

Summary

Units (SI):

Length: m = meter

Time: s = second

Mass: kg = kilogram; atomic mass unit $u = 1.661 \cdot 10^{-27}$ kg = $m(^{12}\text{C})/12$

Velocity: m/s

Acceleration: m/s^2

Momentum: kg m/s

Force: N = Newton = kg m/s^2

Energy: J = Joule = Nm = $\text{kg m}^2/\text{s}^2 = \text{Ws}$; eV = electron-Volt = $1.602 \cdot 10^{-19}$ J

Power: W = Watt = J/s

Charge: C = Coulomb

Currents: A = Ampere = C/s

Electric Field: N/C = V/m

Electric Potential: V = Volt = J/C

Electric Resistance: Ω = Ohm = V/A

Magnetic Field: T = Tesla = $\text{Vs/m}^2 = 10,000$ Gauss

Amount: mol (1 mol = N_A molecules = A gram, where A is atomic mass)

Density ρ : kg/m^3

Pressure: Pa = Pascal = N/m^2 ; 100,000 Pa \approx 1 atmosphere

Temperature: K = Kelvin (C = Celsius, F = Fahrenheit);

$$32^\circ\text{F} = 0^\circ\text{C} = 273.15\text{K}; 212^\circ\text{F} = 100^\circ\text{C} = 373.15\text{K}; +1^\circ\text{C} = +1\text{K} = +1.8^\circ\text{F}$$

Heat: J or 1 calorie = 4.187 J; 1 food calorie = 1000 calories = 4187 J

Frequency: Hz = 1/s

Prefixes:

kilo = k = $10^3 = 1000$ = Thousand

Mega = M = $10^6 = 1,000,000$ = Million

Giga = G = $10^9 = 1,000,000,000$ = Billion

Tera = T = 10^{12} = Trillion, Peta = P = 10^{15} = Quadrillion

centi = c = $10^{-2} = 0.01$ = 1-hundreth

milli = m = $10^{-3} = 0.001$ = 1-thousandth

micro = μ = $10^{-6} = 0.000,001$ = 1-millionth

nano = n = 10^{-9} , pico = p = 10^{-12} , femto = f = 10^{-15}

Useful Constants:

Gravitational constant: $G = 6.67 \cdot 10^{-11} \text{ Nm}^2/\text{kg}^2$

Gravitational acceleration at surface of Earth: $g = 9.81 \text{ m/s}^2$

Mass of Sun: $1.99 \cdot 10^{30} \text{ kg}$

Distance from Earth to Sun: $1.50 \cdot 10^{11} \text{ m}$

Mass of Moon: $7.35 \cdot 10^{22} \text{ kg}$

Distance from Earth to Moon: $3.84 \cdot 10^8 \text{ m}$

Mass of Earth: $5.97 \cdot 10^{24} \text{ kg}$

Radius of Earth: $6.38 \cdot 10^6 \text{ m}$

Earth's magnetic field: about 0.5 Gauss = 0.0005 T ($5 \cdot 10^{-5} \text{ T}$)
(Magnetic north pole near Australia)

Elementary charge: $e = 1.602 \cdot 10^{-19} \text{ C}$

Permittivity constant: $\epsilon_0 = 8.854 \cdot 10^{-12} \text{ F/m}$

Permeability constant: $\mu_0 = 4\pi \cdot 10^{-7} \text{ H/m}$

k (electrostatic force constant) = $1/4\pi\epsilon_0 = 8.99 \cdot 10^9 \text{ Nm}^2/\text{C}^2$

Speed of Light: $c = 2.998 \cdot 10^8 \text{ m/s} = 1/\sqrt{\epsilon_0 \mu_0}$

Avogadro's number:

$N_A = 6.022 \cdot 10^{23} \text{ molecules/mol} = \text{number of } ^{12}\text{C atoms in 12 g of carbon}$

Universal Gas Constant: $R = 8.32 \text{ J/mol/K}$

Density of water: 1000 kg/m^3 , air (sea level): 1.25 kg/m^3 , of iron: 7874 kg/m^3

