Physics 313 Astrophysics Spring 2009

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www.odu.edu/~chyde/Teaching

Homework 6 due Wed March 18, 2009

1) Use the web page www.nndc.bnl.gov/wallet/ (or equivalent) to find the energy release in the following fusion reactions.
   a. $^{4}He + ^{12}C \rightarrow ^{16}O$
   b. $^{12}C + ^{12}C \rightarrow ^{24}Mg$
   c. $p + ^{12}C \rightarrow ^{13}N + \gamma$.
2) **Astrophysics in a Nutshell** Chapter 3 Problem 7.
   The nuclear reaction rate in a star is proportional to

   $$\langle \sigma v \rangle \propto \int_0^\infty f(E)dE$$

   $$f(E) = e^{-E/(kT)} e^{-\sqrt{E_G/E}}$$

   with $E_G = (\pi\alpha)^2 M_p c^2, \ \ \alpha = e^2/(\hbar c) = 1/137...$
   a. Show that the maximum value of *f(E)* occurs at

      $$E = E_0 = [kT/2]^{2/3} E_G^{1/3}$$

   b. Form a Taylor series, to second order in *ln[f(E)]*, to approximate *f(E)* as a Gaussian; i.e. find A and Δ in the approximation

      $$f(E) \approx A e^{-(E-E_0)^2/(2\Delta^2)}$$

   c. Using the Gaussian identities

      $$\int_{-\infty}^{\infty} e^{-x^2/2} dx = \sqrt{2\pi} = \int_{-\infty}^{\infty} x^2 e^{-x^2/2} dx,$$ show that

      $$\int_0^\infty f(E)dE \approx \sqrt{2\pi} f(E_0)\Delta$$ if $E_0 >> \Delta$.
3) The pp chain produces 2 neutrinos for every 26.2 MeV of fusion energy release. The solar flux of visible light reaching the upper atmosphere of the earth is ≈ 1000W/m^2. Assume the typical photon in the solar spectrum is a green photon of energy hv= 2 eV.

a. What is the number flux of visible photons reaching the upper atmosphere of the earth?
b. What is the number flux of neutrinos reaching the earth?
c. If the average energy of these neutrinos is 200 KeV, what fraction of the solar luminosity is carried away by neutrinos?

4) **Astrophysics in a Nutshell** Chapter 3 Problem 8.
The power production per unit mass of the pp->$De^{+}\nu$ reaction is (3.134)

$$\varepsilon = \frac{\rho}{M_H}\frac{2^{2/3}}{\sqrt{3}}\frac{QS_0c}{M_H\sqrt{M_Hc^2}}\frac{E_G^{1/6}}{(kT)^{2/3}}e^{-3\left[E_G/(4kT)\right]^{1/3}}$$

The "astrophysical S-factor" is $S_0=4\bullet10^{-46}$ cm^2 KeV. For the first step in the chain, Q=2.2MeV, but for the whole pp chain Q= 26.2 MeV. Use values from the solar center, ρ=150 g/cm^3.

a. Perform dimensional analysis on the expression for ε to establish it has units of Energy per unit mass per second.
b. Approximate ε as a power law $\varepsilon \approx \varepsilon_0\left(T/T_0\right)^{\beta}$. Hint, make a Taylor series in lnε as a function of $\ln(T/T_0)$ and expand around $\ln(T/T_0)=0$.
c. Evaluate β for kT_0=1.0 KeV and $E_G=500$ KeV.