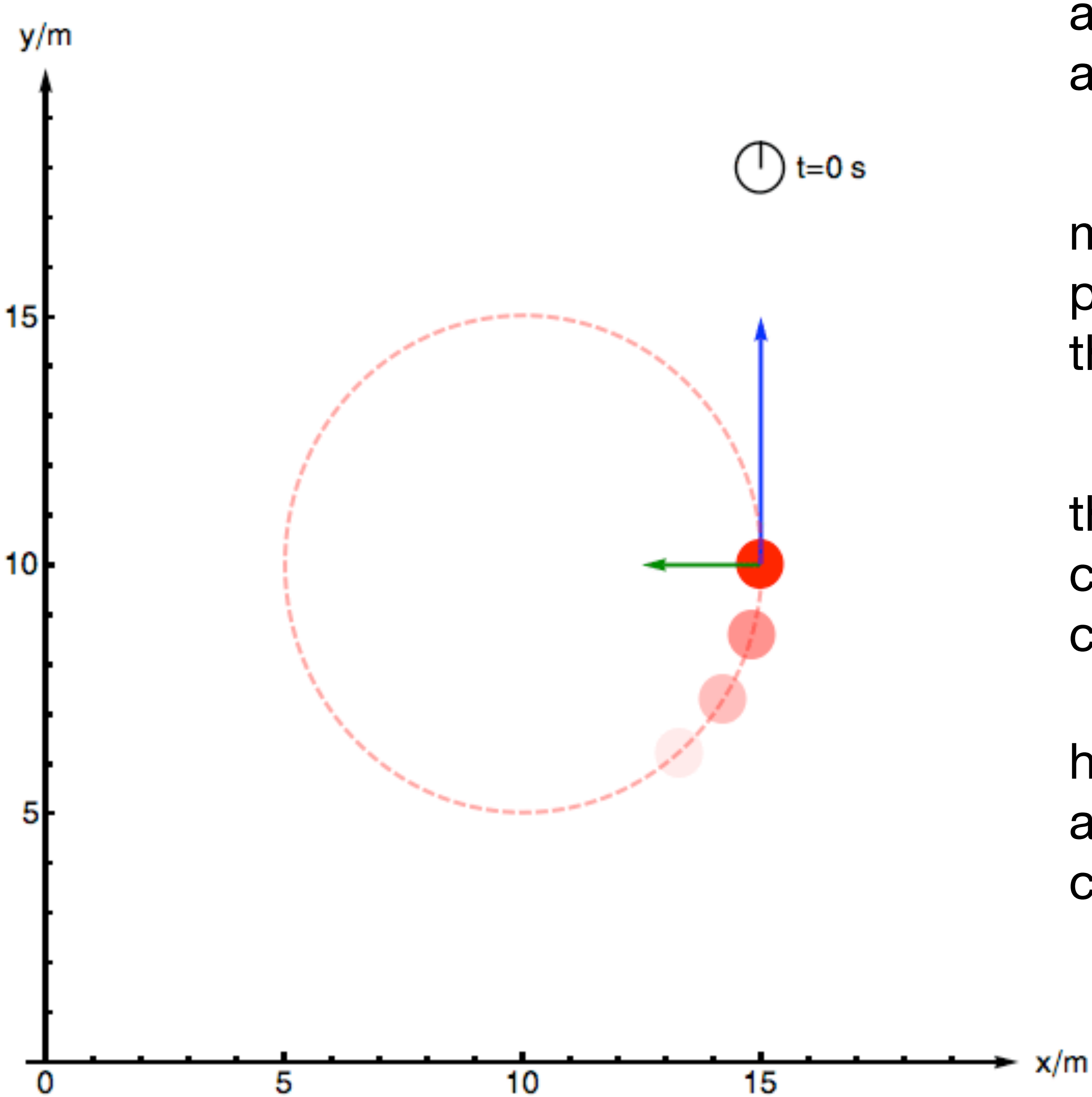
circular motion
& gravitation

uniform circular motion



an object moving around a circle at a constant rate

must have an acceleration always perpendicular to the velocity (else the speed would change)

the velocity is clearly tangent to the circle (or it would move off the circle)

hence the acceleration points always toward the center of the circle - “centripetal acceleration”