Atmospheric pressure at sea level: 101,300 Pa

Typical speed of sound: 330 m/s – 340 m/s in air

Frequency of “middle A” musical note: 440 Hz

Electron mass: $m_e = 9.109 \cdot 10^{-31} \text{ kg}$; $E = mc^2 = 510,999 \text{ eV} = 511 \text{ keV}$

Proton mass: $m_p = 1.673 \cdot 10^{-27} \text{ kg}$; $E = mc^2 = 938,272,030 \text{ eV} = 938.3 \text{ MeV}$

Neutron mass: $m_n = 1.675 \cdot 10^{-27} \text{ kg}$; $E = mc^2 = 939,565,360 \text{ eV} = 939.6 \text{ MeV}$

Planck's constant:

$h = 6.63 \cdot 10^{-34} \text{ Js} = 1240 \text{ eV/c} \times \text{nm} = 4.17 \cdot 10^{-15} \text{ eV} / \text{Hz}$

$\Rightarrow 1/h = 1.51 \cdot 10^{33} \text{ Hz/J} = 2.4 \cdot 10^{14} \text{ Hz/eV}$;

2 eV corresponds to $\lambda = 620 \text{ nm}$ and $f = 4.84 \cdot 10^{14} \text{ Hz}$ (yellow light)

Atomic structure of matter

Size of typical atoms: about 1 Ångström (10^{-10} m); nuclei: 1 fm (10^{-15} m)

Atomic Mass $A = Z+N$ (number of protons and neutrons in nucleus);

Z determines element, N and hence A which isotope of that element

$Z=1$: H ($A = 1, 2$ or 3); $Z = 2$: He ($A = 3,4$) ... $Z = 6$: C ($A = 12, 13,14\dots$)

Molecular Mass = Sum of atomic masses in the molecule (e.g., H_2O : 18)

1 mol of a compound/element = as many grams as molecular/atomic mass
(1 mol $^{12}C = 12$ g, 1 mol of water = 18 g)

1 mol of anything contains $N_A = 6.022 \cdot 10^{23}$ molecules or atoms

Deformation of solids

Elongation or compression: $\Delta L/L \propto F$ (tension) \Rightarrow Hooke's law: $F = -kx$

Bending beam: Outside layer elongated, inside layer compressed

Density

$\rho = \text{mass}/\text{Volume}$ [kg/m^3]

- Air: $\approx 1.25 \text{ kg}/\text{m}^3$ at sea level – decreases continuously with height (50% at 5.6 km)
- Water: $1000 \text{ kg}/\text{m}^3$ (ice a little less)
- Iron: $7,874 \text{ kg}/\text{m}^3$
- Iridium: $22,650 \text{ kg}/\text{m}^3$

Pressure

$P = \text{force}/\text{surface area}$ [$\text{N}/\text{m}^2 = \text{Pa}$]

Hydrostatic pressure in fluid w/ density ρ changes with height: $\Delta p = -\rho g \Delta h$

Air pressure at sea level: 101,300 Pa (weight of ≈ 1 kg or ≈ 10 N on 1 cm^2)

Buoyant force: Equal to weight of displaced fluid (upwards) = $g \cdot V_{\text{objct}} \cdot \rho_{\text{fluid}}$

Pascal's Principle: Same height = same pressure everywhere in a liquid

Boyle's Law: Pressure and density are proportional in (ideal) gases;

$PV = \text{const.}$ (for constant amount of gas and constant temperature)

Bernoulli's principle: Pressure is lower in faster flowing fluid

Pressure in an enclosed gas is proportional to absolute temperature (in K)

Internal Energy (Part of Total Energy of a System)

All types of energy of particles making up an object that are random and un-directed (no overall motion of the object); can be changed through **work** on the object or through **heat** transferred from one object to another.

Temperature

Measure of average internal energy per particle; measured in Kelvin, Celsius or Fahrenheit; tends to even out between different objects or systems by ->

Heat transfer: Conduction (direct contact), Convection (movement of fluid with different temperature) and Radiation (E.M. waves, even in vacuum)

Heat capacity C – amount of internal energy change needed to increase temperature of an object by $1^\circ\text{C} = 1\text{ K}$. Specific heat capacity $c = C/\text{mass}$.

