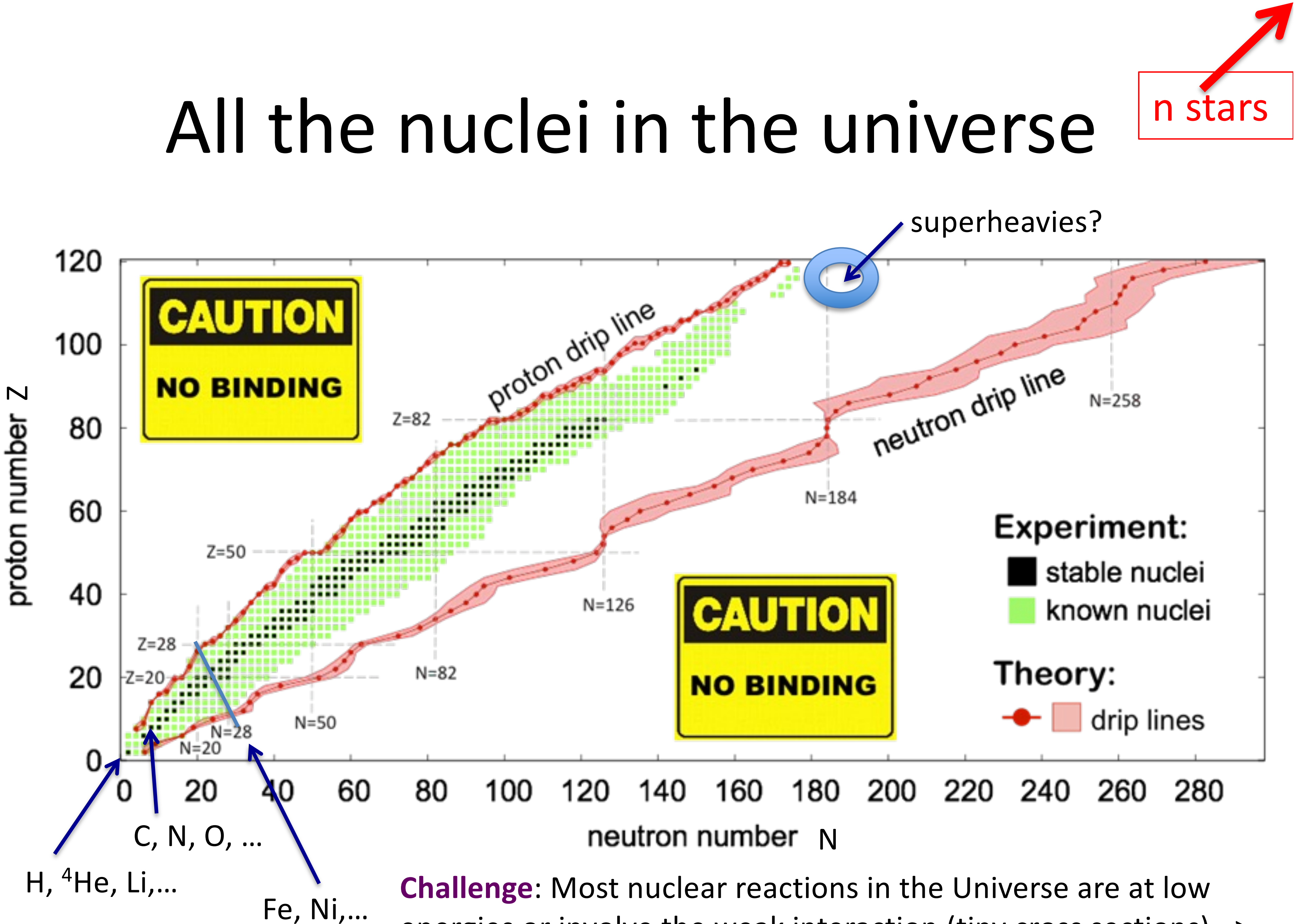
Nuclear Stability and Decay

- Not just any collection of protons and neutrons will form a stable nucleus!
- A (wannabe) nucleus will decay if
  - the combined mass of its decay products is smaller than the initial nuclear mass *)
  - the decay is not forbidden by a law of nature
- Even if allowed, a decay can take a long time (from μs to much longer than the age of the Universe).

*) Another way of saying this: IF the sum of the mass excesses/deficits of the daughters is less than the mass excess/deficit of the parent (including signs!).
WARNING: Some decays require taking electrons into account!

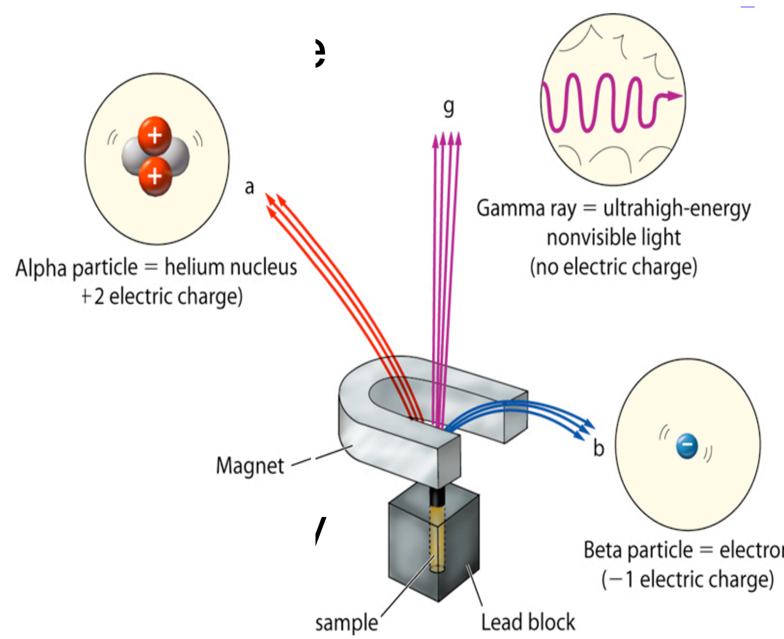
All the nuclei in the universe



Challenge: Most nuclear reactions in the Universe are at low energies or involve the weak interaction (tiny cross sections) => Experiments and Theory are HARD! (subtle effects play big role!)