$$a_{\mathrm{rad}} = \frac{v^2}{r}$$

circular motion

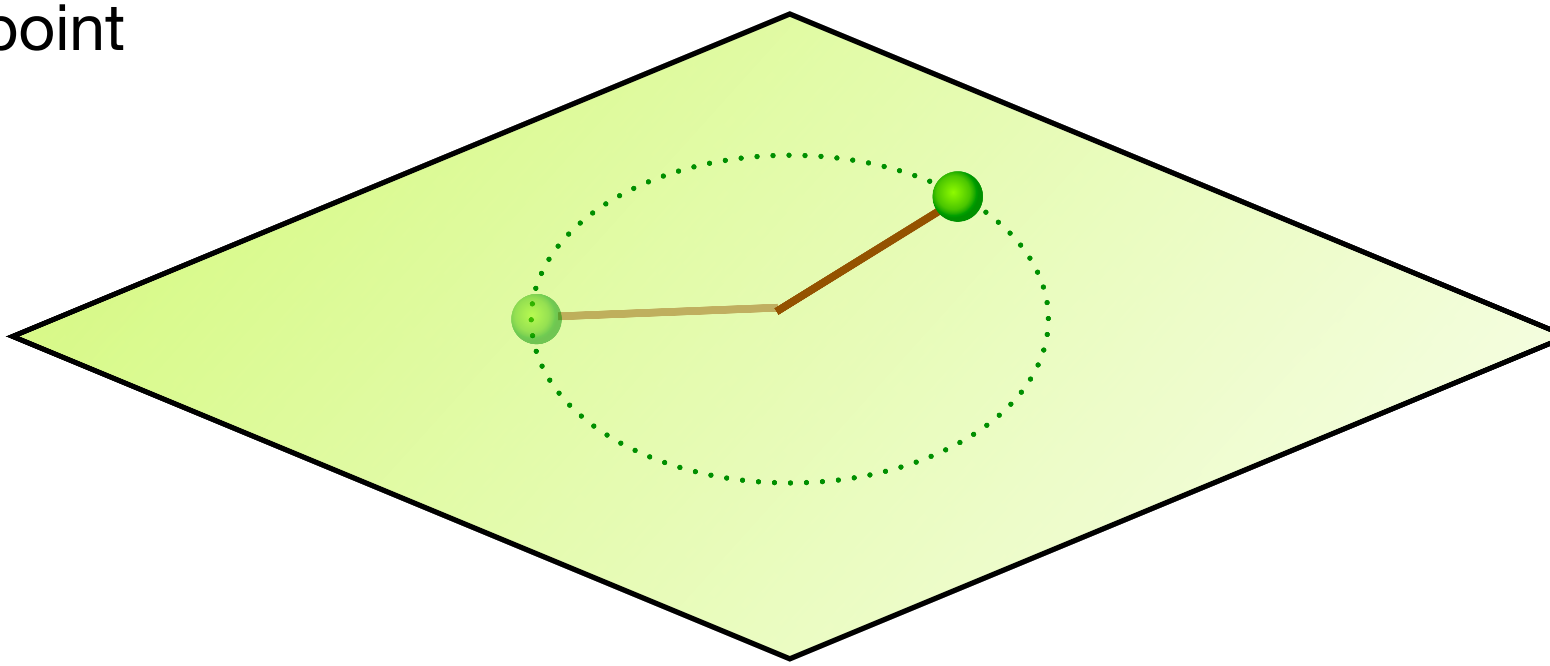
➜ velocity is tangent to the curve - can see it by cutting the rope

uniform circular motion

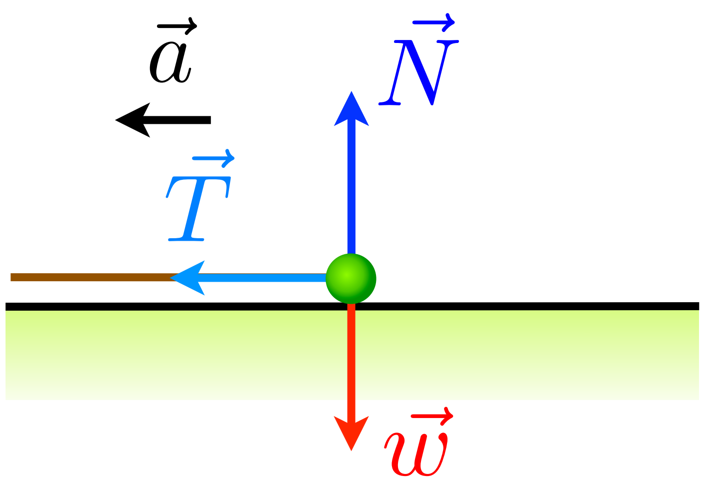
➜ acceleration is of constant magnitude and directed toward the circle's center

➜ **something must provide the force**

e.g. ball moving on a frictionless plane
tethered by a string to a fixed point



viewed side on:



