Scattering

Nuclear Physics 415/515

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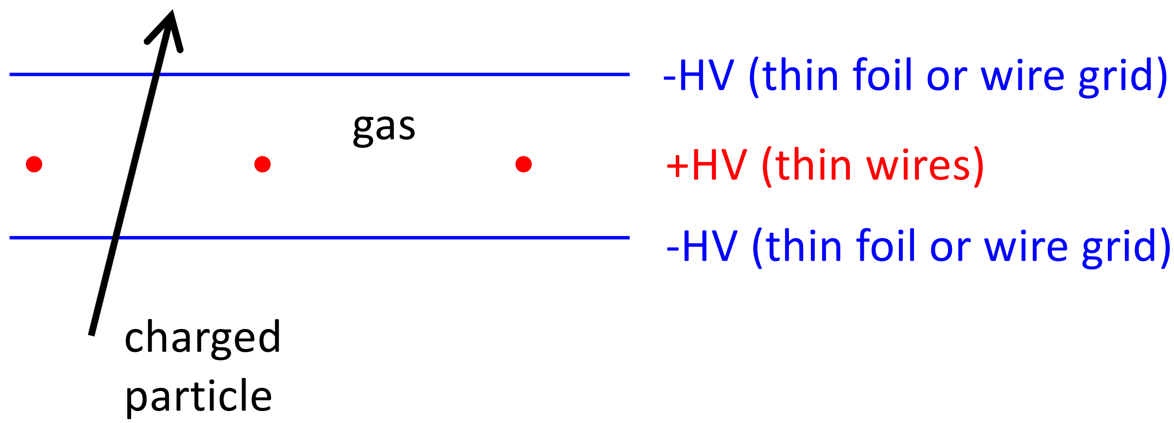
What Do we Need?

- Beam
- Electrons or muons → γ^*; pions, protons, antiprotons, light nuclei, heavy ions…
- Targets (or counterrotating beams)
- Protons, deuterons, ^{3}He, heavier nuclei, heavy ions, antiprotons
- Detectors
- For scattered/produced electrons/muons/… and hadrons/nuclei
- Facilities

Typical Detector Elements

Wire chambers measure position (and angle)



1. Charged particle passes through wire chamber and knocks out electrons from the gas.
2. Electrons drift in the E field to the cathode wire, colliding with gas molecules
3. Close to the wire, the mean free path times the electric field is large enough to ionize the gas molecules. Avalanche!
4. Read the signal on the cathode wire (time gives distance)

Similar: G$_{as}$E$_{lectron}$M$_{ultiplier}$s, μMEGAs,...

Applications: VDC, Multi-layer drift chamber (track → $\vec{p}$), T$_{ime}$P$_{rojection}$C$_{hamber}$

Typical Detector Elements

Scintillators: time ($\Rightarrow \beta \Rightarrow$ particle type) and energy measurement

(typical resolution: down to 50-100 ps for plastic)

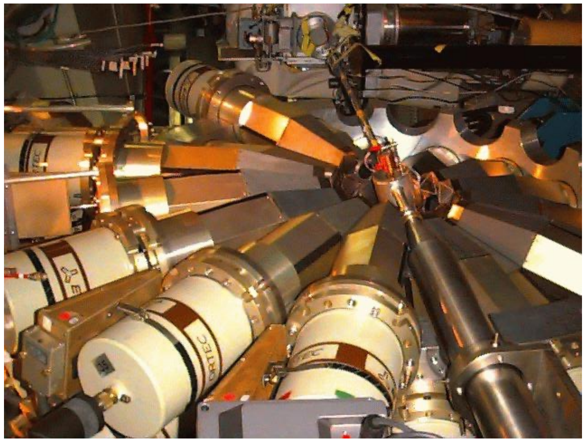
- Typically a doped plastic or crystal (eg: Ge, NaI, BaF_2)
- Charged particle passes through scintillator (or neutral particle interacts) and excites atomic electrons. These de-excite and emit light.
- Minimum energy loss (when $\beta\gamma \approx 1$) is $dE/dx = 2$ MeV/(g/cm^2)

Cherenkov counter: threshold velocity measurement.

- Typically an empty box with smoke (ie: a gas) and mirrors
- Local light speed is $v = 1/n < c$
- Particles travelling faster than v will emit Cherenkov light (an electromagnetic 'sonic boom') $\Rightarrow$ threshold CC (yes/no)
- The opening angle of the Cherenkov cone is related to the particle's velocity $\Rightarrow$ R$_{\text{ing}}$I$_{\text{maging}}$CH$_{\text{erenkov}}$ (measure $\beta \Rightarrow$ particle type)

Also: T$_{\text{ransition}}$R$_{\text{adiation}}$D$_{\text{etector}}$, DIRC, Typical Detector Elements

Photon Counters ->

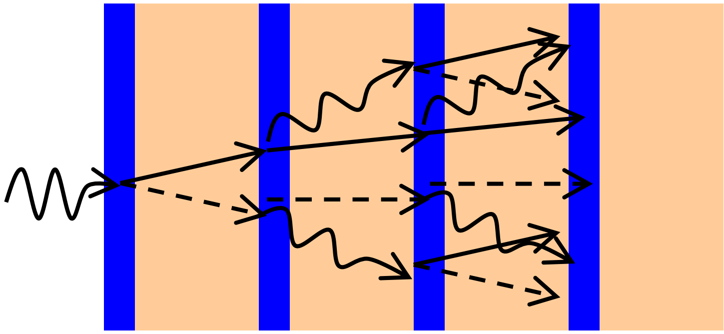


Electromagnetic shower counters:
measure energy (+ time), discriminate electrons and detect neutral particles

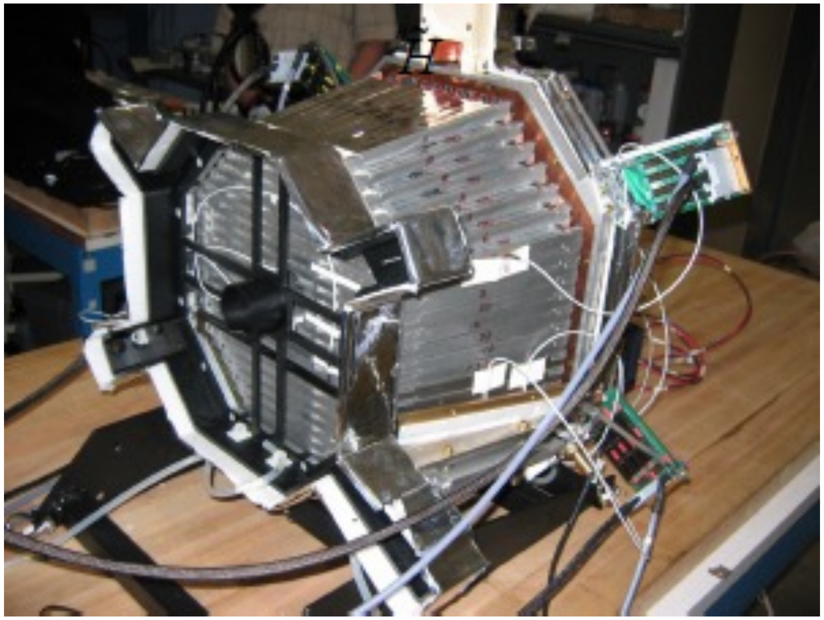
- Electrons and photons passing through material shower
- After one radiation length of material on average:
  - Electrons emit a bremsstrahlung photon
  - Photons convert to an electron/positron pair or Compton-scatter
- After ≈10 radiation lengths, one e^- or γ is now ~1000 particles
- Simple design: alternating layers of lead (R_L = 6 mm) and scintillant

Higher resolution: Heavy metal glass (Pb glass, $PbWO_4$) combine both

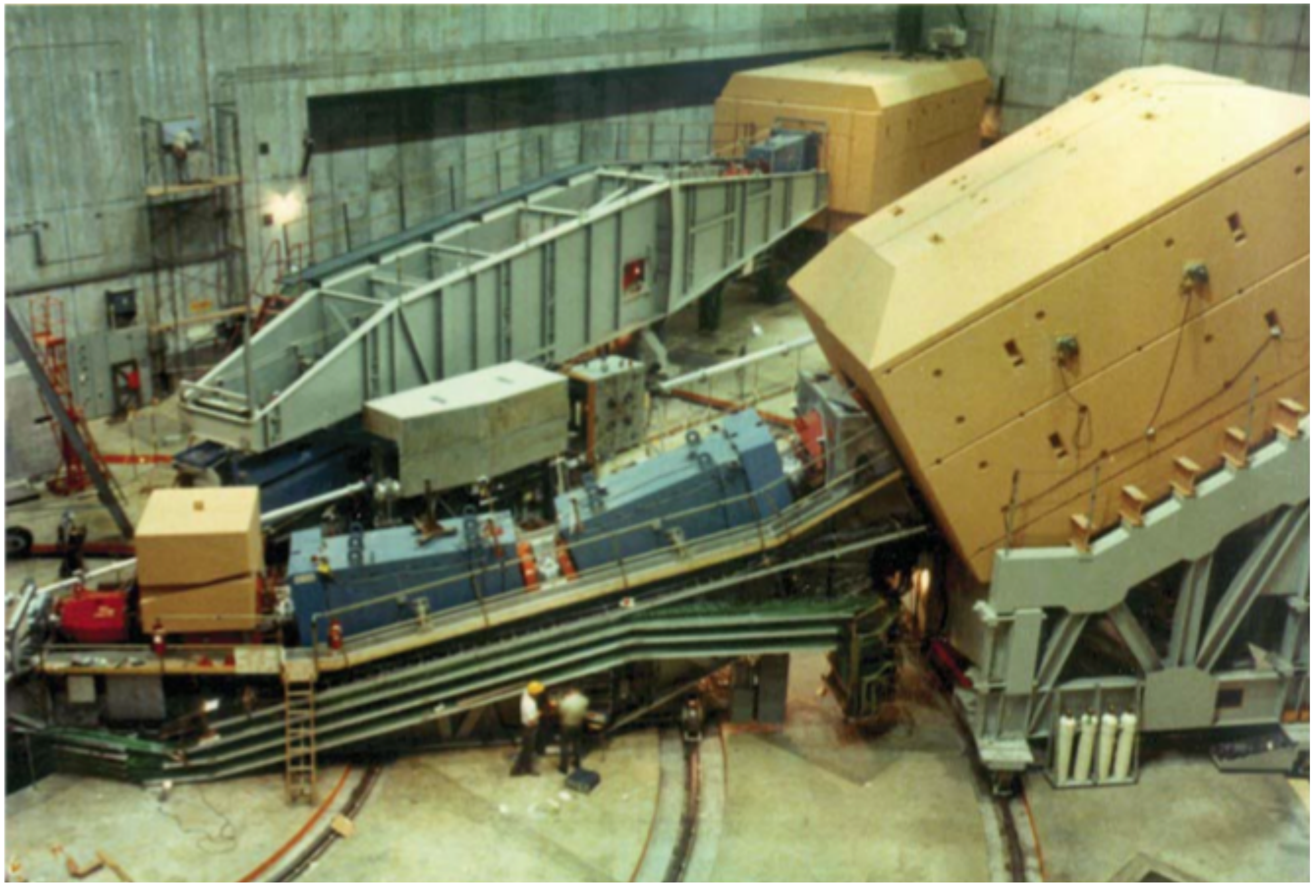
- Particles shower in the lead
- Charged particles deposit energy in the scintillant