Radioactivity (Nuclei going “kapoom”)

- Around late 1890s, people noticed that some nuclei emit extremely energetic, penetrating radiation
  - Can darken photographic film
  - Can make scintillators flash (fluorescence!)
  - Can cause discharge in high voltage gas tube
- Several different types
  - Proton or neutron emission (drip lines)
  - alpha radiation:
    whole ^{4}He nuclei being sent off
    at a few % of *c*
  - next letter in greek alphabet: beta radiation
    = high-energy electrons or positrons
  - gamma radiation =
    high-frequency electromagnetic waves
    (high energy photons)
  - More general breakup: Fission

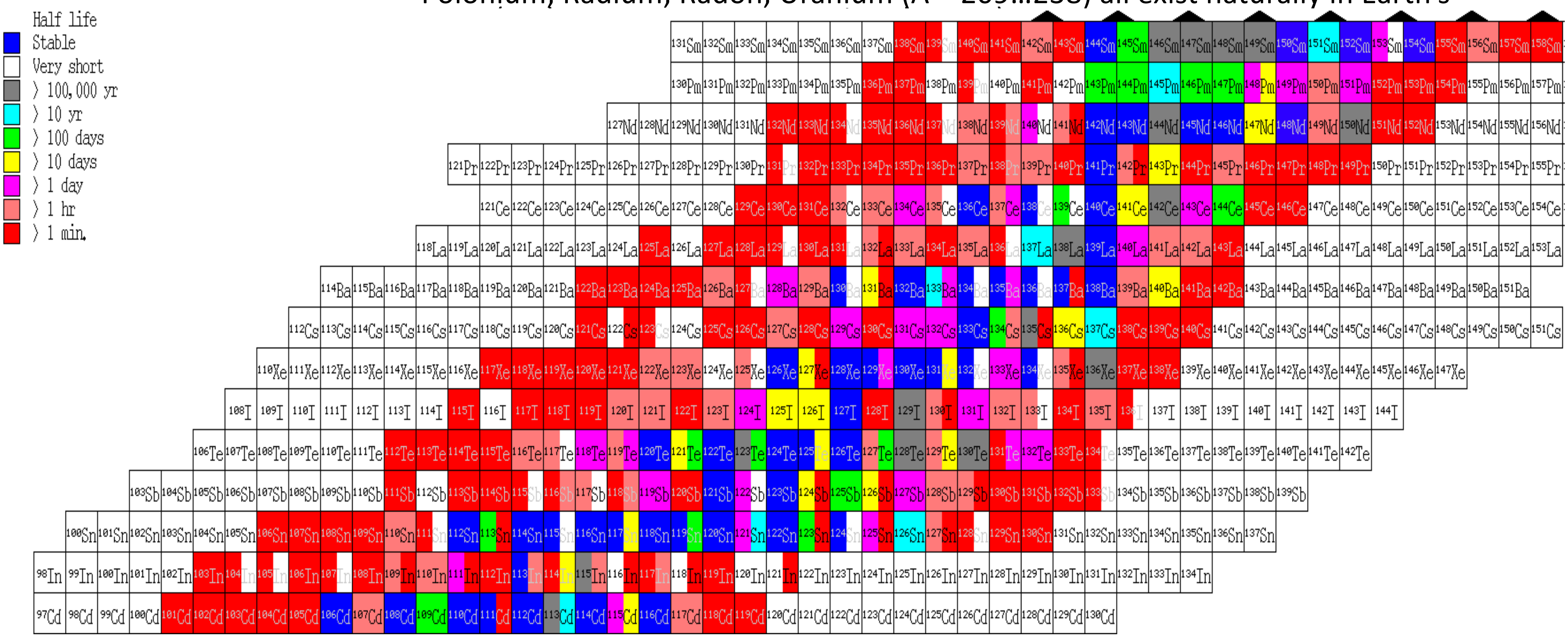


Some Examples for Radioactive Nuclei

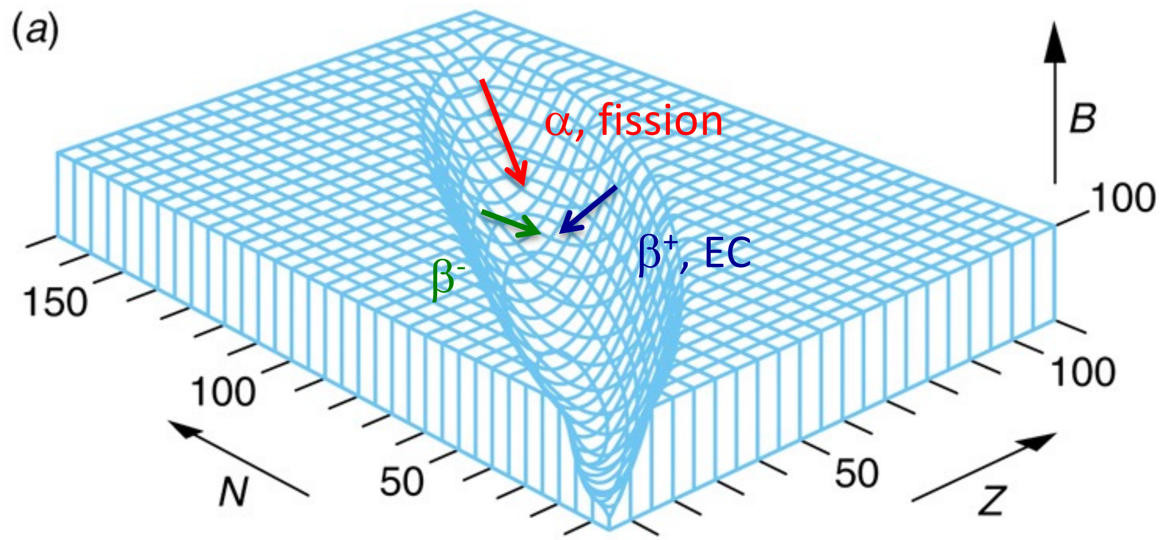
- All Nuclei heavier than lead (Pb):
  - Mostly emit alphas
  - Examples:
    - Polonium, Radium, Radon, Uranium (A = 209…238) all exist naturally in Earth’s crust (Radon, as a gas, can seep into houses)
      - Energy liberated in decay warms Earth’s interior!
    - Even heavier ones can be created artificially but don’t survive very long (Neptunium, Plutonium, Americium… up to A = 292)
  - Reason for decay: get rid of too much charge (electrostatic repulsion)
- Many isotopes throughout the periodic table emit betas:
  - ^{3}H = tritium (artificial), ^{14}C (atmosphere), ^{40}K (your bones)…
  - Reason for decay: re-balance ratio protons/neutrons
- Many nuclear alpha and beta decays are followed by gammas
  - Remove excess energy (just like electron jumping from higher to lower orbit after excitation)