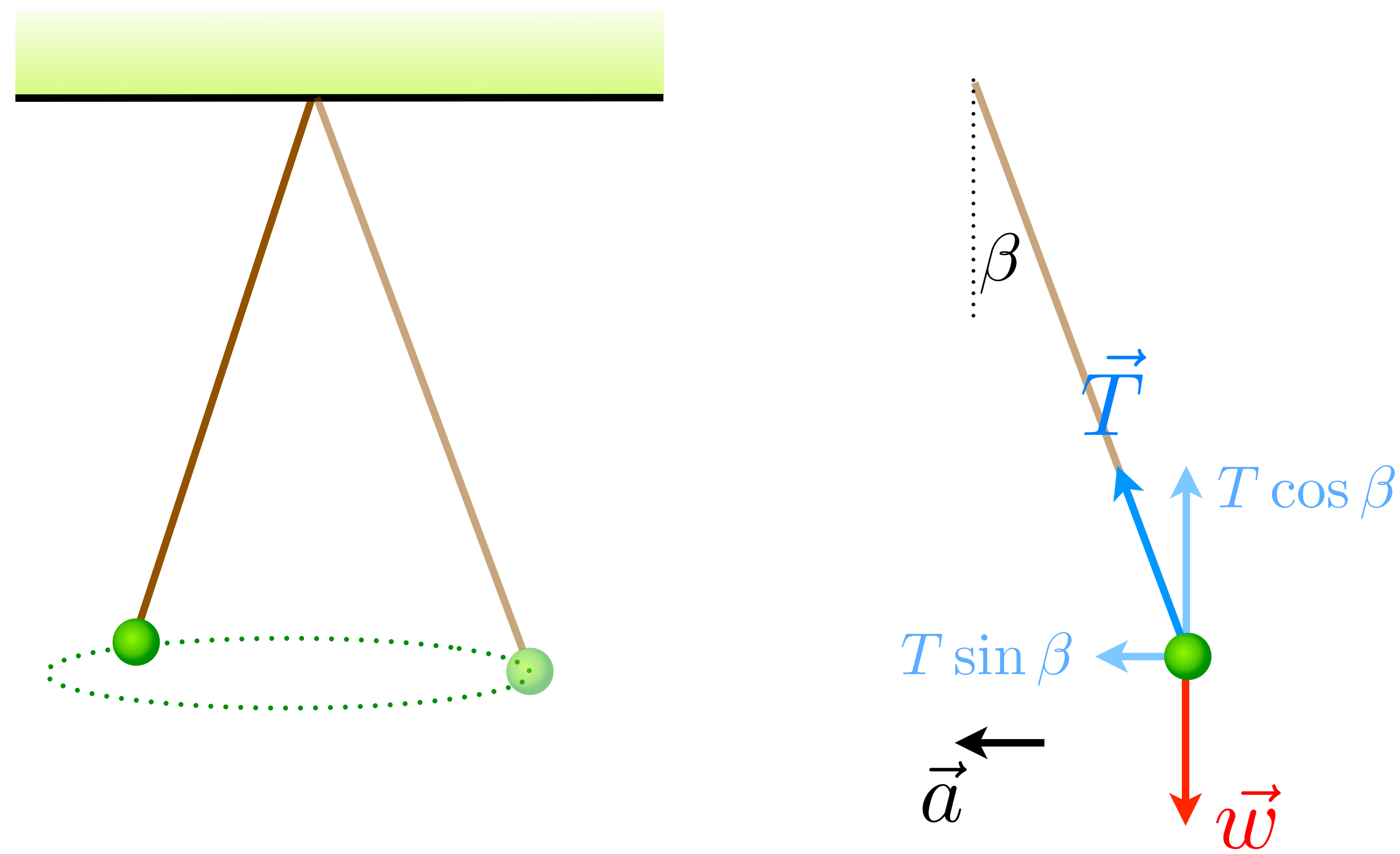
tension in the string provides a force always pointing toward the center of the circle

uniform circular motion

➜ acceleration is of constant magnitude and directed toward the circle's center

➜ **something must provide the force**

e.g. a conical pendulum



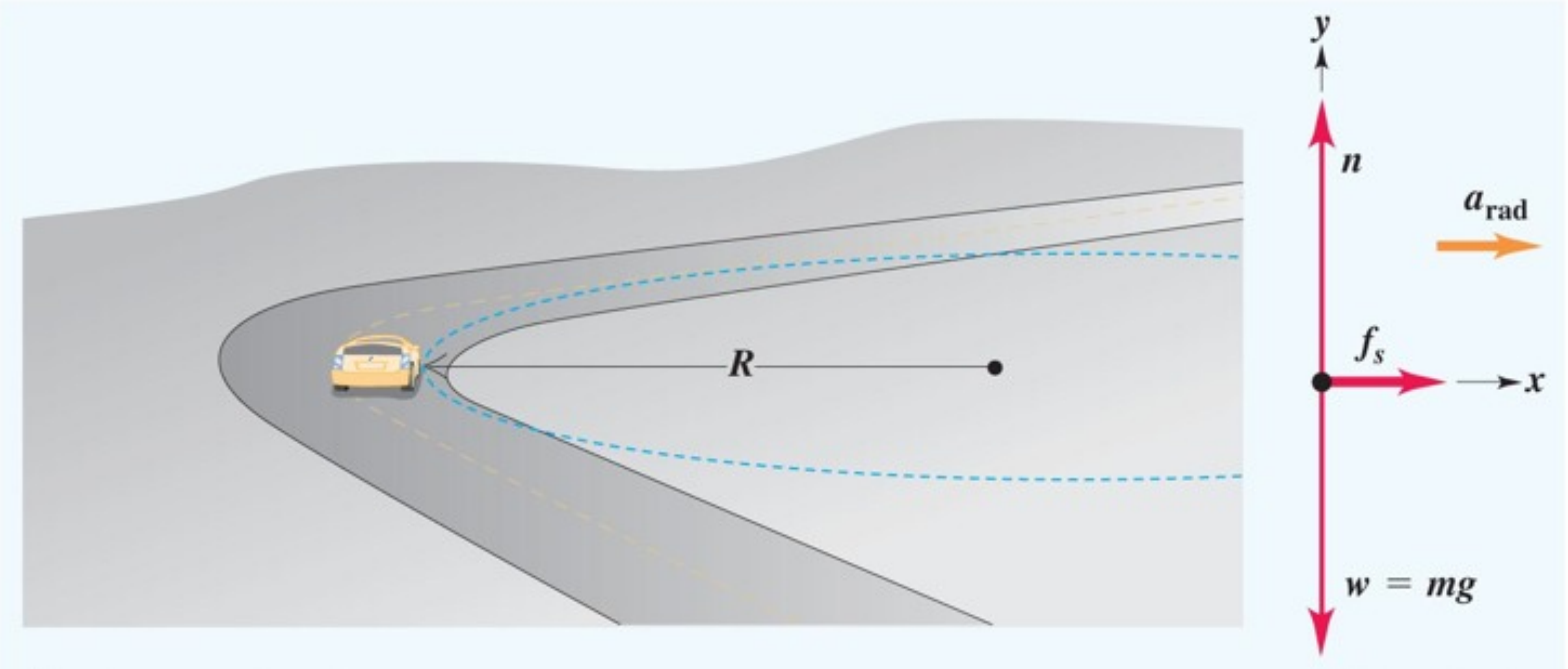
the horizontal component of **tension** in the string provides a force always pointing toward the center of the circle