Ex.: water has specific heat capacity $c = 4200\text{ J/kg/}^\circ\text{C} = 1\text{ cal/g/}^\circ\text{C}$

Newton's Law of cooling/heating: Heat transfer rate proportional to temperature difference => will slow as temperatures approach each other.

Phase Change

Change in internal energy without change in temperature (e.g., transition solid -> liquid -> gas -> plasma ->...).

Example: Water

“Latent heat of melting”: 80 cal (335 J) melts 1 g of ice at 0°C

100 cal (420 J) increase temperature of 1 g of water from 0°C to 100°C

“Latent heat of evaporation”: 540 cal (2255 J) convert 1 g of water to vapor at 100°C ; Evaporation can occur at any temperature (if vapor pressure at that temperature exceeds pressure of surrounding water vapor). Vapor pressure increases with temperature and is 1 atm (10^5 Pa) at 100°C =>

Boiling: gas can form **everywhere** (*bubbles*), not only on surface if vapor pressure > surrounding pressure.

Triple point: All 3 phases exist together (0.01°C , 600 Pa)

First Law of Thermodynamics:

$\Delta E_{\text{internal}} = (\text{Work done on system}) + (\text{Heat added}) - (\text{Work done by system}) - (\text{Heat removed})$ [assuming no other form of energy changes in system]

Energy conservation! (“You cannot win”)

For ideal gases:

$$E_{\text{internal}} = \frac{3}{2} nRT$$

$PV = nRT$ [P = pressure in Pascal, V = volume in m^3 , n = number of mols, R = gas constant = 8.3 J/mol/K , T = temperature in K]

=> Volume of 1 mol of gas at 0°C and normal atmospheric pressure = 22.4 liters

Ideal Heat Engine (Carnot Machine):

Remove Heat Q_1 from reservoir at higher temperature T_{hot}

Exhaust less heat $Q_2 = T_{cool}/T_{hot} \times Q_1$ at lower temperature T_{cool}

Get work $\Delta W = (1 - T_{cool}/T_{hot}) \times Q_1$ out – efficiency $e = 1 - T_{cool}/T_{hot}$

Refrigerator or heat pump => works in reverse: Remove Heat Q_1 from cold reservoir at T_{cool} ; put in mechanical work $\Delta W = (T_{hot} / T_{cool} - 1) \times Q_1$ to exhaust more heat $Q_2 = T_{hot} / T_{cool} \times Q_1$ at higher temperature T_{hot} .

Second Law of Thermodynamics:

- No machine can simply convert heat into work without exhausting some of the heat into a colder reservoir
- No machine can beat the Carnot machine’s efficiency
- Heat can never flow spontaneously from cold to warm without external input of work
- Entropy (disorder) can never decrease within a closed system – any local decrease must be compensated by an equally large or larger increase somewhere else; it always tends to increase over time (e.g. when heat flows from warm to cold)

Oscillations:

Period: T [sec]; Frequency: $f = 1/T$, unit Hertz (1 Hz = 1 oscillation/sec)

Harmonic Oscillator: $x(t) = A \sin(2\pi ft)$; f independent of amplitude
 [x = excursion, A = amplitude=maximum excursion from equilibrium/rest]

Pendulum of length l : $f = \frac{1}{2\pi} \sqrt{g/l}$

Mass m on spring (spring constant k): $f = \frac{1}{2\pi} \sqrt{k/m}$

Oscillator will react most strongly to external periodic disturbance near its own intrinsic frequency (**resonance**) -> amplitude can build up quickly.

Waves:

Wave velocity v_{Wave} , wavelength λ (distance between crests), frequency f :

$$v_{wave} = \lambda/T = \lambda f ; \Rightarrow \lambda = v_{wave} / f$$

Waves on a string: wave velocity increases with string tension

Other examples: Water waves, “slinky waves”, sound waves, light...