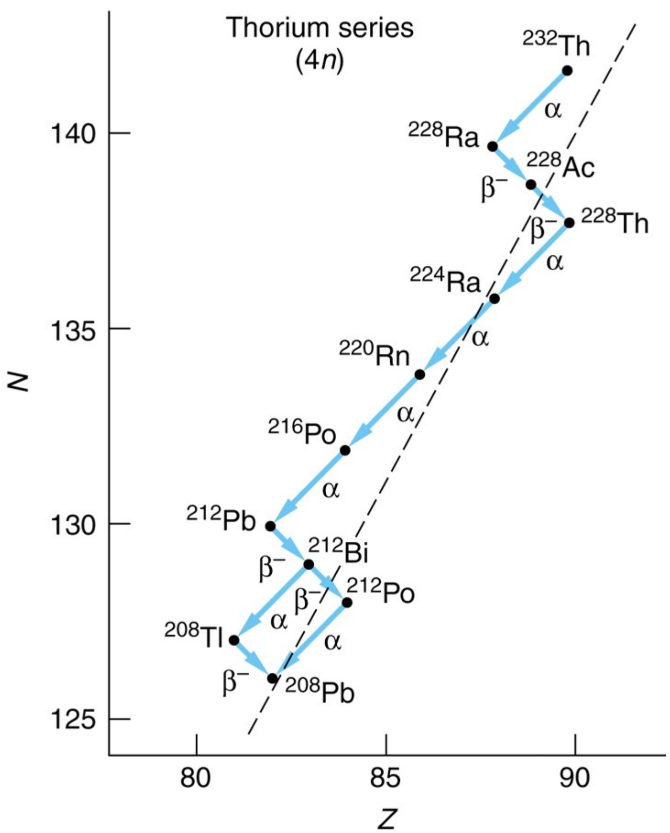
Some Examples for Radioactive Nuclei

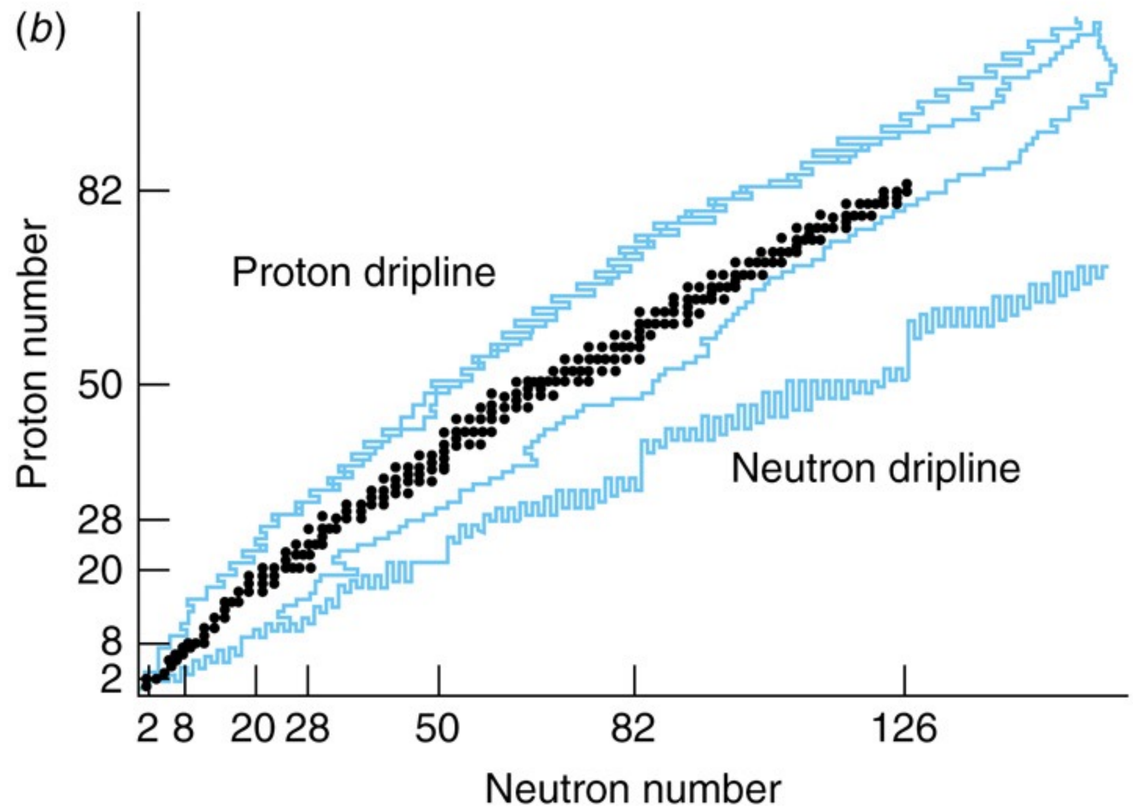
- All Nuclei heavier than lead (Pb):
  - Mostly emit alphas
  - Examples:
    - Polonium, Radium, Radon, Uranium (A = 