uniform circular motion

➜ acceleration is of constant magnitude and directed toward the circle's center

➜ **something must provide the force**

e.g. car rounding a curve



(a) Car rounding flat curve

(b) Free-body diagram of car

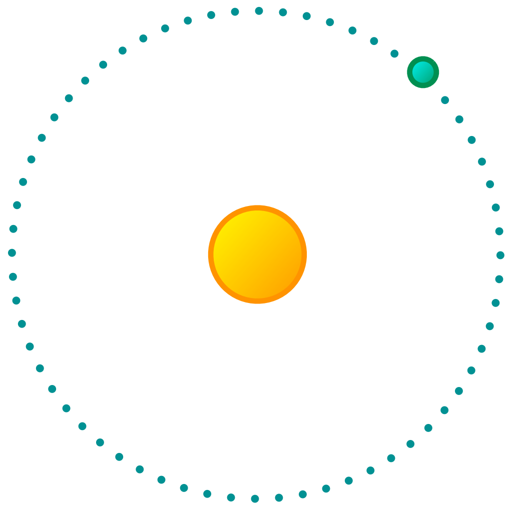
friction between tires & road provides a force pointing toward the center of the circle

in orbit

the Earth is in an orbit around the Sun that is very close to a circle

but there is no string joining the Earth to the Sun
nor is there anything to have friction against

what force is holding the Earth in a circular orbit ?

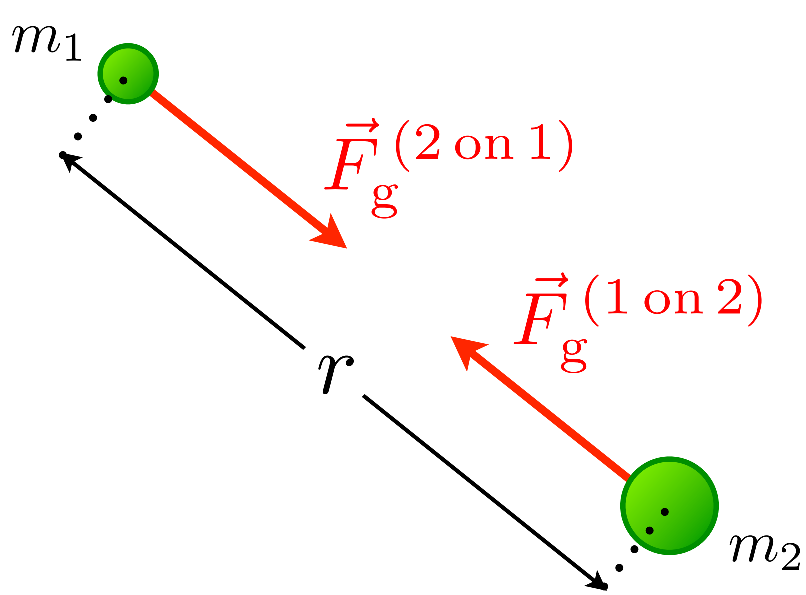


gravity

Newton's law of Gravitation

From astronomical observations and precise lab measurements we infer that the force of gravity between two bodies of mass m_1 and m_2 whose centers are separated by a distance r is

$$F_\mathrm{g} = G\frac{m_1 m_2}{r^2}$$



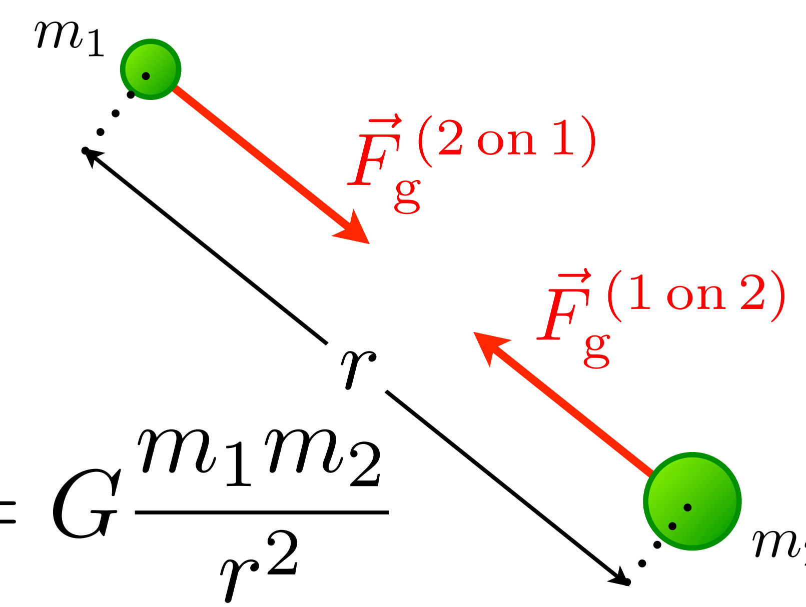
where there is a universal constant that controls the strength of gravitational attraction

$$G = 6.674 \times 10^{-11}\,\mathrm{Nm}^2/\mathrm{kg}^2$$

notice Newton's third law at work here

the scale of gravitation

Consider two spheres having mass 5.00 g and 1.00 kg whose centers are separated by 10.00 cm. Two such spheres could be used in a lab experiment called a Cavendish balance.