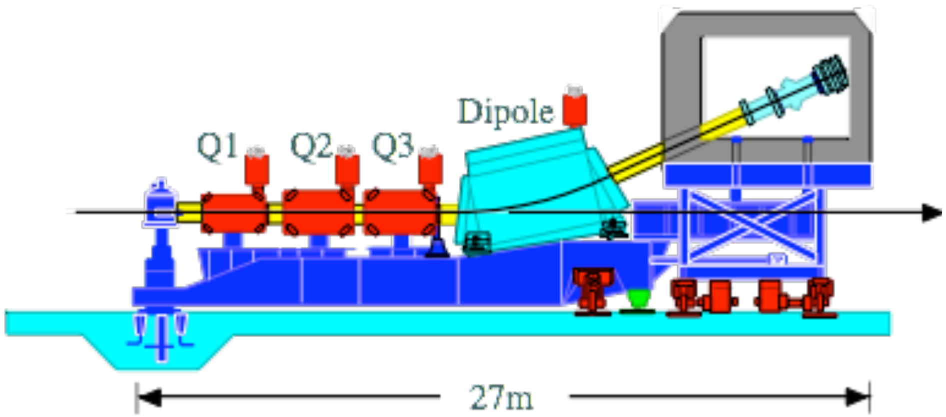
Also: Hadronic Calorimeter, μ counter…



Detectors+Magnets = Spectrometers

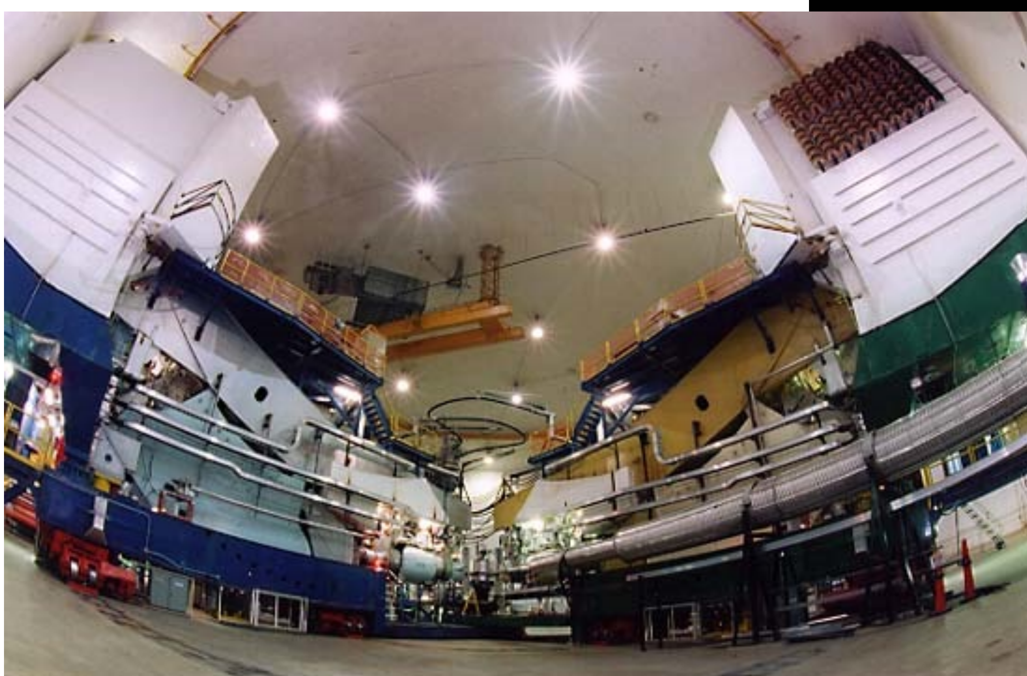


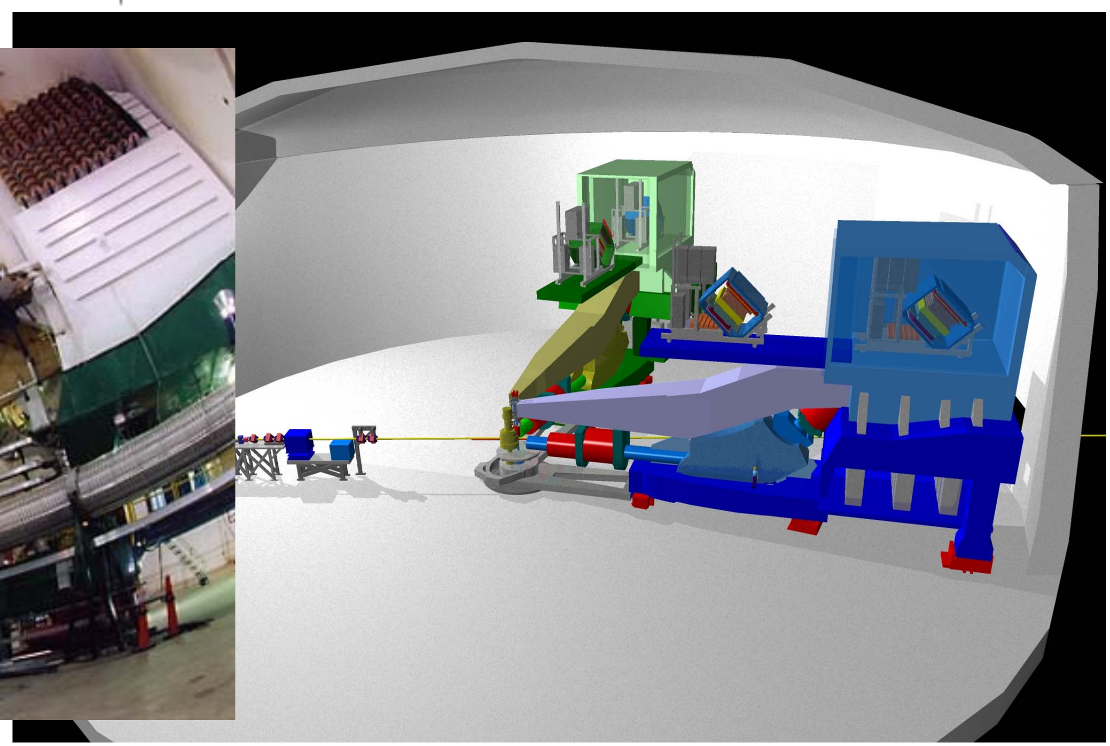
SLAC End Station A – where the quarks were discovered experimentally



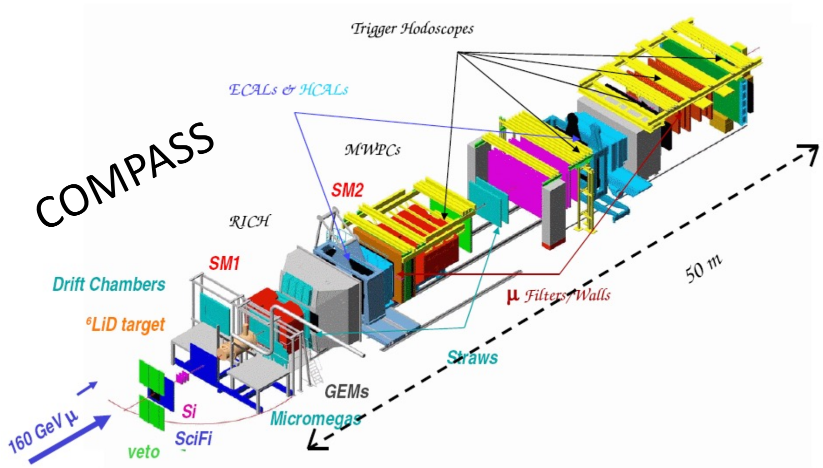
Jefferson Lab Hall A (Hall C similar)

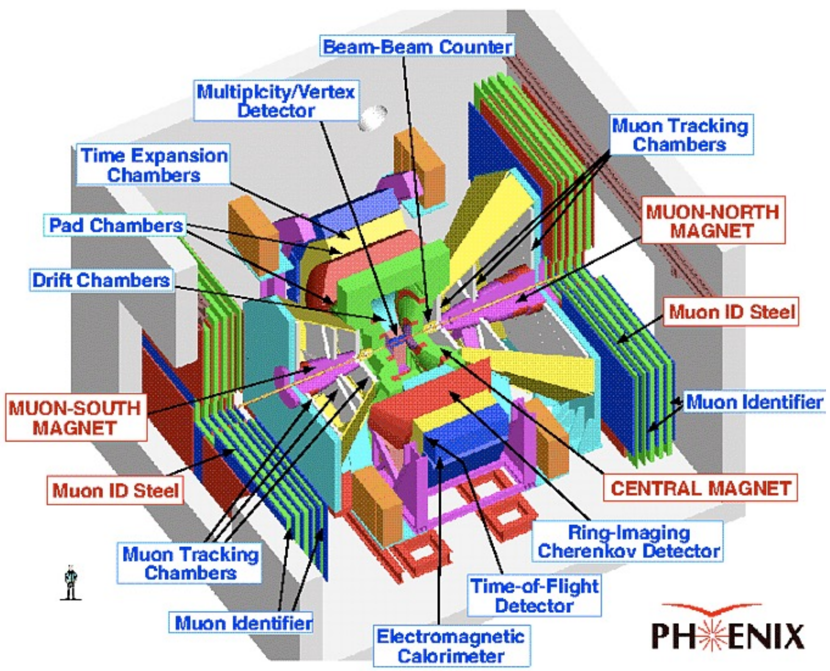
Typically, small acceptance but high resolution, very good shielding
(→ high luminosity)