Excursions **perpendicular** to propagation: **Transverse** wave (2 polarization directions); Examples: Water waves, waves on a string, light.

Excursions in direction of propagation: **Longitudinal** wave; Examples: “Slinky waves”, sound waves.

Doppler effect: Emitter or receiver moving through medium – λ shortened and f increased if moving towards each other, else λ increased and f smaller.

Interference: Excursions of 2 overlapping waves add up; can lead to destructive interference (out of phase; less net excursion) or to constructive interference (in phase; more net excursion). Extreme case: Complete extinction (2 waves with equal amplitude, 180 degrees out of phase).

Standing wave: Wave interfering with its reflection in medium of length L ; wavelength λ must be integer fraction of a fundamental (either $2L$ or $4L$):

Nodes at both ends (string,...): $\lambda_n = 2L/n$, $n = 1, 2, 3, \dots$ (harmonics)

Node at one end only (air column,...): $\lambda_n = 4L/n$, $n = 1, 2, 3, \dots$

$n-1$ nodes between n maxima; resonant frequencies at $f_n = v_{\text{wave}} / \lambda_n$

Refraction: Waves change direction when wave velocity changes (transition from one medium to another); bend toward normal in “slower” medium

Diffraction: Each point along a wave is source of new (spherical/circular) wave (Huygen’s principle); waves can spread out and bend “behind” corners and obstructions.

Sound and Music:

Sound: longitudinal wave, oscillations in density (pressure in fluids or position of molecules in solids). Requires medium – solid, liquid, gas.

Wave speed in air: 330 m/s (0°C) – 340 m/s (20°C).

Intensity (audible range): 10^{-12} W/m² (0 decibel) – 1 W/m² (120 decibel – pain threshold!); Intensity = Amplitude squared

10x more intensity: add 10 decibel; 10x more amplitude: add 20 decibel

Beat frequency: fluctuation of sound with frequency $f_1 - f_2$ if 2 sound waves with nearby frequencies f_1, f_2 interfere.

Concert A: $f = 440$ Hz; 1 octave = factor 2 in f

Electromagnetism II: Induction

Wire moving in magnetic field \rightarrow force qvB on each electron inside the wire
 \rightarrow pushing a current along wire.

\rightarrow Working principle of electric generator (wire loop rotating in magnetic field)

Magnet moving relative to a “fixed” wire \rightarrow same force, but cause is electric field \mathbf{E} induced by changing magnetic field \Rightarrow **Faraday’s Law:**

Changing magnetic field \mathbf{B} due to *any* cause: (non-static) electric field \mathbf{E}

Induced field \mathbf{E} will circle “flux” of magnetic field, perpendicular to \mathbf{B}

Lenz’s Law: IF the induced electric field can push free charges around (conductor present), the resulting current will oppose the change in the magnetic field strength. \Rightarrow jumping ring on magnet, sparks after disconnecting a plug, electric braking with eddy currents...

Induced EMF (= energy gain per loop per unit charge) is proportional to area filled with magnetic field times the rate of change of the magnetic field.

Coil with several loops in changing magnetic field: Induced EMF is proportional to number of loops.

Transformers: Output voltage/Input voltage = # of secondary coil loops/# of primary coil loops.

Electromagnetic Waves

Maxwell’s Law: Changing electric field creates magnetic field circling it (perpendicular to \mathbf{B}).

Chain reaction: Changing $\mathbf{E} \rightarrow \mathbf{B}$ (Maxwell); Changing $\mathbf{B} \rightarrow \mathbf{E}$ (Faraday);

Changing $\mathbf{E} \dots \Rightarrow$ Electromagnetic waves! Wave speed in vacuum = c

\mathbf{E} perpendicular to \mathbf{B} , \mathbf{E} and \mathbf{B} perpendicular to propagation, $B = E/c$

Due to accelerated charges (antenna, oscillating molecules/atoms/nuclei...)