$$F_g = G\frac{m_1 m_2}{r^2}$$

$$G = 6.674 \times 10^{-11}\,\mathrm{Nm^2/kg^2}$$

$$\begin{aligned} F_g &= 6.674 \times 10^{-11}\,\mathrm{Nm^2/kg^2}\,\frac{5.00 \times 10^{-3}\,\mathrm{kg} \times 1.00\,\mathrm{kg}}{\left(10.00 \times 10^{-2}\,\mathrm{m}\right)^2} \\ &= 33.37 \times 10^{-11-3+2}\,\mathrm{N} \\ &= 3.34 \times 10^{-11}\,\mathrm{N} \end{aligned}$$

the force is extremely small !

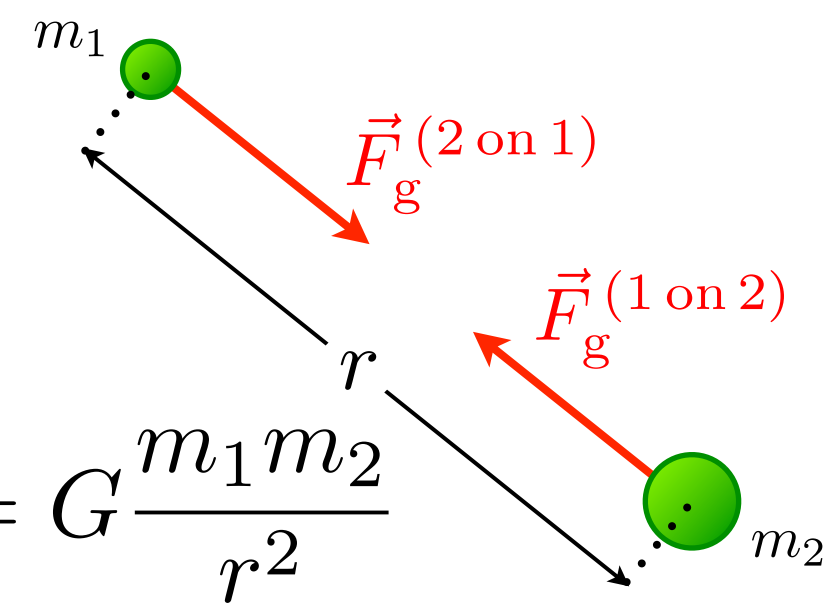
gravity is a very weak force, but it always adds up - the more mass a body has, the larger the gravitational pull it can exert on other masses

thus gravity becomes important if at least one of the two bodies under consideration is very massive

e.g. the Earth & you
e.g. the Sun and the Earth

weight

recall that earlier we said that all objects free-fall with an acceleration of g=9.80 m/s^2 due to their weight of magnitude $w=mg$



$$F_g = G\frac{m_1 m_2}{r^2}$$

but now we have a more complete formalism for gravitation can we see where this comes from ?

$$G = 6.674 \times 10^{-11}\,\mathrm{Nm^2/kg^2}$$

gravitational force from the Earth on an object of mass m located close to the surface of the Earth,

$$F_g = G\frac{m\, m_E}{R_E^2}$$

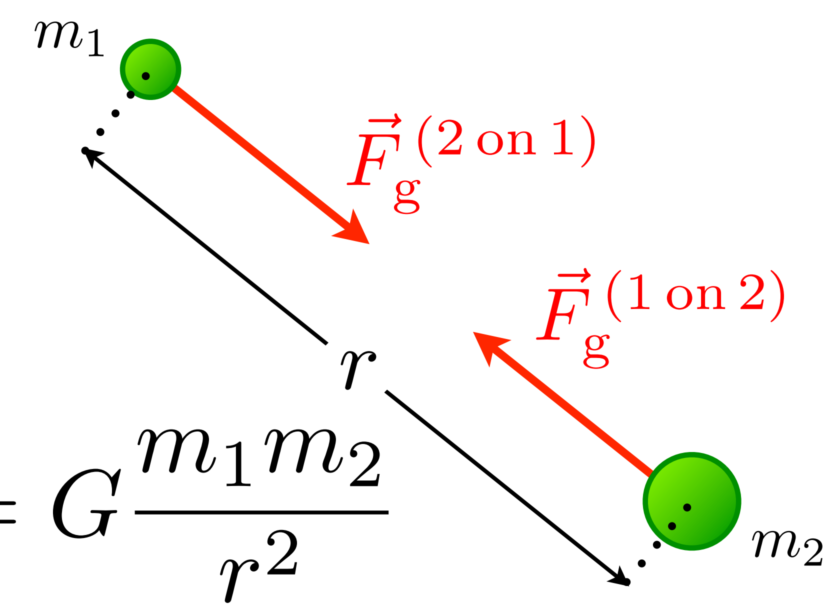
but this is the **weight** force $w = mg = F_g$

thus $g = \dfrac{G m_E}{R_E^2}$ which is independent of the mass of the body & depends only on the mass and radius of the Earth

but now we see that as we get further away from the surface of the Earth, the weight will get smaller !

