Physics 313 Astrophysics Spring 2009

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Homework 7 due Wed March 25, 2009

1) The speed of light in a medium of index of refraction n is c/n, where c is the speed of light in vacuum. Relativistic charged particles can travel in the medium with a velocity v such that $c/n < v < c$. In this case, the particle will emit visible light all along its trajectory. The light is emitted at an angle θ relative to the particle direction, with $\cos(\theta) = c/(nv)$. The index of refraction of water is 1.33 .

   The relativistic energy E of a particle of rest mass m is

   $E=mc^2\gamma$ $\gamma = [1-\beta^2]^{-1/2}$ $\beta=v/c$.

   a. The rest mass of the electron is $mc^2$=0.511 MeV. Find the β value (close to 1) of an electron of energy 5.11 MeV.
   b. Find the Cerenkov angle for this electron in water.
   c. What is the lowest energy electron that will produce Cerenkov light in water (this limit is $\beta=1/n$ since the cosine function cannot be >1).

2) An ultra-high energy cosmic ray proton can be slowed down by the following inelastic collision with the photons of the 3° K cosmic black body radiation (CBR)

   $\gamma+p \rightarrow \Delta \rightarrow p+\pi^0$.

   The photons in the CBR have a typical energy of $2.5 \bullet 10^{-4}$ eV. The proton has a rest mass of $Mc^2=938 \bullet 10^6$ eV and the Δ-particle has a mass $M_\Delta c^2=1232 \bullet 10^6$ eV. Consider just the head on collision $\gamma+p \rightarrow \Delta$. Energy and momentum must be conserved in this reaction.

   Energy Conservation $k+E=E_\Delta$.

   Momentum Conservation $Pc-k=P_\Delta c$,

   where k is the photon energy (and momentum times c), E is the proton energy, P is the proton momentum, $E_\Delta$ is the Delta energy and $P_\Delta$ is the Δ momentum. The following relativistic relation holds for any particle of mass m:

   $E^2 = (pc)^2 + (mc^2)^2$.

   a. Using the energy and momentum conservation equations, as well as the energy-momentum-mass relation for the proton and Δ, find the minimum proton energy E such that the reaction $\gamma+p \rightarrow \Delta$ is allowed.