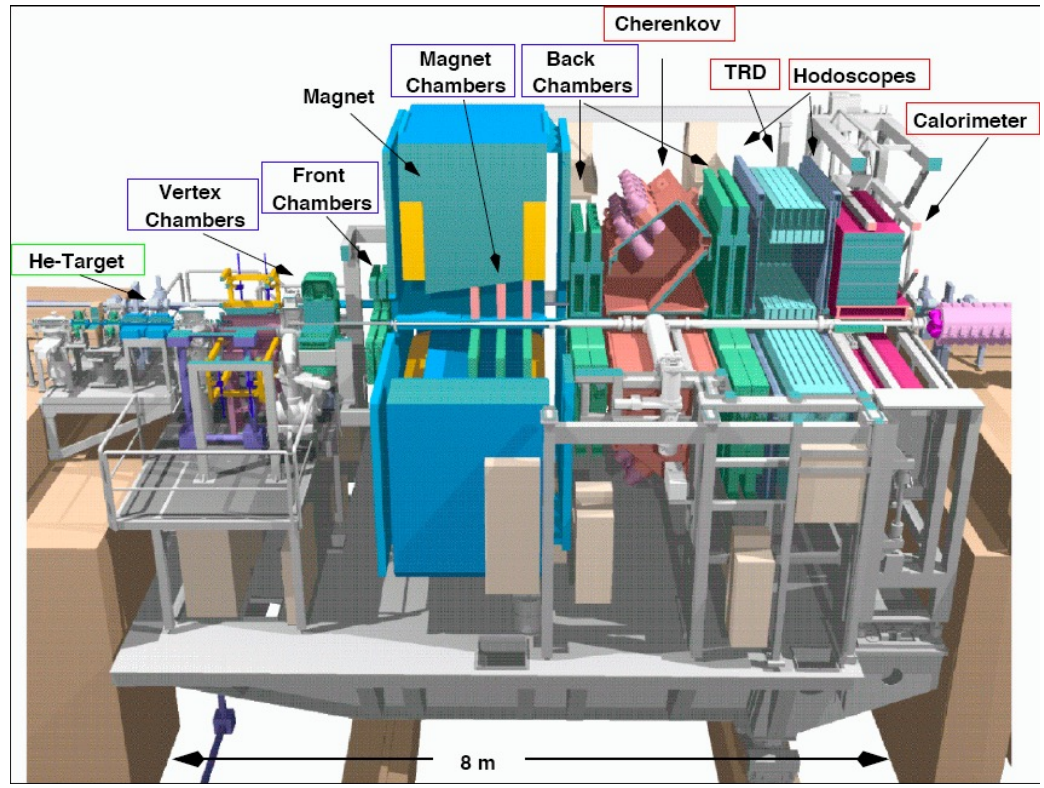




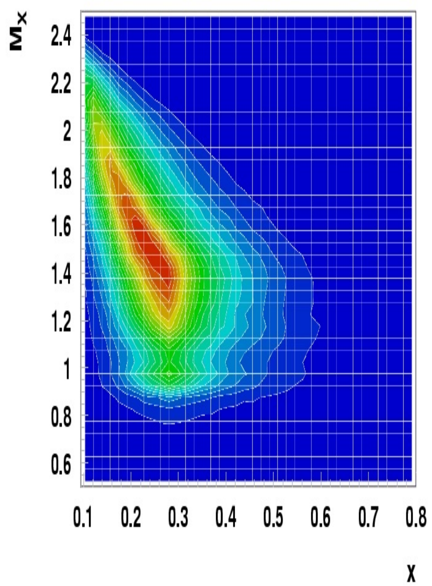
Large Acceptance Spectrometers



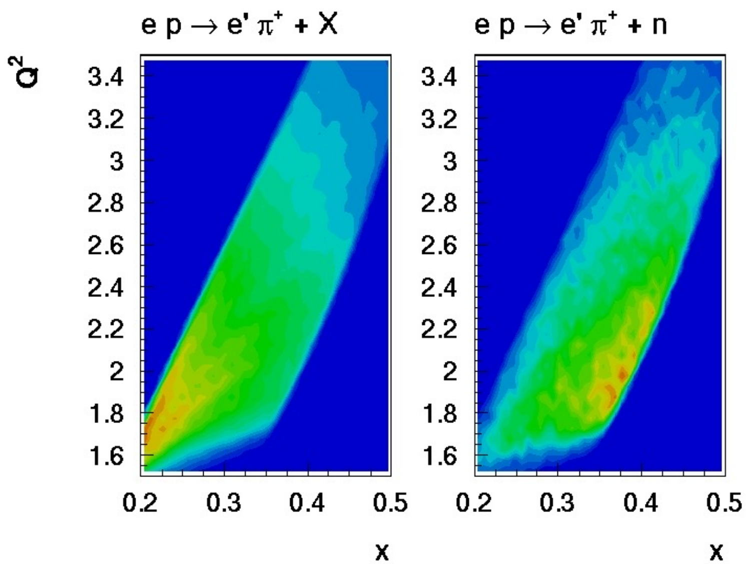




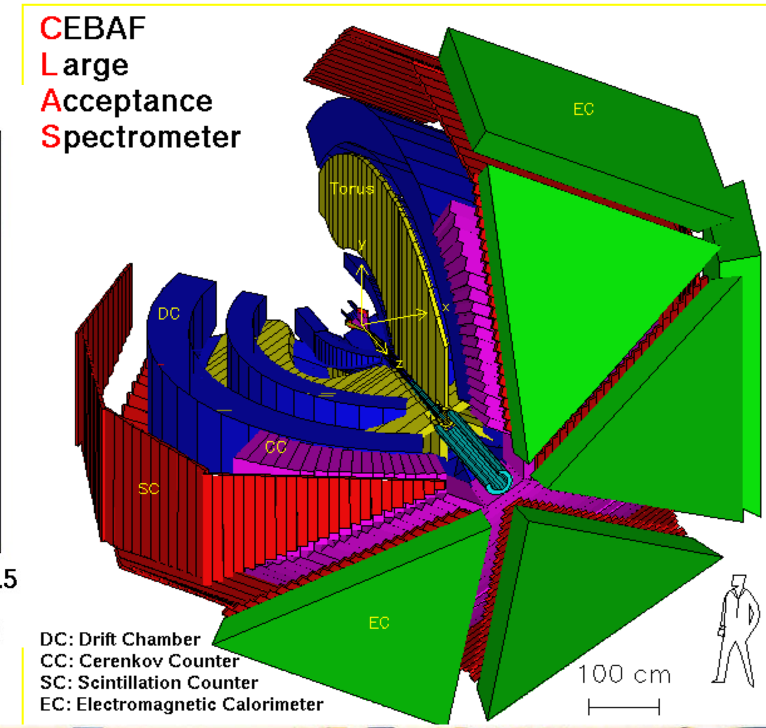
HERMES



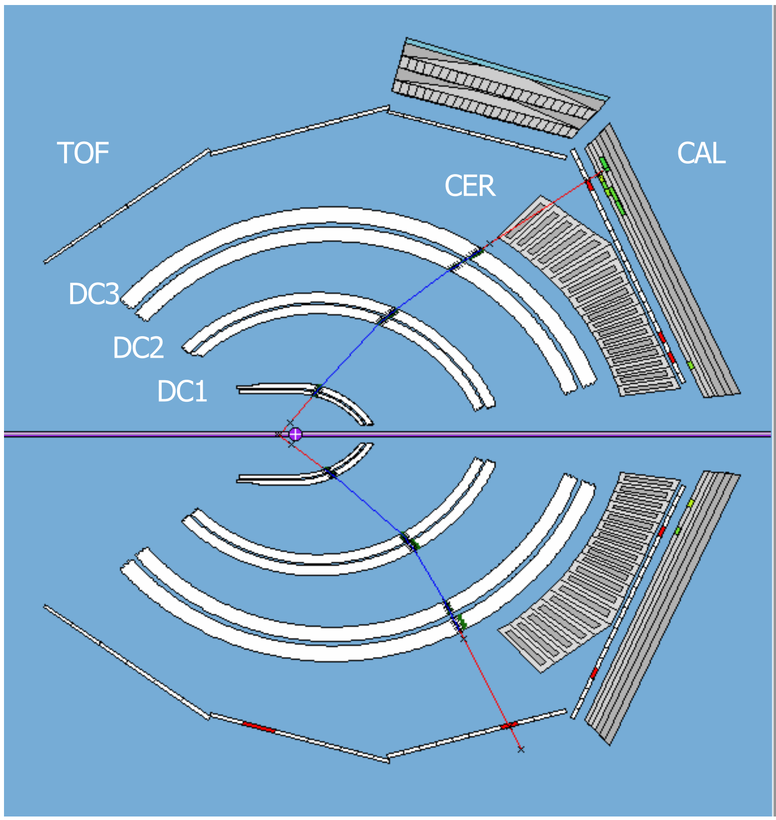
M_x
2.4
2.2
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8
x