satellites

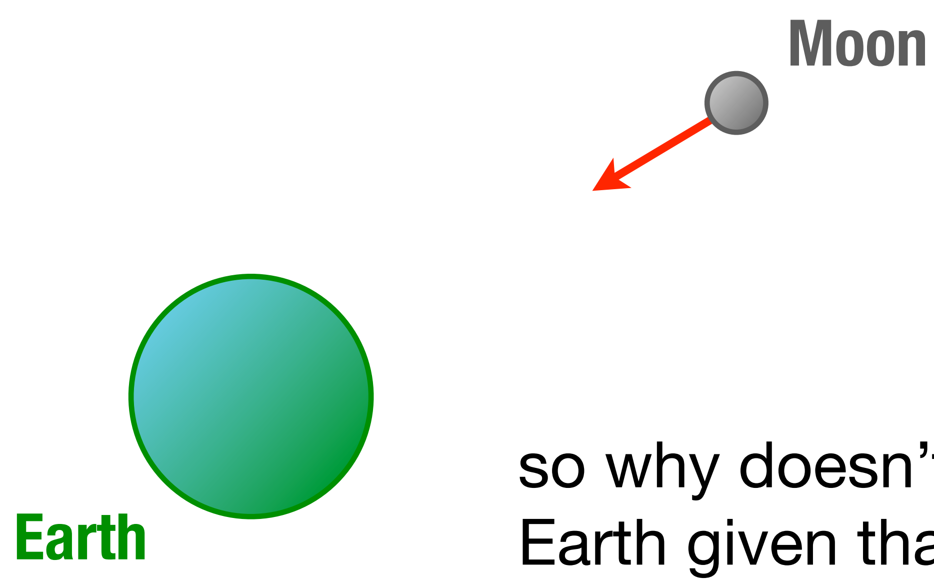
the Earth orbits the Sun, the Moon orbits the Earth,
GPS, TV, spy... satellites orbit the Earth ...



$$F_g = G\frac{m_1 m_2}{r^2}$$

$$G = 6.674 \times 10^{-11}\,\mathrm{Nm^2/kg^2}$$

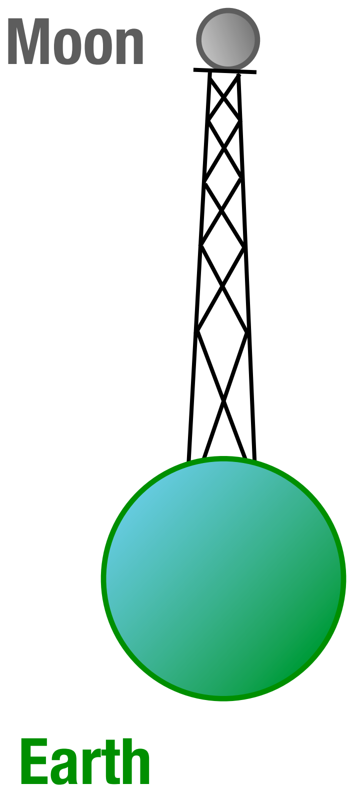
they are all examples of satellites where the only important force is gravitational attraction



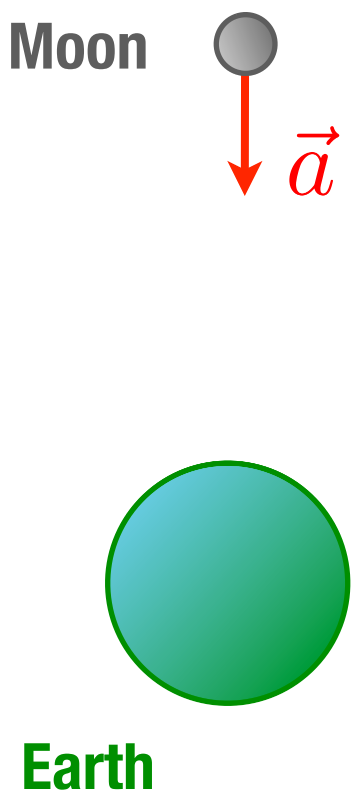
so why doesn't the moon plummet toward the Earth given that it is accelerating toward it ?

satellites

consider a 'thought' experiment where the moon got into its orbit by being launched from a huge platform



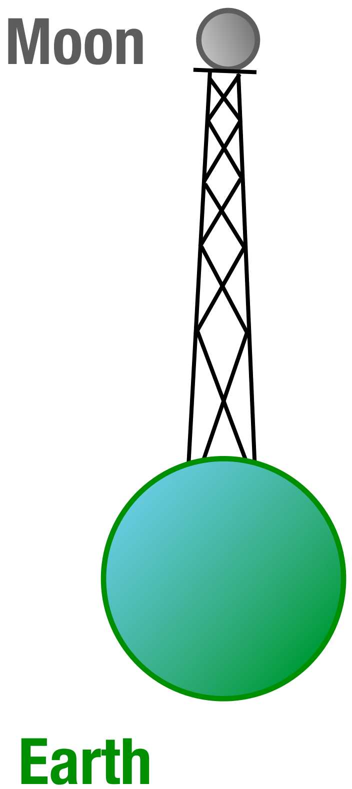
suppose we 'dropped' the moon with negligible tangential velocity