Typical wave lengths/frequencies

- Radio waves: $\lambda = 1\text{ m} - 1000\text{ m} \mid f = 300\text{ kHz} - 300\text{ MHz}$
- Microwave: $\lambda = 1\text{ mm} - 1\text{ m} \mid f = 300\text{ MHz} - 300\text{ GHz}$ (GHz = 10^9 Hz)
- Infrared: $\lambda = 0.75\mu\text{m} - 1\text{ mm} \mid f = 300\text{ GHz} - 400\text{ THz}$ (THz = 10^{12} Hz)
- Visible Light: $\lambda = 375\text{ nm} - 750\text{ nm} \mid f = 400 - 800\text{ THz}$
- Ultraviolet: $\lambda = 10\text{ nm} - 375\text{ nm} \mid f = 0.8\text{ PHz} - 30\text{ PHz}$ (PHz = 10^{15} Hz)
- X-ray: $\lambda = 10\text{ pm} - 10\text{ nm} \mid f = 30\text{ PHz} - 30,000\text{ PHz}$
- γ -ray: $\lambda < 10\text{ pm} \mid f > 30,000\text{ PHz}$

All objects/materials emit electromagnetic radiation over the full spectrum; maximum intensity centered on frequency which is proportional to temperature (ice-cold universe at 2.7 K: microwaves; room temperature: Infrared; sun at 6000K: visible light around 550 nm = greenish-yellow, etc.)

Color of Light:

Determined by mix of wave lengths/frequencies in light waves (spectrum)

“Pure” colors: 400 nm = violet, 500nm = blue-green, 600 nm = yellow, 700 nm = red

Color perception: Primary colors = red (600-700 nm), green (500- 600), blue (400- 500)

Additive colors: combination of relative intensities in these 3 ranges – e.g., yellow = red + green, cyan = green + blue, magenta = blue + red; white = equal mix of all 3

Subtractive (complementary) colors (cyan, magenta, yellow) remove intensity from reflected light – e.g. cyan removes red.

Color of light can get altered by scattering (wave-length dependent transmission vs. sideways scattering), absorption (removal of certain wave lengths in transmission or reflection) and dispersion (splitting up refracted light into individual wave lengths -> prism); also interference and diffraction.

Geometrical Optics (follows from Fermat’s Principle of least time¹):

Propagation: constant medium => wave propagation follows straight lines = “rays” => Shadows (umbra/penumbra), pin hole camera

Reflection: Incoming angle = outgoing angle (on opposite side of normal)

Absorption: Wave energy dissipates, “ray” is stopped (often color-selective).

Refraction: Change in wave velocity $v_{\text{wave}} = c/n$ (n = index of refraction) =>

- going from small n to large n : refracted part of wave bending towards normal
($n_1 \sin \theta_1 = n_2 \sin \theta_2$);
- going from large n towards small n : bending away from normal, beyond maximum incoming angle: total internal reflection.

Dispersion: n increases with shorter λ – blue more deflected than red -> Prism, rainbow, red or blue fringes in photos,...

¹ Light “rays” travel from A to B along that path which requires the least time

Scattering: Waves are absorbed by small particles and reemitted in all directions => decrease of intensity in the direction of the ray, diffuse background light in all other directions. Shorter-wavelength waves (blue) usually are scattered more. -> Blue sky, red sunset, brownish color due to air pollution

Image: Where light rays reaching your eye *seem* to be coming from

Real image: Formed by actual light rays converging at image position; can be captured by screen or photosensor or cones + rods in your retina

Virtual image: Cannot be captured; appears to be at a point where no actual light rays converge.

Flat Mirrors: Upright, equal-sized virtual image equally far from mirror (on other side) as object; interchanged front and back

Convex curved mirror: Smaller, upright virtual image on other side of mirror

Concave curved mirror: Object close => magnified, upright virtual image on other side of mirror; Object further away => inverted real image in front

Object in denser medium: Appears closer to surface than it actually is

Light going through parallel-surface plate at an angle: Sideways offset

Lenses: Described by focal length f (point where parallel rays are focused)

Concavely curved (dispersing) lens: Smaller, upright virtual image on object side of lens; used to correct near-sightedness (defocuses); virtual focal point.