209…238) all exist naturally in Earth's



Nuclear Stability and Decays

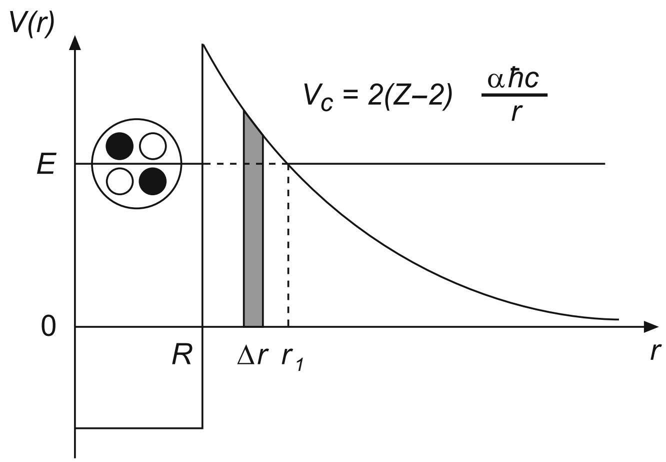






Alpha Decay

M(A-4, Z-2) + M(4He) < M(Z,A)



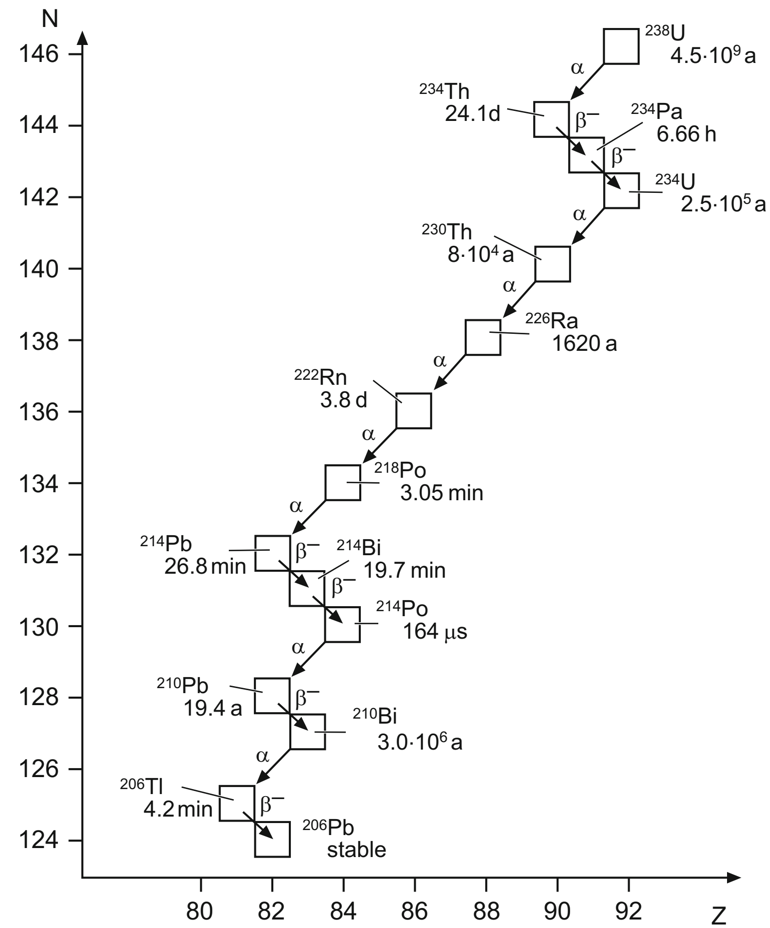
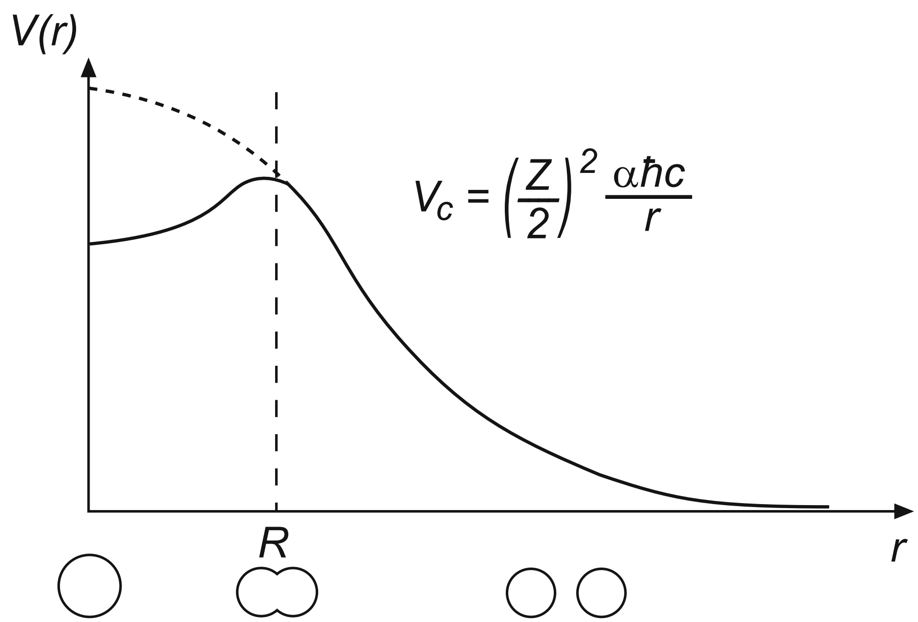