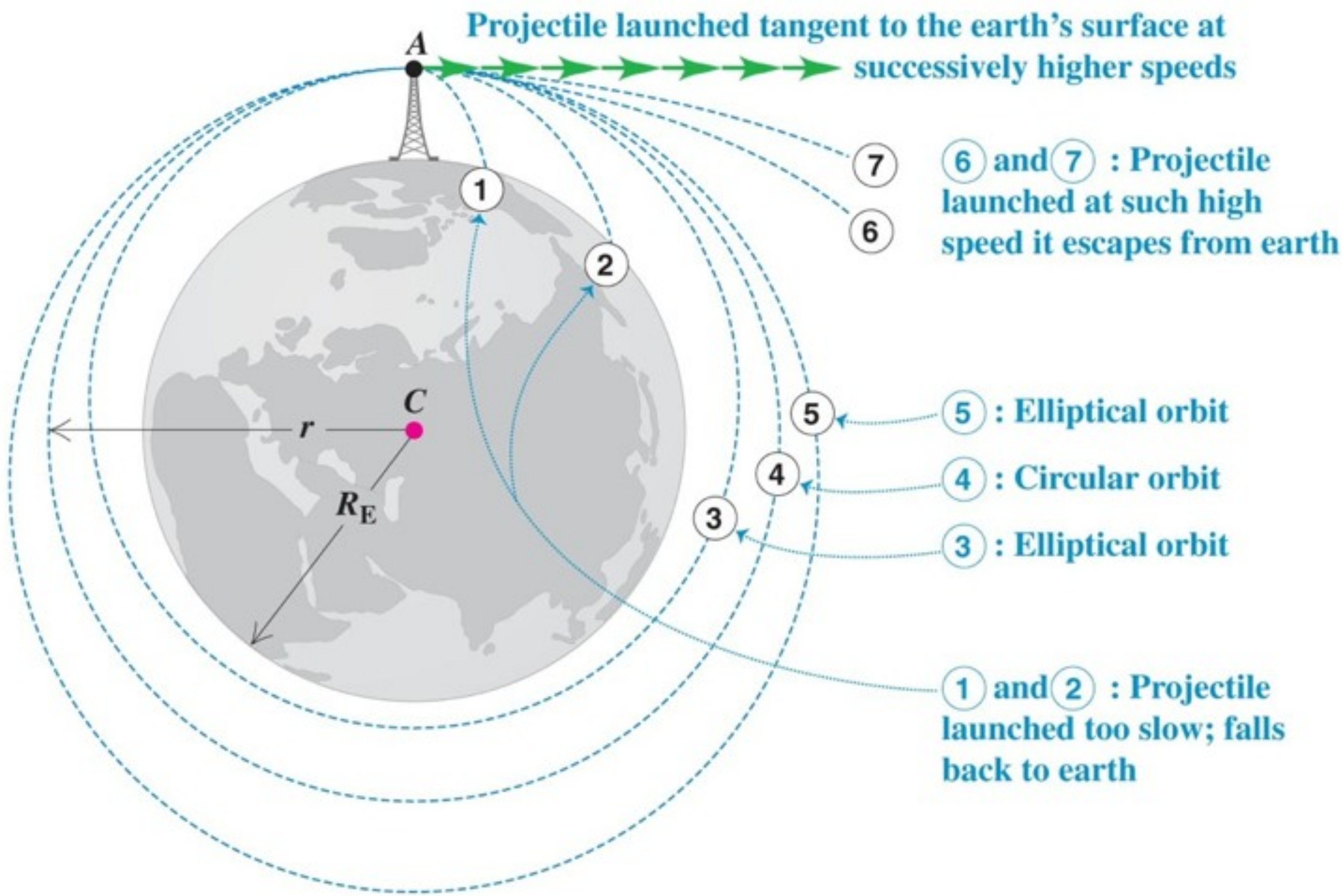
the moon would fall straight 'down' to the Earth

satellites

consider a ‘thought’ experiment where the moon got into its orbit by being launched from a huge platform