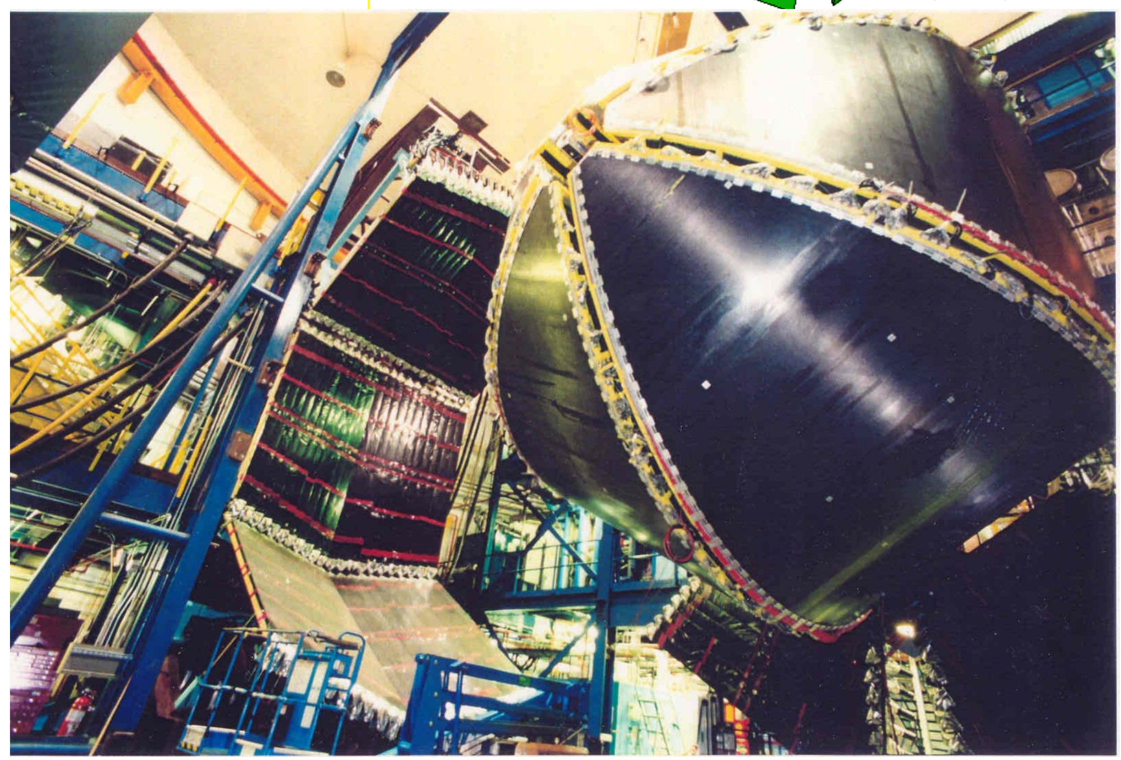
Q^2
$e\,p \rightarrow e'\,\pi^+ + X$
$e\,p \rightarrow e'\,\pi^+ + n$
3.4
3.2
3
2.8
2.6
2.4
2.2
2
1.8
1.6
0.2 0.3 0.4 0.5
x



CEBAF
Large
Acceptance
Spectrometer
EC
Torus
DC
CC
SC
EC
DC: Drift Chamber
CC: Cerenkov Counter
SC: Scintillation Counter
EC: Electromagnetic Calorimeter
100 cm



TOF
CER
CAL
DC3
DC2
DC1



CLAS12

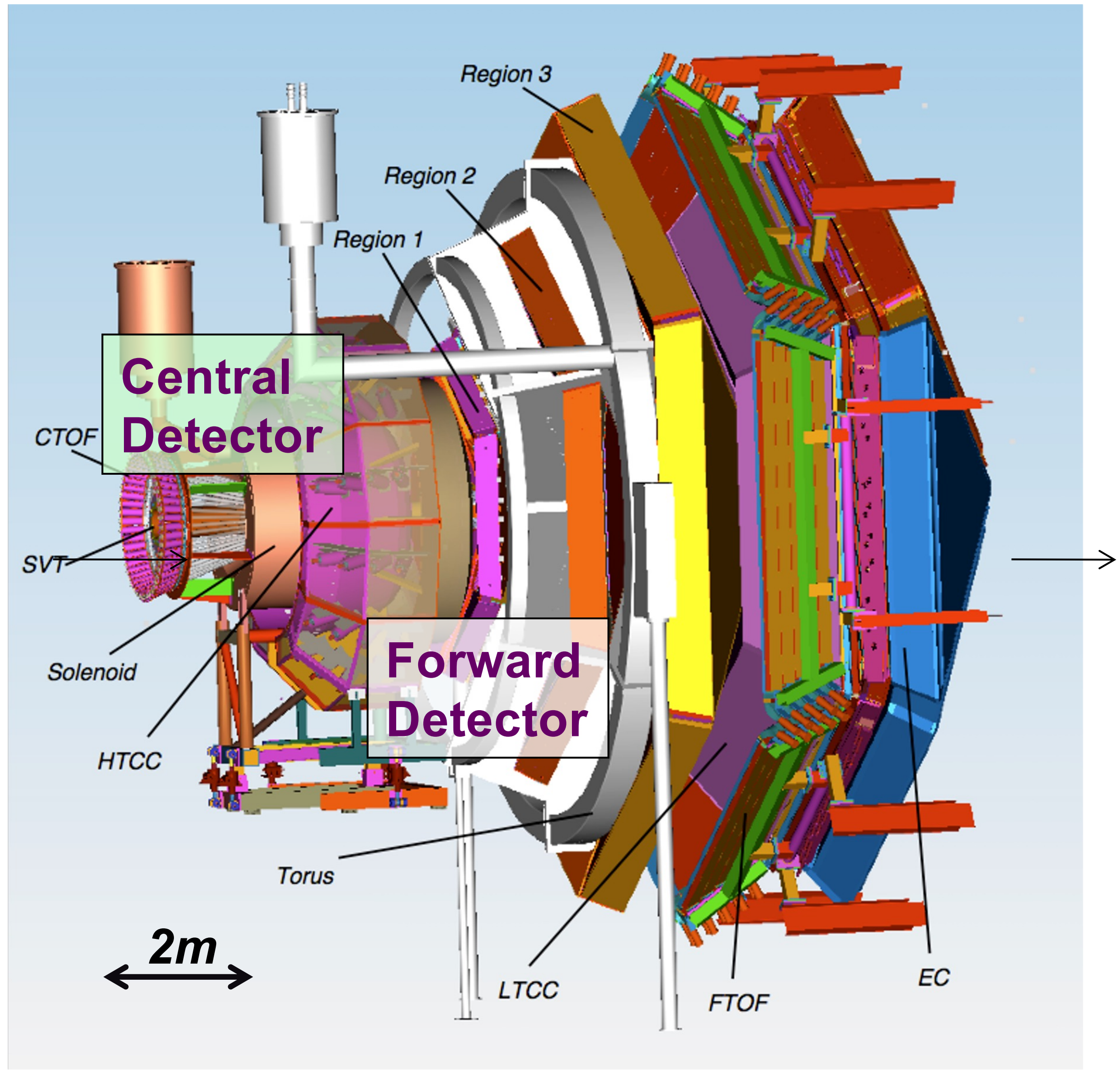
Another nearly 4π detector in Hall D (GLUEX)

Base equipment

- **Forward Detector**
- TORUS magnet
- Forward vertex tracker
- HT Cherenkov Counter
- Drift chamber system
- LT Cherenkov Counter
- Forward ToF System
- Preshower calorimeter
- E.M. calorimeter
- **Central Detector**
- SOLENOID magnet
- Barrel Silicon Tracker
- Central Time-of-Flight

Additional equipment

- Micromegas (CD & FD)
- RICH counter (FD)
- Neutron detector (CD)
- Small angle tagger (FD)



Cross Section and Reaction Rate

- Incoming "current": $\dot{n}_b$ (beam particles/s)
- Target areal density: $n_T L$ = number of nuclei per unit surface area
- Cross section $\Delta\sigma$ for a specific reaction to happen
- => number of times this reaction happens per second (event rate): $\dot{N} = \dot{n}_b n_T L \Delta\sigma$
- Call $\mathcal{L} = \dot{n}_b n_T L$ the luminosity of the experiment

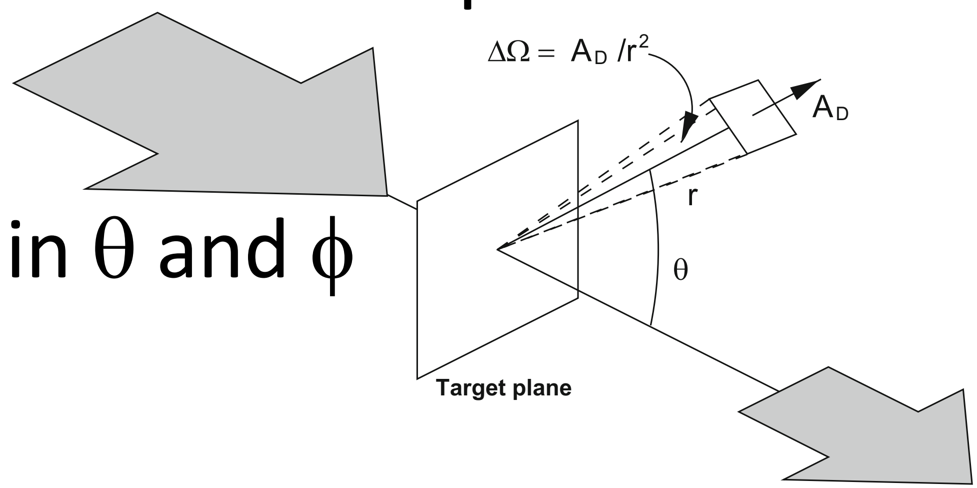
$$N = \int \mathcal{L} \, \Delta\sigma \, dt$$

Example: Luminosity and cross sections

- On white board
- Remember: If atomic mass is *A*, then 1 g of the material contains 1/*A* mol
- 1 mol = $6.022 \cdot 10^{23}$ atoms (and hence nuclei)
- 1μA of electrons contain $10^{-6}\ {}^{C}/_{s} \;/\; 1.6 \cdot 10^{-19}$ C = $6.25 \cdot 10^{12}$ e/s
- 1 "barn" 1 b = 10^{-24} cm^2
  - mb(arn), μb, nb, pb,…

Example partial cross section: scattering into a detector

Look only at events where the beam particle is scattered into a specific detector area = a specific angular range in θ and ϕ
=> Solid angle $\Delta\Omega$



$\Delta\sigma$ proportional to $\Delta\Omega$

=> Use ratio $\Delta\sigma/\Delta\Omega$ to express the "intrinsic" scattering strength (independent of detector used)

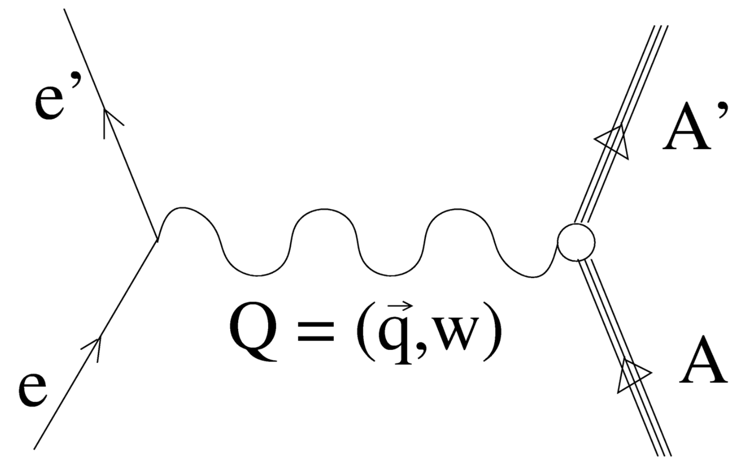
$$\dot{N}(E, \theta, \Delta\Omega) = \mathcal{L} \cdot \frac{\mathrm{d}\sigma(E,\theta)}{\mathrm{d}\Omega} \Delta\Omega$$

Why use electrons and photons?