Fig. 3.7 Illustration of the ^{238}U decay chain in the *N-Z* plane. The half-life of each of the nuclides is given together with its decay mode

Fission

(A,Z) -> (A1,Z1) + (A2,Z2) + a few n's
M(A,Z) > M (A1,Z1) + M(A2,Z2) + M(n's)



However: Fission barrier – combined effect of Coulomb barrier and deformation penalty (surface term!)
=> only happens (in finite time) for very heavy nuclei
HOWEVER: "lighter" nuclei (e.g., U, Pu, Th,…) can be INDUCED to fission if given a little extra energy.
One way to do it: Odd neutron number -> add 1 n -> extra binding energy -> excite nucleus -> kaboom!

Note the potentially self-sustaining chain reaction due to the "few n's" that are released!
=> Nuclear reactors, explosives,…

Beta- decay

Liquid Drop Model: Quadratic in Z

$$M(A,Z) = \alpha \cdot A - \beta \cdot Z + \gamma \cdot Z^2 + \frac{\delta}{A^{1/2}}$$

where
$$\alpha = M_n - a_v + a_s A^{-1/3} + \frac{a_a}{4},$$
$$\beta = a_a + (M_n - M_p - m_e),$$
$$\gamma = \frac{a_a}{A} + \frac{a_c}{A^{1/3}},$$
$$\delta = \text{as in (2.8)}.$$

$\Rightarrow$ There should be a minimum!
Any nucleus away from that minimum could lower its mass by converting n into p (or vice versa).
Underlying process:

$$n \rightarrow p + e^- + \overline{\nu}_e$$

$M(A,Z) > M(A,Z+1)$ "Beta -"

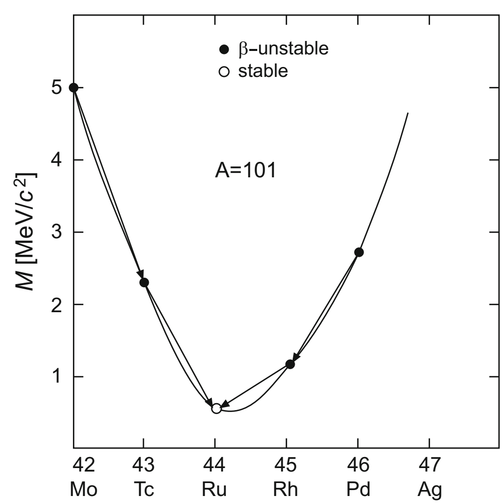
$p \rightarrow n + e^+ + \nu_e$ (only bound p's!)

M(A,Z) > M(A,Z - 1) "Beta +"

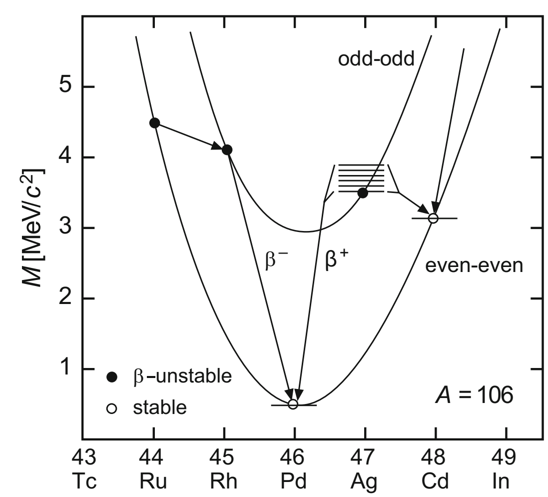
Example:

$$^{101}_{42}\text{Mo} \rightarrow {}^{101}_{43}\text{Tc} + e^- + \overline{\nu}_e,$$
$$^{101}_{43}\text{Tc} \rightarrow {}^{101}_{44}\text{Ru} + e^- + \overline{\nu}_e$$