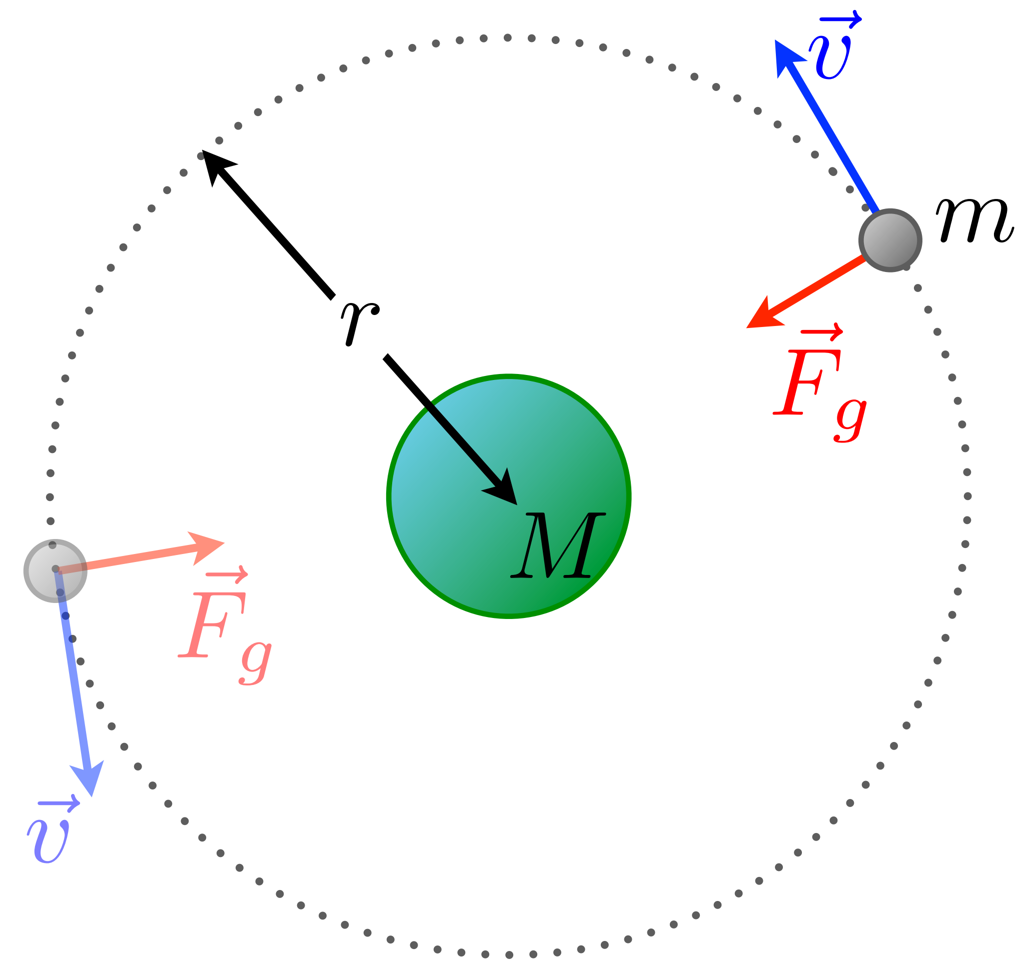
suppose we fire the moon with increasing tangential velocity



satellite in a circular orbit

$$\sum \vec{F} = m\vec{a}$$

suppose a satellite is found to be in a circular orbit, what do Newton's laws say about the motion ?



forces: (just gravity)

$$\sum \vec{F} = \vec{F}_{\mathrm{g}} \quad \text{directed radially inward}$$

$$F_g = G\frac{mM}{r^2}$$

acceleration: (in circular motion)

$$a_{\mathrm{rad}} = \frac{v^2}{r}$$

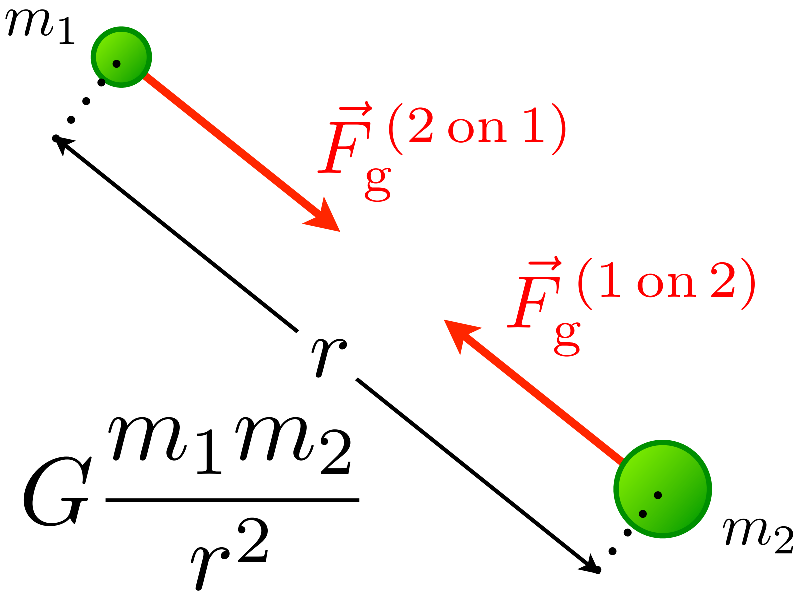
$$G\frac{mM}{r^2} = m\frac{v^2}{r} \implies v = \sqrt{\frac{GM}{r}}$$

relation to the orbital period: $v = \dfrac{2\pi r}{T}$

$$\implies T = \frac{2\pi}{\sqrt{GM}} r^{3/2}$$

weightless in the ISS

the International Space Station orbits the Earth every 91 minutes at a distance of 353 km above the surface of the Earth



$$F_g = G\frac{m_1 m_2}{r^2}$$

$$G = 6.674 \times 10^{-11}\ \mathrm{Nm^2/kg^2}$$

$$R_E = 6380\,\mathrm{km}$$
$$m_E = 5.98 \times 10^{24}\ \mathrm{kg}$$