Convexly curved (focusing) lens: If object closer than f => Virtual, upright, magnified image on same side as object; if object farther than f => real, inverted image on opposite side of lens than object (magnified if distance less than twice f); real focal point. Used for far-sightedness, projection, cameras; focusing lens in eye makes real image on retina for visual perception

Diffraction + Interference

Thin film interference: reflected waves from both sides of film interfere => pattern of bright light (constructive interference; path length difference = $n\lambda$) and dark (destructive interference; path length difference = $\lambda/2$ or $n\lambda + \lambda/2$); depends on wave length => different colors are removed at different thickness (bands, splotches or rings of varying color). Example: Soap bubbles, oil sheen on water.

Larger distance (cavities bounded by 2 mirrors) -> standing waves: interferometers (Fabry-Perot, Michelson); lasers

Double slit interference: Dark and bright stripes of equal thickness; spacing proportional to λ/s (s = slit separation) and to distance from slits to screen

Single slit: Broad central maximum, dim fringes; spacing of first minimum (diffraction limit) proportional to λ/w (w = slit width)

Diffraction grating: Many slits, narrow maxima separated by wide dark bands (spacing proportional to λ divided by slit separation); spectroscopy

Polarization: E field oscillates in one single direction (x or y only)

Polarizer/analyzer: transmits light oscillating only along its axis; if incoming light is polarized, amount (intensity) transmitted is $I/I_0 = \cos^2(\phi)$

(ϕ = angle between direction of polarization and axis of polarizer)

Reflected light is typically polarized parallel to the reflecting plane

Emission and Absorption

Classical Physics: Electromagnetic radiation due to accelerated charges, e.g., antennae, “bremsstrahlung” (= braking radiation due to charges slowing down), synchrotron radiation (charges moving in circles), hot objects.

Atomic Physics: Electromagnetic radiation due to transition between different electron orbits of different (fixed) energy; energy difference ΔE carried away by a “photon” of frequency $f = \Delta E/h$ (h = Planck’s constant)

Free atoms (dilute gas): Sharp, widely separated frequencies (emission after exciting atoms into higher orbits) with unique color (wave length) pattern for each type of atom/molecule (“spectroscopic fingerprint”); identical absorption lines (removing photons of same frequency from passing light beam)

2-step processes: Fluorescence (immediate re-emission at lower frequency) and Phosphorescence (re-emission of light after a while)

Stimulated emission (photon doubling) -> LASER

Incandescence: Broad, overlapping spectral lines -> continuous spectrum (due to interactions and Doppler broadening in dense materials); typical thermal radiation with peak frequency $f_{\max} \propto$ Temperature.

Example: The peak frequency of the light emitted by the sun (surface temperature $T \approx 6000\text{K}$) corresponds to green-yellow light ($\lambda = 500\text{-}550\text{ nm}$)

Particle-Wave Duality – Quantum Mechanics

Microscopic particles travel like waves (\Rightarrow interference, diffraction, standing waves, etc.) with wavelength $\lambda=h/p$ (momentum) and frequency $f=E/h$; waves interact with microscopic objects like particles carrying energy $E=hf$ and momentum $p=h/\lambda$. Examples: light travels as electromagnetic wave and interacts as point-like photons; particles like electrons travel as “matter waves” and interact like point-like objects, etc. Interaction is always “all or nothing” (= quantized).

“Matter waves” are oscillations of an abstract “probability amplitude” ψ ; its **intensity** = amplitude squared $|\psi|^2$ at some point gives probability of finding particle at that point. Stable orbits correspond to standing “matter waves”.

Photo-electric effect: **energy** of emitted electrons depends on **frequency** f of absorbed light: $E = hf - W$ (W = work required to eject electron from metal; no emission if $hf < W$); **number** of emitted electrons proportional to light intensity

Heisenberg’s uncertainty principle: $\Delta p_x \Delta x \geq h/2\pi$ (you cannot measure position in some direction and momentum in the same direction simultaneously with infinite precision). QM predicts probabilities for individual particles!