A = odd



A = even



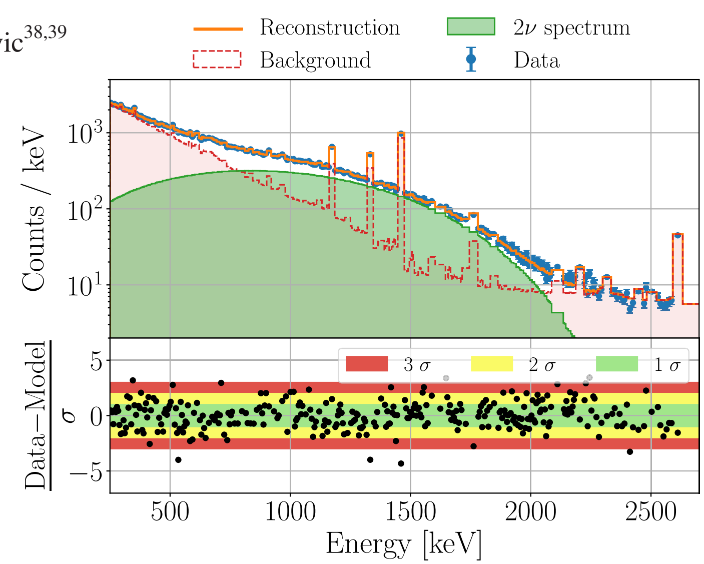
PHYSICAL REVIEW LETTERS **135,** 082501 (2025)

Half-Life and Precision Shape Measurement of the $2\nu\beta\beta$ Decay of ^{130}Te

D. Q. Adams,[1] C. Alduino,[1] K. Alfonso,[2] A. Armatol,[3] F. T. Avignone III,[1] O. Azzolini,[4] G. Bari,[5] F. Bellini,[6,7] G. Benat
M. Beretta,[10,11] M. Biassoni,[11] A. Branca,[10,11] C. Brofferio,[10,11] C. Bucci,[9] J. Camilleri,[2] A. Caminata,[12] A. Campani,
J. Cao,[14] C. Capelli,[3] S. Capelli,[10,11] L. Cappelli,[9] L. Cardani,[7] P. Carniti,[10,11] N. Casali,[7] E. Celi,[8,9] D. Chiesa,[10,
M. Clemenza,[11] S. Copello,[15] O. Cremonesi,[11] R. J. Creswick,[1] A. D'Addabbo,[9] I. Dafinei,[7] S. Dell'Oro,[10,11]
S. Di Domizio,[13,12] S. Di Lorenzo,[9] T. Dixon,[16] D. Q. Fang,[14] M. Faverzani,[10,11] E. Ferri,[11] F. Ferroni,[8,7] E. Fiorini,[10
M. A. Franceschi,[17] S. J. Freedman,[3,18,*] S. H. Fu,[9,14] B. K. Fujikawa,[3] S. Ghislandi,[8,9] A. Giachero,[10,11] M. Girola,
L. Gironi,[10,11] A. Giuliani,[16] P. Gorla,[9] C. Gotti,[11] P. V. Guillaumon,[9,†] T. D. Gutierrez,[19] K. Han,[20] E. V. Hansen,
K. M. Heeger,[21] D. L. Helis,[9] H. Z. Huang,[22] M. T. Hurst,[23] G. Keppel,[4] Yu. G. Kolomensky,[18,3] R. Kowalski,[24] R. Li
L. Ma,[14,22] Y. G. Ma,[14] L. Marini,[9] R. H. Maruyama,[21] D. Mayer,[18,3,25] Y. Mei,[3] M. N. Moore,[21] T. Napolitano,[1
M. Nastasi,[10,11] C. Nones,[26] E. B. Norman,[27] A. Nucciotti,[10,11] I. Nutini,[11] T. O'Donnell,[28] M. Olmi,[9] B. T. Oregui
S. Pagan,[21] C. E. Pagliarone,[9,29] L. Pagnanini,[8,9] M. Pallavicini,[13,12] L. Pattavina,[10,11] M. Pavan,[10,11] G. Pessina,[1
V. Pettinacci,[7] C. Pira,[4] S. Pirro,[9] E. G. Pottebaum,[21] S. Pozzi,[11] E. Previtali,[10,11] A. Puiu,[9] S. Quitadamo,[8,9] A. Res
C. Rosenfeld,[1] B. Schmidt,[26] R. Serino,[16,10] A. Shaikina,[8,9] V. Sharma,[23] V. Singh,[18] M. Sisti,[11] D. Speller,[24]
P. T. Surukuchi,[23] L. Taffarello,[30] C. Tomei,[7] A. Torres,[2] J. A. Torres,[21] K. J. Vetter,[25,18,3] M. Vignati,[6,7]
S. L. Wagaarachchi,[18,3] B. Welliver,[18,3] J. Wilson,[1] K. Wilson,[1] L. A. Winslow,[25]
F. Xie,[14] T. Zhu,[18] S. Zimmermann,[31] S. Zucchelli,[32,5]

(CUORE Collaboration)

D. Castillo,[33,34] J. Kotila,[35,36] J. Menéndez,[33,34] O. Niţescu,[36,37] and F. Šimkovic[38,39]



Double-beta decay example – within the lower parabola for *A* – even isobars (ΔZ = 2)

Beta + decay and e$^-$ capture

Underlying event: $\mathrm{p} \rightarrow \mathrm{n} + \mathrm{e}^+ + \nu_\mathrm{e}$ $M(A,Z) > M(A,Z-1) + 2m_\mathrm{e}$ *)

or $\mathrm{p} + \mathrm{e}^- \rightarrow \mathrm{n} + \nu_\mathrm{e}$ $M(A,Z) > M(A,Z-1)$ note: mostly in big nuclei with high Z – why?

