theta_h \cos\phi_h, \sin\theta_h \sin\phi_h)$$

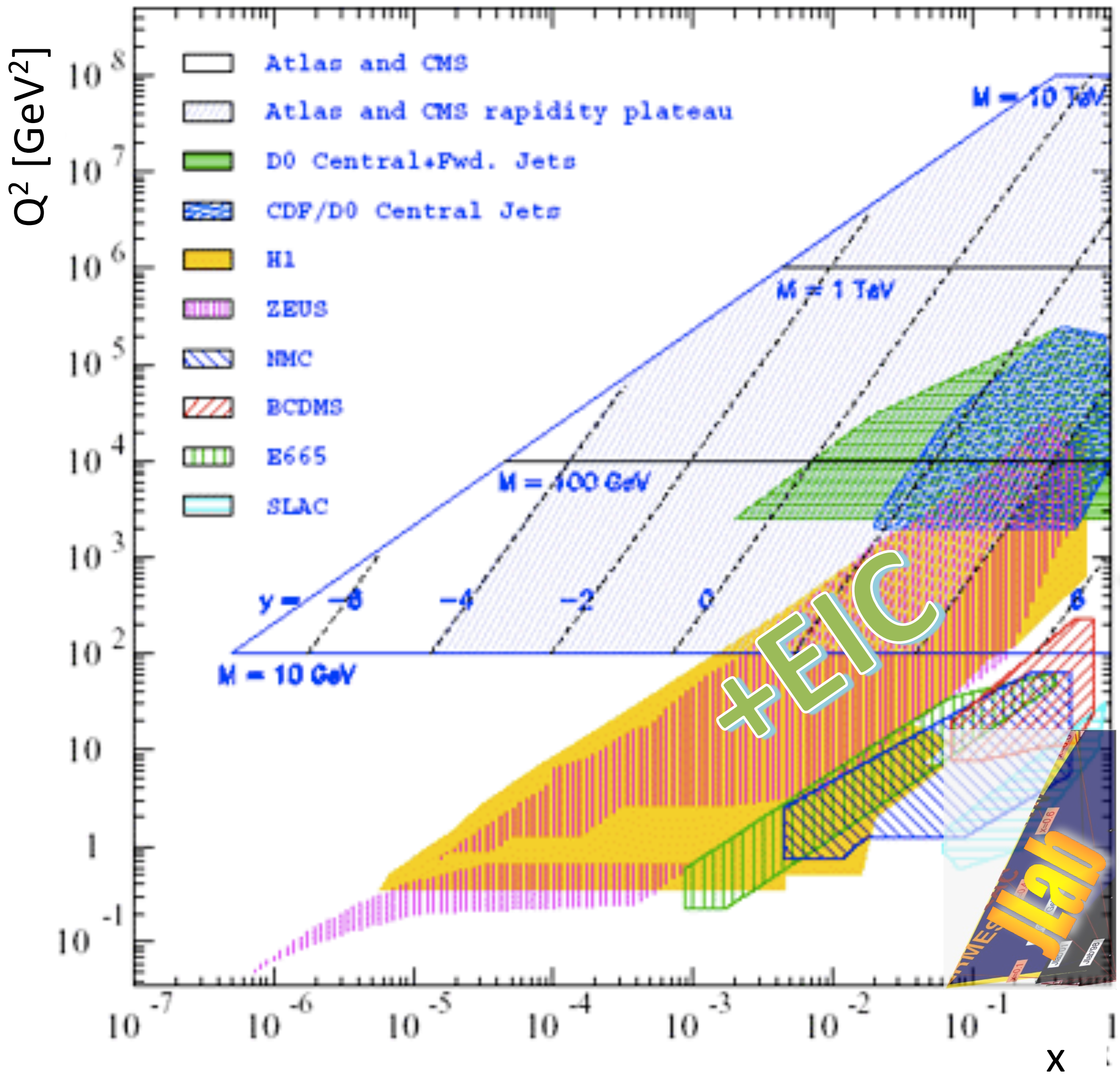
$$x_F = \frac{\vec{P}_h^{c.m.} \cdot \hat{q}}{|\vec{P}_h^{c.m.}|(\max)}$$

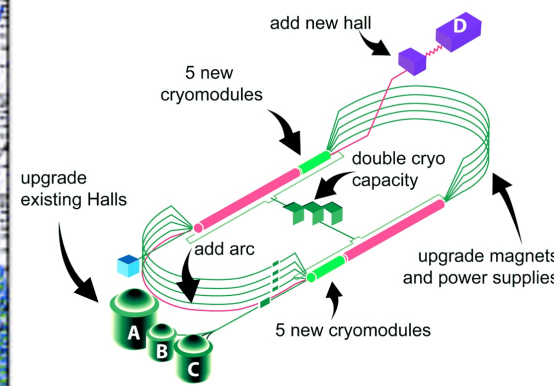


Rapidity $\quad \operatorname{artanh}\frac{|\mathbf{p}|c}{E} = \frac{1}{2}\ln\frac{E + |\mathbf{p}|c}{E - |\mathbf{p}|c}$

Pseudo-Rapidity $\quad \eta = -\ln\left[\tan\left(\frac{\theta}{2}\right)\right]$

Jefferson Lab in Perspective





JLab DIS

Now: 12 GeV

$Q^2 = 1 \ldots 13\ \mathrm{GeV}^2$

$x = 0.06 \ldots 0.8$

$W < 4$ GeV

The future landscape of Nuclear Physics

1. Study how nucleons are made up from quarks ("flavor", **p**, **L**, **S** -> 3D tomography)
2. Study how hadronic quark structure is influenced by the nuclear environment
3. Understand nuclear structure and dynamics in terms of quark degrees of freedom
4. Study extreme forms of nuclear matter: high energy (Quark-Gluon plasma), high density (short range correlations, n stars, "color glass condensates",…), non-zero strangeness (hypernuclei, strangelets, …), large n/p imbalance (radioactive beams)…
5. Study fundamental symmetries, neutrinos, nuclei in the universe
6. Develop new applications in medicine, energy, materials, homeland security, …

Hadron Machines

FAIR

J-PARC

LHC

Electron machines

Jlab

Electron-Ion-Collider (2025?)