Atomic Physics

Stable “orbits” have fixed energies, are standing “electron waves” which have $(n-1)$ nodes ($n = 1,2,3,\dots$). n = principle quantum number; $n = 1$ ground state and $n = 2, 3,\dots$ are excited states (“higher harmonics”).

Hydrogen atom: average radii are $r = \frac{h^2 n^2 \epsilon_0}{\pi m e^2} = n^2 \cdot 0.53 \cdot 10^{-10}$ m

and corresponding energy levels (kinetic+electrostatic) of

$$E = -\frac{m e^4}{8 \epsilon_0^2 h^2} \frac{1}{n^2} = \frac{-13.6 \text{ eV}}{n^2}$$

Possible photon frequencies are $f = \Delta E/h$ where ΔE is the energy difference between any two of these orbits; for hydrogen $\Delta E = 13.6 \text{ eV} \times (1/n_{\text{final}}^2 - 1/n_{\text{initial}}^2)$ Ex.: Balmer line series: $n_{\text{final}} = 1$ (all UV); $n_{\text{final}} = 2$ are visible

Number of electrons per orbit limited (Pauli Exclusion Principle): any given orbit can have only 2 electrons (1 “spin up”, 1 “spin down”)

At higher energy, several orbits can have exact same energy (“degeneracy”)
 => the possible number of electrons in each **energy level** is 2 for $n=1$, $2 + 6$
 for $n=2$, $2 + 6 + 10 = 18$ for $n=3$, etc. (=> Periodic Table of Elements!)

Total number of electrons = total number of protons in nucleus = Z
 => determines element (chemical properties)

The larger Z , the smaller each individual orbit

Higher n orbits cover larger distances from nucleus

In reality, orbits are smeared-out 3-dimensional waves (“electron clouds”)

Nuclear Physics

Nucleus contains roughly 99.97% of the mass of an atom, but occupies only 10^{-15} of its volume (radius: $1 \dots 7 \cdot 10^{-15}$ m = $1 \dots 7$ fm)

Mass number A = proton (element) number Z + neutron number N ;

EINSTEIN (“ $m = E/c^2$ ”):

$$m_{\text{Nucleus}} < Zm_{\text{proton}} + Nm_{\text{neutron}} \text{ (negative binding energy } \Delta E \rightarrow \Delta m = \Delta E/c^2 \text{)}$$

Most binding energy per nucleon ($\Delta E / A = 8$ MeV = 0.9% of its mass) for $A \approx 56$ (iron); Smaller A : less $\Delta E / A$ (missing neighbors at surface);

Larger A : less $\Delta E / A$ because of electrostatic repulsion between protons.

Transmutations of nuclei:

Change Z (different element) and/or N (different isotope)

Fusion: two light nuclei combine; energy gain if final $A \leq 56$

Requires initial energy to overcome Coulomb repulsion between ingredients

Fission (often initiated by neutron capture): one very heavy nucleus breaks into 2 pieces; energy gain (due to electrostatic repulsion between fragments) if final pieces both have $A > 56$; usually additional neutrons are liberated

Chain reaction if enough nuclei around (critical mass) for neutrons to initiate new fission

Alpha (α) decay: Heavy nucleus emits ${}^4\text{He}$ nucleus ($Z \rightarrow Z-2$, $A \rightarrow A-4$)

Beta (β) decay: Isotope emits electron (and anti-neutrino; β^-) – $Z \rightarrow Z+1$;
 or positron (and neutrino, β^+) – $Z \rightarrow Z-1$ (A stays constant, $\Delta N = -\Delta Z$)

γ decay: photon emission when excited nucleus goes from higher to lower energy state (Z , N , A don't change).

Radioactivity: Spontaneous transmutation of a given isotope

α , β , γ decay, spontaneous fission, etc.