Example:

$$^{101}_{46}\mathrm{Pd} \rightarrow {}^{101}_{45}\mathrm{Rh} + \mathrm{e}^+ + \nu_\mathrm{e}$$

$$^{101}_{45}\mathrm{Rh} \rightarrow {}^{101}_{44}\mathrm{Ru} + \mathrm{e}^+ + \nu_\mathrm{e}$$

Example for double beta+ decay:

$$^{106}_{48}\mathrm{Cd} \rightarrow {}^{106}_{46}\mathrm{Pd} + 2\mathrm{e}^+ + 2\nu_\mathrm{e}$$

Some odd-odd nuclei can do BOTH!

*) Note: beta– decay produces the extra electron for the neutral atom "automatically", so we can compare the atomic masses to see if energy is available.
Beta+ decay requires 2 extra electron masses – one for the positron, and one for the extra electron left over from a neutral daughter. NOT needed with electron capture (EC) => only neutral atoms involved

$$M(A,Z) > M(A,Z-1) + \varepsilon$$

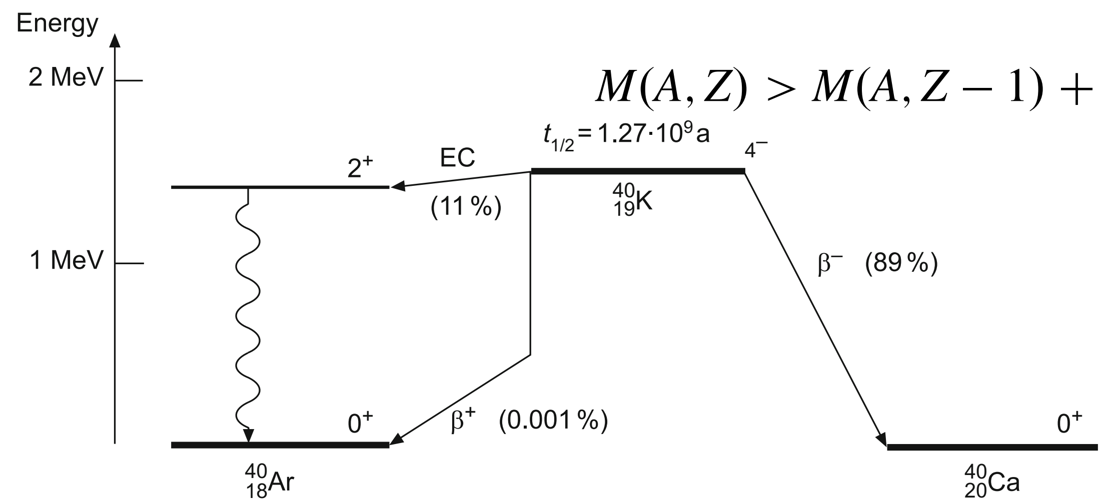


Fig. 3.4 The β-decay of ^{40}K. In this nuclear conversion, β^-- and β^+-decay as well as electron capture (EC) compete with each other. The relative frequency of these decays is given in parentheses. The bent arrow in β^+-decay indicates that the production of an e$^+$ and the presence of the surplus electron in the ^{40}Ar atom requires 1.022 MeV, and the remainder is carried off as kinetic energy by the positron and the neutrino. The excited state of ^{40}Ar produced in the electron capture reaction decays by photon emission into its ground state

Gamma decay

- Due to internal excitation of the nucleus
- Bound states of the nucleus (eigenstates of the Hamiltonian WITHOUT E&M)
- Similar to atomic light emission
- Characteristic "spectra" specific to each nucleus
- Energies from keV to multi-MeV

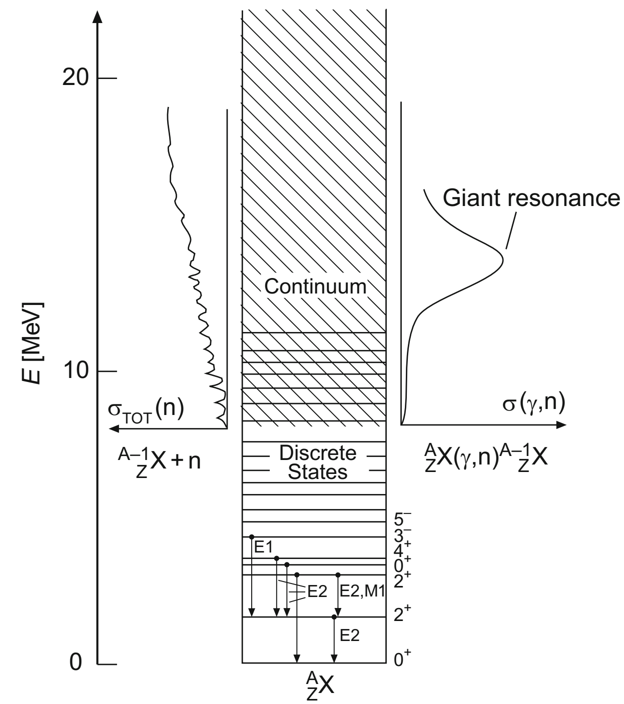


Fig. 3.10 Sketch of typical nuclear energy levels. The example shows an even-even nucleus whose ground state has the quantum numbers 0^+. To the left the total cross-section for the reaction of the nucleus ${}^{A-1}_{Z}X$ with neutrons (elastic scattering, inelastic scattering, capture) is shown; to the right the total cross-section for γ-induced neutron emission ${}^{A}_{Z}X + \gamma \rightarrow {}^{A-1}_{Z}X + n$