- Probe structure understood (point particles)
- Electromagnetic interaction understood (QED)
- Interaction is weak (α = 1/137)
  - Perturbation theory works!
    - First Born Approx / one photon exchange
  - Probe interacts only once
  - Study the entire nuclear volume

BUT:

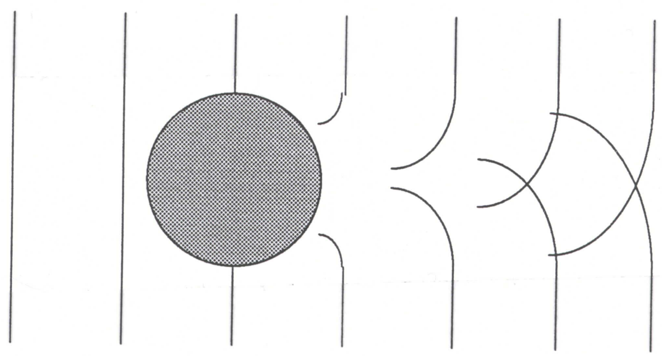
- Cross sections are small
- Electrons radiate



Electrons as Waves

Scattering process is quantum mechanical

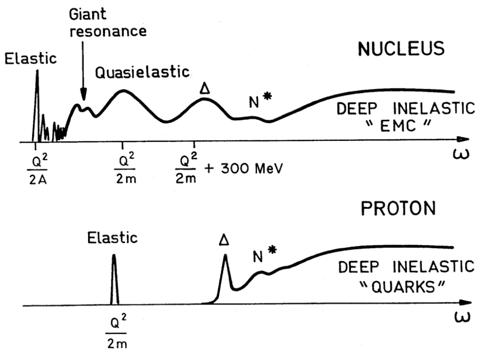
De broglie wavelength: $\lambda = \frac{h}{p}$



Electron energy: $E_e \approx pc$

λ resolving "scale": $\lambda = \frac{2\pi(197\ \mathrm{MeV}\cdot\mathrm{fm})}{E_e}$

Experimental goals:



- Elastic scattering
  - structure of the nucleus
    - Form factors, charge distributions, spin dependent FF
- Quasielastic (QE) scattering
  - Shell structure
    - Momentum distributions
    - Occupancies
  - Short Range Correlated nucleon pairs
  - Nuclear transparency and color transparency
- Deep Inelastic Scattering (DIS)
  - The EMC Effect and Nucleon modification in nuclei
  - Quark hadronization in nuclei

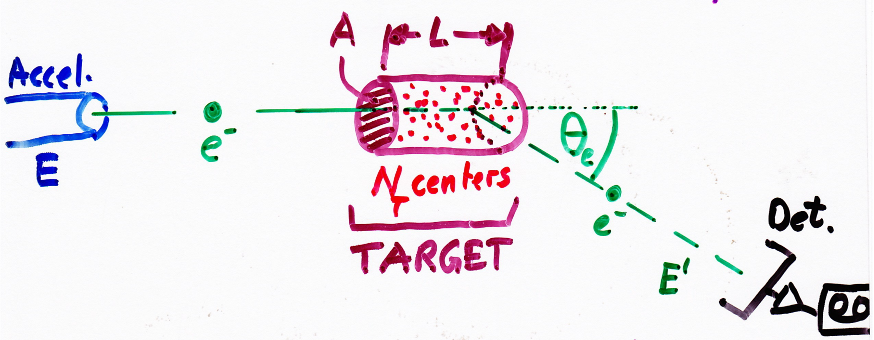
Energy vs length

Select spatial resolution and excitation energy independently
- Photon energy ν determines excitation energy
- Photon momentum q determines spatial resolution: $\lambda \approx \dfrac{\hbar}{q}$

Three cases:

- Low q
  - Photon wavelength λ larger than the nucleon size (R_p)
- Medium q: $0.2 < q < 1$ GeV/c
  - $\lambda \sim R_p$
  - Nucleons resolvable
- High *q*: $q > 1$ GeV/c
  - $\lambda < R_p$
  - Nucleon structure resolvable

Electron Scattering – what can we measure?



What is the likelihood to find the electron scattered into the detector?

$$P \sim n_T \cdot L = \frac{N_T}{A\,L} \cdot L = \frac{N_T}{A}\ ^*$$

$\Rightarrow$ call $\Delta\sigma = P/\left(\frac{N_T}{A}\right)$ (cross section)

$\Delta\sigma$ DEPENDS on the kinematics (E, E', θ_e) and is $\approx$ proportional to SIZE of kinematic bin spanned by the detector

*Note: $\frac{N_T}{A} = \rho\left[\frac{g}{cm^3}\right] \cdot L\,[cm] \cdot \frac{\text{Avogadro}}{\text{Atomic Weight [u]}}$

Count rate (L = luminosity):

$$\dot{N} = P \cdot \dot{n}_{el} = \Delta\sigma \cdot \frac{N_T}{A}\dot{n}_{el} = \Delta\sigma \cdot \frac{N_T}{A}\frac{I}{e} = \Delta\sigma \cdot L$$

In general:

$$\Delta\sigma = \Delta\sigma\left(E'...E'+\Delta E', \theta_e...\theta_e+\Delta\theta_e, \varphi...\varphi+\Delta\varphi\right)$$

Limit of infinitesimal acceptance:

$$\Delta\sigma = \frac{d^3\sigma}{dE'\,d\theta_e\,d\varphi}\left(E',\theta_e,\varphi\right)\Delta E'\Delta\theta_e\Delta\varphi = \frac{d\sigma}{dE'\,d\Omega}\Delta E'\Delta\Omega$$

(use Jacobian to transform variables)

In case of more particles/observables and finite phase space:

$$\Delta\sigma = \iiint_{\substack{Phase\\Space}} \frac{d^n\sigma}{dk_1 dk_2...dk_n}\left(k_1...k_n\right) Acc\left(k_1...k_n\right) dk_1 dk_2...dk_n$$

where Acceptance is defined by detector boundaries and bin sizes as well as number of observed particles

(inclusive = only 1, semi-inclusive, exclusive)

Electron Scattering – what can we measure?

What is the likelihood to find the electron scattered into the detector?

$$P \sim n_T \cdot L = \frac{N_T}{A\,L} \cdot L = \frac{N_T}{A} \; {}^{*}$$

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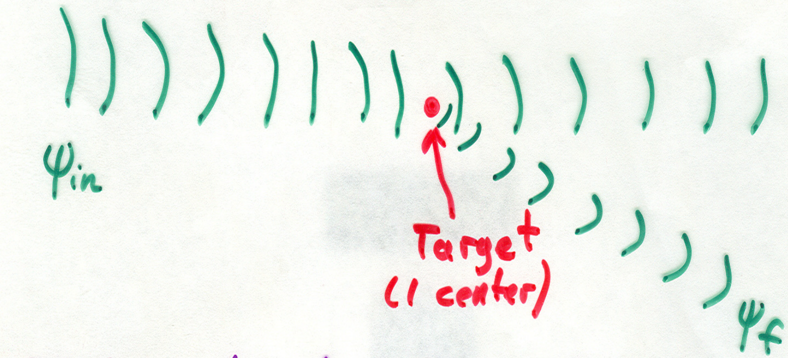
*Note: $\frac{N_T}{A} = \rho\left[\frac{g}{cm^3}\right] \cdot L\,[cm] \cdot \frac{\text{Avogadro}}{\text{Atomic Weight [u]}}$

Electron Scattering – Theorist's View



What is the transition rate $W_{i\to f}$?

$$\dot{N}_{e,f} = \dot{N}_{e,in} \cdot P(i\to f) = I_{e,in} \cdot \frac{N_T}{A} \cdot \Delta\sigma$$

$$= \frac{I_{e,in}}{A} \cdot N_T \cdot \Delta\sigma = \left(\vec{j}_{e,in}\right)_z \cdot N_T \cdot \Delta\sigma$$

$$\Rightarrow W_{i\to f} = j_{in} \cdot \Delta\sigma$$

Fermi's GOLDEN Rule:

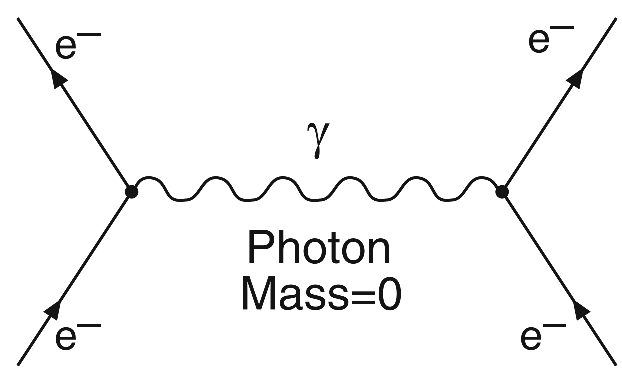
$$W_{i\to f} = \frac{2\pi}{\hbar} \left| \mathcal{M}_{fi} \right|^2 \Delta\phi$$ ← Phase space spanned by detector/kinematic bin

$$\mathcal{M}_{fi} = \langle \psi_f \,|\, H_{int} \,|\, \psi_{in} \rangle$$

How do we calculate cross sections? Feynman diagrams

- Theoretical ansatz: Look at single scattering centers, incoming beam = current density j_b. Event rate $\dot{N} = \Delta\sigma \cdot j_b$.
- "Infinitesimal" cross section: ${}^{d\sigma}/_{d\Omega}(\theta,\phi)$.
- Differential cross section depends only on physics of interaction (potential…) and available final state "phase space".
- Interaction often depicted with Feynman diagrams.

$$\sigma = \frac{2\pi}{\hbar \cdot v_a} \left|\mathcal{M}_{fi}\right|^2 \cdot \varrho\,(E')$$



Recap: Relativistic Kinematics

- Often in high energy nuclear/particle physics, particles move with close to the speed of light, *c*, hence we have to use special relativity
- Recall: $\gamma = (1-v^2/c^2)^{-1/2}$, $\beta = v/c$, $E = \gamma Mc^2$, $p=\gamma Mv$. (Note: we'll simplify our lives by often ignoring factors of *c*.)
- 4-vectors: $v^\mu = (v^0, v^1, v^2, v^3)$. $x^\mu = (ct, x, y, z)$.
  $P^\mu = (E/c, \vec{p})$ ($\vec{p}$ = "3-vector part" of P^μ).
- General transformation: Lorentz Matrix
- Very useful in relativistic kinematics: Invariants (same in all coordinate systems). *E.g.*:
  Scalar product $P^\mu P_\mu = (P^0)^2 - \vec{p}^{\,2} = M^2c^2$.

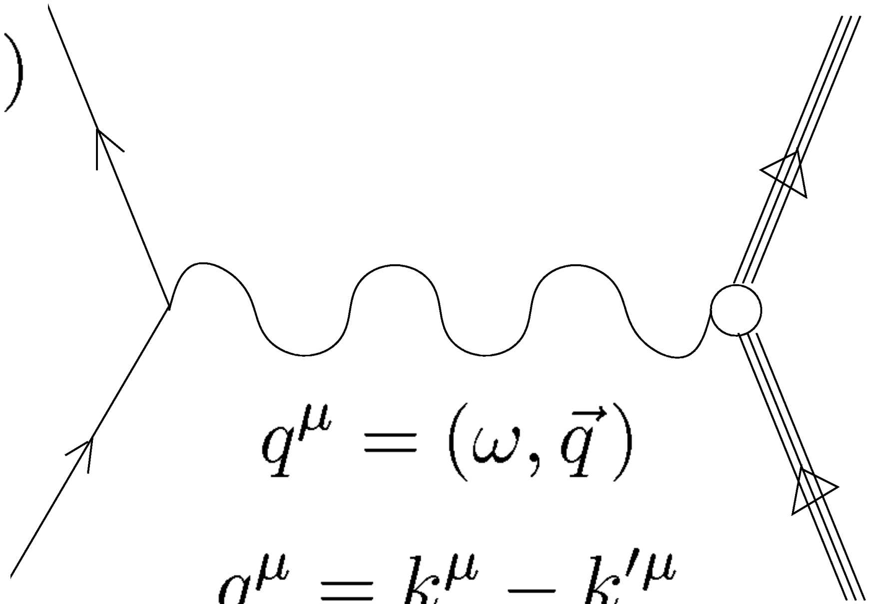
Inclusive electron scattering (e,e')

$k'^{\mu} = (E', \vec{k}\,')$

$p'^{\mu} = (E_p, \vec{p}_p)$

(not detected)

Lab frame kinematics



$q^{\mu} = (\omega, \vec{q})$

$k^{\mu} = (E, \vec{k}\,)$

$q^{\mu} = k^{\mu} - k'^{\mu}$

$p^{\mu} = (M, \vec{0}\,)$

Invariants:

$$p^{\mu} p_{\mu} = M^2 \qquad p_{\mu} q^{\mu} = M\omega$$

$$Q^2 = -q^{\mu} q_{\mu} = |\vec{q}|^2 - \omega^2 \qquad W^2 = (q^{\mu} + p^{\mu})^2 = p'_{\mu} p'^{\mu}$$

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Some kinematics – in the lab system

$$k^\mu = (E, 0, 0, E) = (E, \vec{k})$$

$$k'^\mu = (E', E'\sin\theta, 0, E'\cos\theta) = (E', \vec{k}')$$

$$q^\mu = (E-E', -E'\sin\theta, 0, E-E'\cos\theta) = (\nu, \vec{q})$$

$$Q^2 = -q^\mu q_\mu = -\nu^2 + \vec{q}^{\,2} = (E-E'\cos\theta)^2 + E'^2\sin^2\theta - (E-E')$$

$$= E^2 - 2EE'\cos\theta + E'^2 - E^2 - E'^2 + 2EE'$$

$$= 2EE'(1-\cos\theta) = 4EE'\sin^2\frac{\theta}{2}$$

$$p^\mu = (M, 0, 0, 0) = (M, \vec{0})$$

$$p'^\mu = p^\mu + q^\mu = (M+\nu, \vec{q})$$

$$p'^\mu p'_\mu =: W^2 = M^2 + 2M\nu + \nu^2 - \vec{q}^{\,2}$$

$$= M^2 + 2M\nu - Q^2$$

Elastic Scattering: $W^2 \stackrel{!}{=} M^2$

$$\Rightarrow \nu_{el} \stackrel{!}{=} \frac{Q^2}{2M} \quad \text{or} \quad x_{el} = \frac{Q^2}{2M\nu_{el}} \stackrel{!}{=} 1$$

Elastic Cross section – final form

$$\Delta\sigma = \frac{4z^2\alpha^2(\hbar c)^2}{Q^4}\cos^2\frac{\theta}{2}\left[\underbrace{\frac{G_E^2 + \tau G_M^2}{1+\tau}}_{\substack{\text{LONGITUDINAL}\\ \text{(charge)}}} + \underbrace{2\tau\tan^2\frac{\theta}{2}G_M^2}_{\substack{\text{TRANSVERSE}\\ \text{(magnetic)}}}\right] \cdot E'^2\Delta\Omega\underbrace{\frac{E'}{E}}_{\text{Recoil}}$$

$\tau = \frac{\nu^2}{Q^2}$, $G_E(Q^2)$, $G_M(Q^2)$: Form Factors

Dirac Particle: $G_E = G_M = 1$ (const.)

Anomalous magnetic moment: $G_M \approx (1+\kappa)G_E$

Extended Charge distribution:

$G_E(Q^2) \simeq$ Fourier transform of $\rho(r)$

Ex: $\rho(r) \simeq \frac{a^3}{8\pi}e^{-ar} \Rightarrow G_E(Q^2) \simeq \left(\frac{1}{1+\frac{Q^2}{a^2}}\right)^2$

(Dipole Form). $p: a^2 = 0.71\ \text{GeV}^2$

Elastic cross section ($p'^2 = m^2$)

Recoil factor

Form factors

$$\frac{d\sigma}{d\Omega} = \sigma_M \frac{E'}{E}\left\{\left[F_1^2(Q^2) + \frac{Q^2}{4M^2}\kappa^2 F_2^2(Q^2)\right] + \frac{Q^2}{2M^2}[F_1(Q^2) + \kappa F_2(Q^2)]^2 \tan^2\frac{\theta}{2}\right\}$$

$$= \sigma_M \frac{E'}{E}\left[\frac{G_E^2(Q^2) + \tau G_M^2(Q^2)}{1+\tau} + 2\tau \tan^2\frac{\theta}{2} G_M^2(Q^2)\right]$$

$$= \sigma_M \frac{E'}{E}\left[\frac{Q^4}{\vec{q}^{\,4}} R_L(Q^2) + \left(\frac{Q^2}{2\vec{q}^{\,2}} + \tan^2\frac{\theta}{2}\right) R_T(Q^2)\right]$$

Mott cross section

$$\sigma_M = \frac{\alpha^2 \cos^2\left(\frac{\theta_e}{2}\right)}{4E^2 \sin^4\left(\frac{\theta_e}{2}\right)}$$

F_1, F_2: Dirac and Pauli form factors

G_E, G_M: Sachs form factors (electric and magnetic)