Half-life $T_{1/2}$: Time it takes for $1/2$ of initial radioactive nuclei to decay

One nucleus: 50/50 chance to survive after time $T_{1/2}$ passes

Number of nuclei left = 1/2, 1/4, 1/8, 1/16,... after 1, 2, 3, 4,... half-lives

Typical α emitter: ^{238}U , $T_{1/2} = 4.5 \cdot 10^9$ years (age of Earth); decay delayed by barrier (alpha particle has to first move away far enough from rest nucleus)

Typical β emitter: ^{14}C , $T_{1/2} = 5700$ years (age of civilization); decay delayed because weak interaction takes a long time

Typical γ emitter: $^{99}\text{Tc}^m$, $T_{1/2} = 6$ hours (metastable state, useful for nuclear medicine).

Radioactive isotopes used for dating (ratio of number of nuclei left N divided by initial number of nuclei $N_0 \Rightarrow$ derive number of half lives expired)

Effects of radiation:

α : highly ionizing (large damage), but easy to shield (don't inhale! Radon!!)

β : moderately ionizing (moderate damage), need thicker shielding (Al sheet)

γ : less ionizing, but has larger range; most difficult to shield (lead!)

n: very penetrating, highly effective interacting with hydrogen.

Nuclear Force:

“Left-over” effect from strong force between quarks; attractive at 1-2 fm distance, repellent at shorter distance, disappears at large distance; only acts on near neighbors in nucleus \rightarrow liquid drop-like behavior of nuclei.

Particle Physics

Matter (Fermions)		Interaction (Gauge bosons)	
Spin $1/2$, mass > 0 , Pauli exclusion		Spin 0,1,2, “massless”, can aggregate	
Leptons	Quarks	Spin 1	Spin 0, 2
$e, \mu, \tau, 3$ neutrinos	u, d, s, c, b, t	$\gamma, W, Z, \text{Gluon}$	Higgs, graviton

Protons, neutrons, ... = baryons and pions,... = mesons made of quarks

Leptons: Electrons in atoms, muons and taus in cosmic rays and accelerators

Neutrinos are siblings without charge (each lepton has a neutrino partner)

Each particle has an anti-particle partner with opposite charge

γ = photon = carrier of electromagnetic interaction

W, Z = carrier of weak interaction

Gluon = carrier of strong interaction

All particles gain mass from Higgs field (discovered at CERN in 2012)

Charge, energy, momentum and angular momentum absolutely conserved

Special Relativity

There is no absolute motion – all inertial reference frames are equivalent
 The highest speed possible for any object is c (speed of light), and it is the same as measured in any reference frame.

2 events separated in space may appear simultaneous in one reference frame but one after the other in a different frame (moving relative to the first one)

Fast-moving² yard sticks are measured to be shorter (length contraction: $1/\gamma$)

Fast-moving² clocks are measured to run slower (time dilation: $1/\gamma$)

Apparent mass (inertia) increases for a fast-moving² object; this increase is equal to T_{kin}/c^2 ; **all** energy contributes to mass of object ($\Delta m = \Delta E/c^2$)

=> Mass at rest represents energy $E = mc^2$ (Energy can be converted into mass; vice versa conversion of mass into energy in particle-antiparticle annihilation), total energy $E = mc^2 + T_{\text{kin}} = \gamma mc^2$; total momentum $p = \gamma mv$.

$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$ (all of these effects are tiny if v is much slower than c)

General Relativity

Object in gravitational field behaves like object in accelerating reference frame; new definition of Inertial System: “Freely falling” => $\mathbf{F} = m\mathbf{a}$ holds

All gravitational effects can be expressed through geometric distortions of space and time (curvature proportional to mass density)

Results: Changes in planet’s orbits, bending of light rays passing stars, red-shifting (energy loss) of light traveling away from heavy masses, clocks on Earth running slow relative to GPS satellites... Most extreme case: black hole – spacetime curves in upon itself (no escape beyond “event horizon”)

Cosmology

Universe started 13.8 billion years ago at extremely high density and temperature => expansion and cooling off – elementary particles, then nucleons, then nuclei (mostly ${}^4\text{He}$) formed, finally galaxies, stars (making more elements through fusion), planets (Earth: 4.5 billion years ago). 80% of all mass in universe is “dark” (unknown to particle physics), 70% of all ENERGY in universe is dark energy (accelerating expansion) – also unknown origin.

² Relative to an observer, as measured in that observer’s reference frame