Recap - Half Life

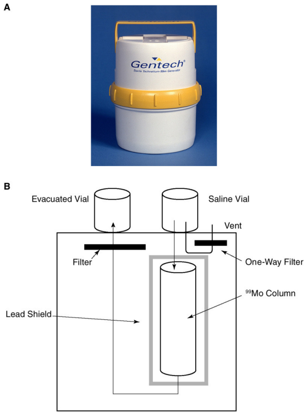
- Decays are **allowed** by energy conservation…
- …**but** still don't proceed instantly – why not?
  - Alpha decay: Barrier
    - Example: Water in a glass - energy lower if instead a puddle on the table; but can't get there
    - Alpha emitter: First have to remove the alpha particle a few fm before electric repulsion can take over (tunneling)
  - Beta decay: New type of interaction involved: WEAK interaction (hence long wait…)
    - Think of a fly pushing a truck - without friction it will move **eventually**
  - Gamma decay: Can be fast or slow - depends on particular energy levels (orbits) involved, just like for atoms

Half Life cont’d

- Quantum Mechanics: Cannot predict WHEN decay will occur, only the probability per unit time
  - Example: Play lottery every week for millions of years - **eventually** you will win, but you cannot predict **when**
- Concept: Half-life = the time that has to elapse before you have a 50-50 chance for a given nucleus to decay
  - If the chance of winning the lottery is 1 in 100 million, you have to play for 50 million weeks to get a 50-50 chance to win
  - Brain teaser: What if you didn’t win in the first 50 million weeks? Will you be guaranteed to win in the next 50 million weeks?
NO, the chances are again only 50-50!
- Start with a large number of identical radioactive nuclei
  - After 1 half-life, about 1/2 will have decayed
    - After 2nd half-life, about 1/4 more (one half of the rest) will be gone
      - After 3rd half-life, only 1/8 left over
        - » After 4th half-life, only 1/16 left
          - …

Half Life cont'd

- Measure Half-Life: Count number of nuclei you start with and how many decay in a given time interval
  - Example: Start with 10^6. After 1 hour, 10 have decayed => Half Life must be of order 50,000 hours (69315, to be precise).
- Half-lifes can be all over the map:
  - some beta emitters live only a fraction of a second
  - others (^{40}K) over 1 billion years!
  - ^{14}C in between: 5700 yr
  - Tc has only radioactive isotopes (up to 1 million years)
  - neutron itself lives only 15 min unless bound in nucleus!
  - alpha emitters can live even longer: ^{238}U has $T_{1/2}$ = 4.5 billion years (the age of our solar system!), but the heaviest nuclei live only seconds
- Can use radioactive nuclei to measure age:
  - "Radiocarbon dating"
  - Age of our solar system from U, K



Half Life cont’d

- Example: Technetium
- The U.S. medical community depends on a reliable supply of the radioisotope Mo-99 for nuclear medical diagnostic procedures. Mo-99's decay product, technetium-99m (Tc-99m), is used in over 40,000 medical procedures in the United States each day to diagnose heart disease and cancer, to study organ structure and function, and to perform other important medical applications. For example, patients undergoing a common procedure—the cardiac “stress test”—likely have benefited from Tc-99m.
- Doctors or trained nuclear medicine health professionals will administer Tc-99m radiotracers to patients before a diagnostic test, usually by injection, to help diagnose medical conditions. As the half-life of Tc-99m is only six hours, it does not stay in the human body long. Nuclear medicine scans are safe and are a widely used imaging test.
- Molybdenum: Naturally occurring element, except for Mo-99. The latter can be produced through fission or through neutron capture by Mo-98.
- Because of its relatively short half-life (66 hours), Mo-99 cannot be stockpiled for use. It must be made on a weekly or more frequent basis to ensure continuous availability. The processes for producing Mo-99 and technetium generators and delivering them to customers are tightly scheduled and highly time dependent.
- Tc-99 has a half-life of about 6 hours and emits 140 keV photons when it decays to Tc-99, a radioactive isotope with about a 214,000-year half-life. This photon energy is ideally suited for efficient detection by scintillation instruments such as gamma cameras.

Effect of Radiation

- Nuclear decay products can be very high-energy => penetrating, able to ionize atoms or molecules (ionizing radiation)
  - alpha particles can kick out lots of electrons along their flight path but can be (therefore) stopped easily (don't inhale Radon!)
  - beta particles have less effect, but can travel further
  - Xrays and gammas are most penetrating (don't stop all after the same distance)
- Create "radicals" that can destroy or alter cells and their function
    - possible cause of cancer or genetic modifications…
    - …but can also be used to destroy cancer cells
    - harmful or deadly in large doses…
    - …but can be used to kill germs in food