$G_E(Q^2) = F_1(Q^2) - \tau\kappa F_2(Q^2)$ $\qquad \tau = Q^2/4M^2$

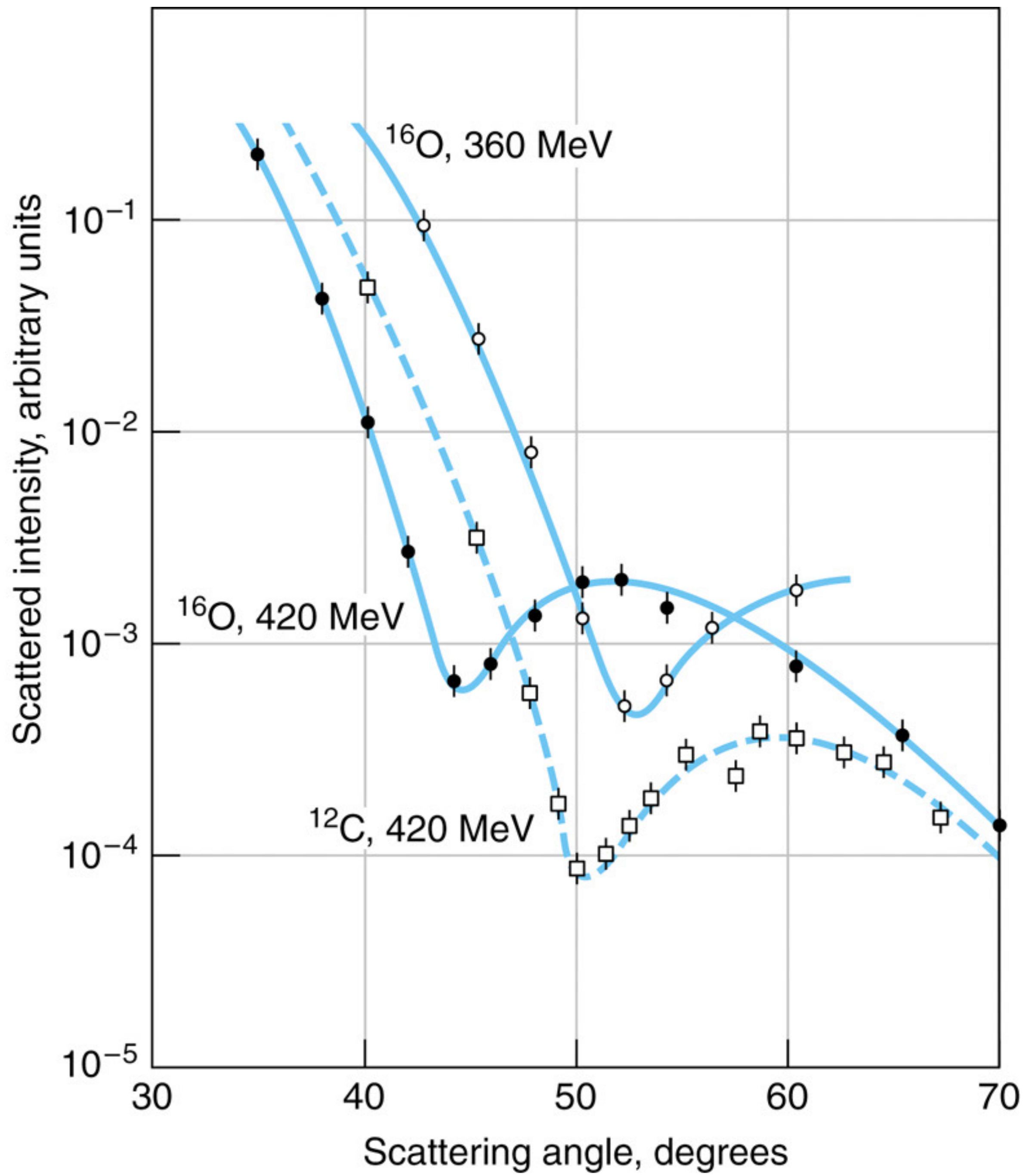
$G_M(Q^2) = F_1(Q^2) + \kappa F_2(Q^2)$ $\qquad \kappa$ = anomalous magnetic moment

R_L, R_T: Longitudinal and transverse response fn

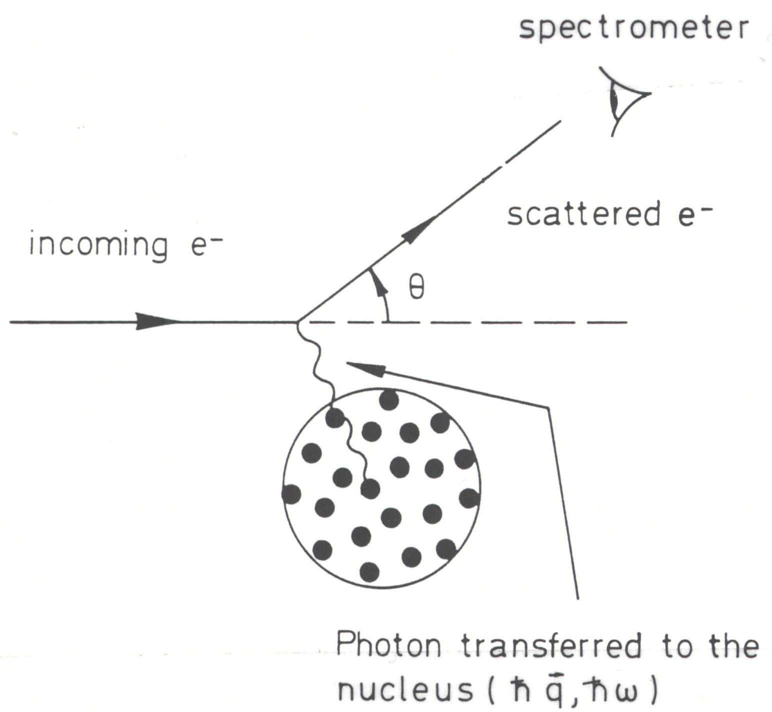
Notes on form factors

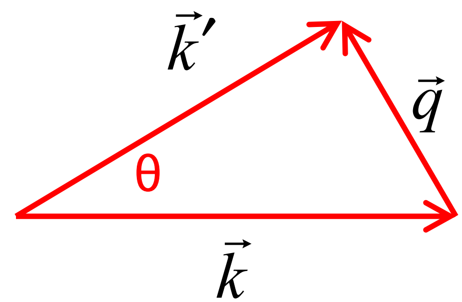
- G_E, G_M, F_1 and F_2 refer to nucleons
  - $F_1^p(0) = 1$, $F_2^p(0) = \kappa_p = 1.79$
  - $F_1^n(0) = 0$, $F_2^n(0) = \kappa_n = -1.91$
  - $G_E^p(0) = 1$, $G_M^p(0) = 1 + \kappa_p = 2.79$
  - $G_E^n(0) = 0$, $G_M^n(0) = \kappa_n = -1.91$
- R_L and R_T refer to nuclei

Electron Scattering



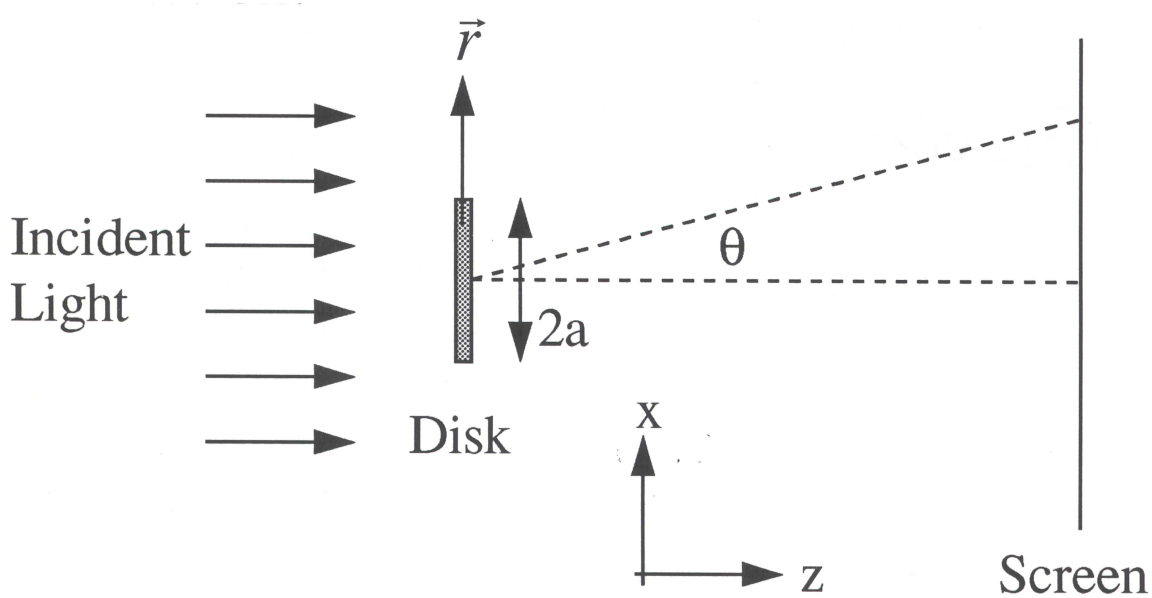
Analogy between elastic electron scattering and diffraction





Simple analogy for elastic electron scattering....

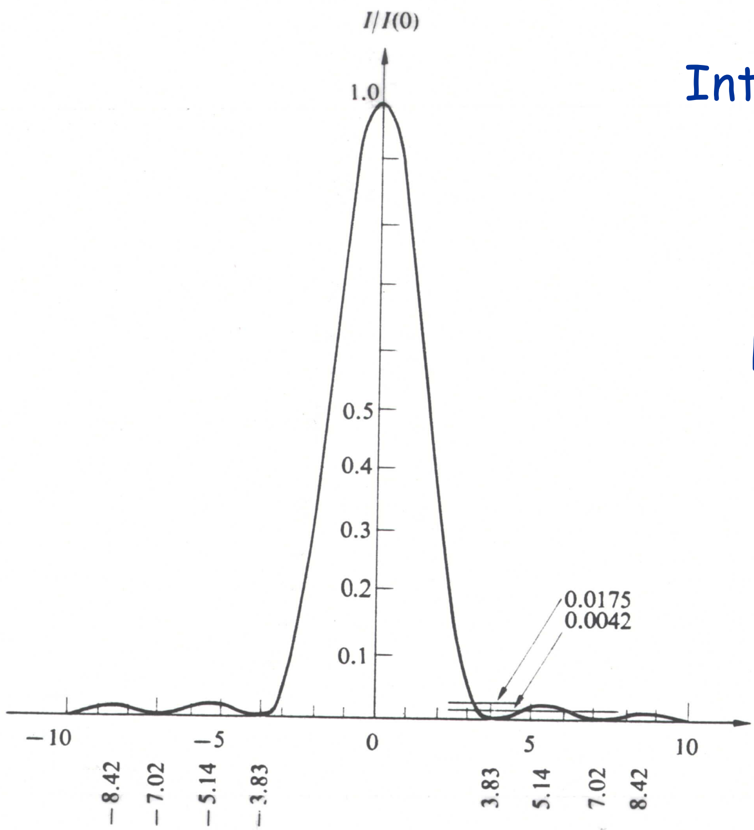
Classical Fraunhofer Diffraction



Amplitude of wave at screen:

$$\Phi \propto \int_0^a \int_0^{2\pi} \exp(ibr\cos\phi)\, r\, d\phi\, dr$$

Classical Fraunhofer Diffraction



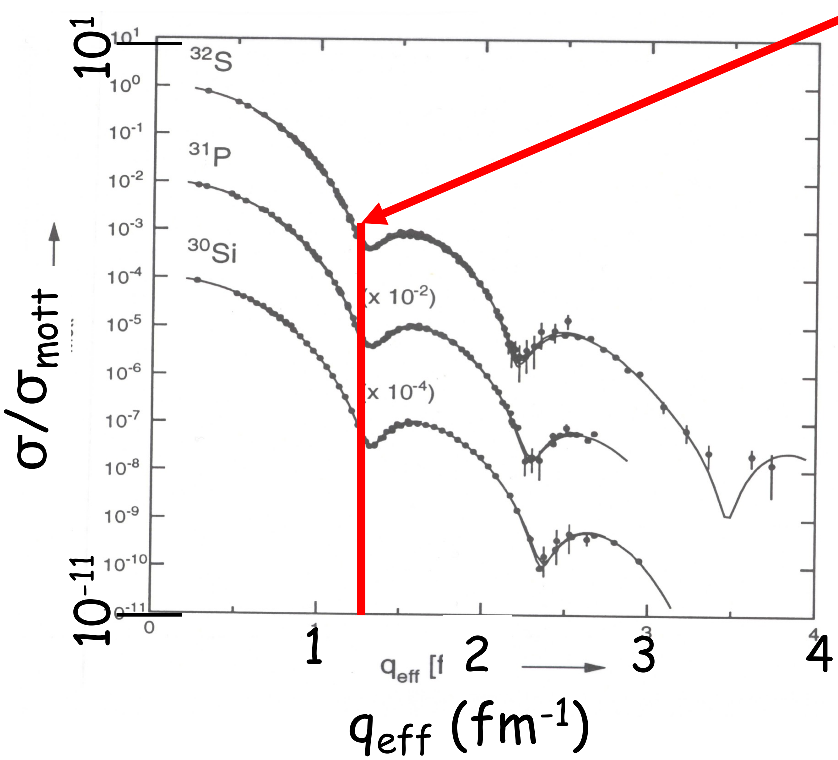
Intensity: $$\Phi^2 \propto \left(\frac{J_1((2\pi a/\lambda)\sin\theta)}{\sin\theta} \right)^2$$

Minima occur at zeroes of Bessel function. 1st zero: x = 3.8317

…some algebra…

Hence $$2a \approx \frac{1.22\lambda}{\sin\theta}$$

Example: $^{30}Si(e,e')$



1st minimum = 1.3 fm^{-1}

➔ $\theta = 32.8^o$

Electron energy = 454.3 MeV

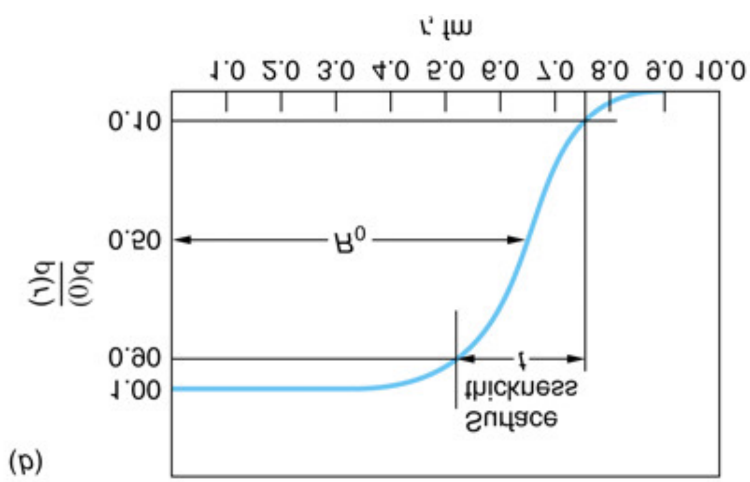
➔ $\lambda = 2.73$ fm

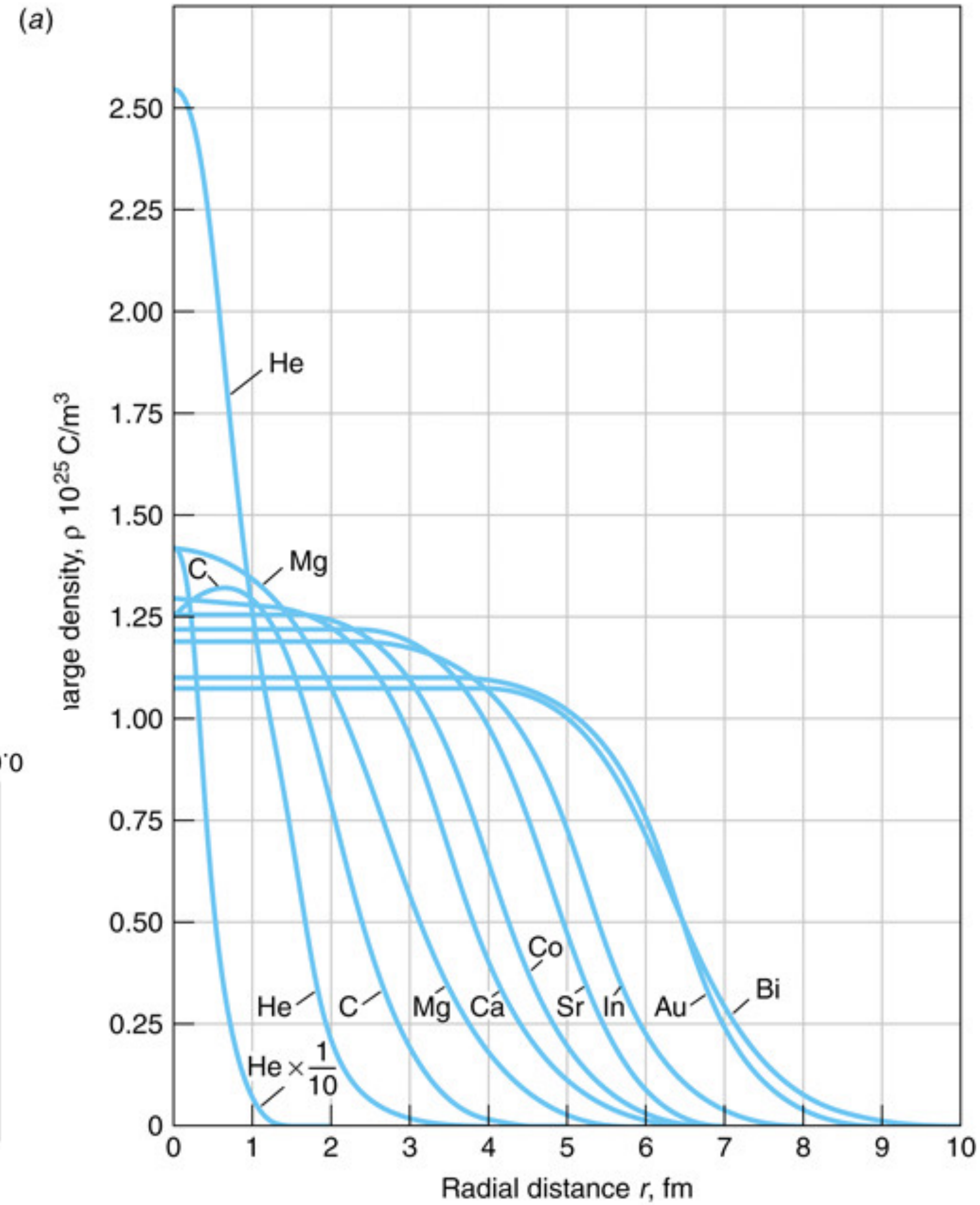
Calculated radius = 3.07 fm

Measured rms radius = 3.19 fm
(from fit to entire curve)

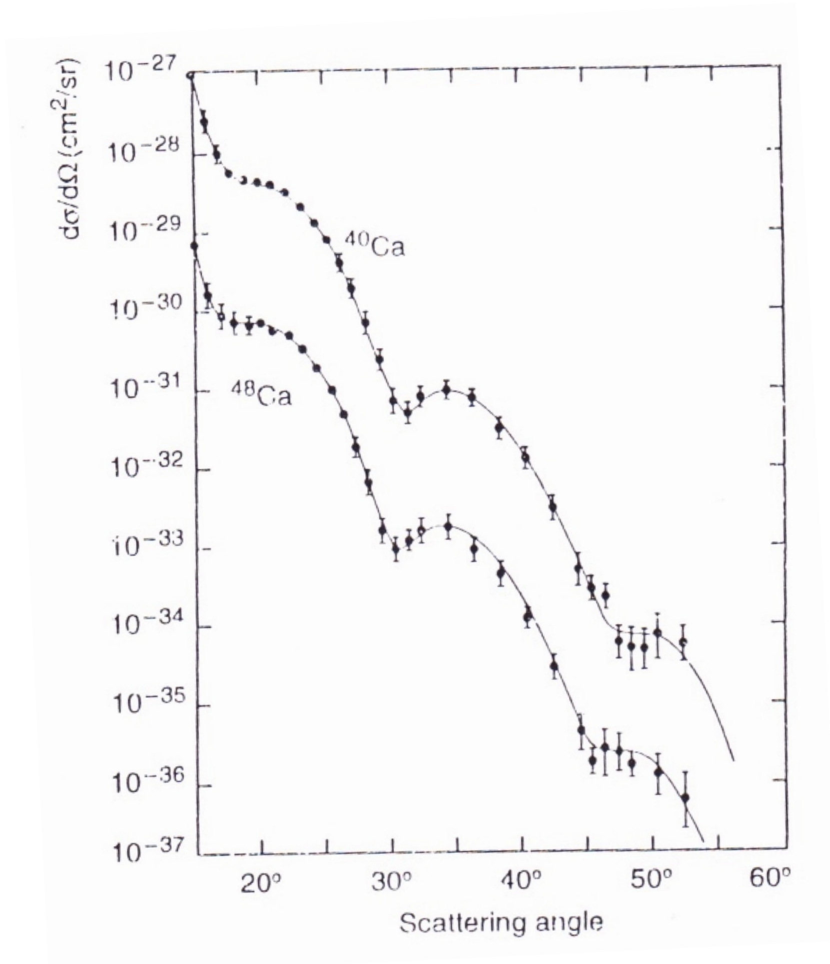
(1 fm^{-1} = 197 MeV/c)

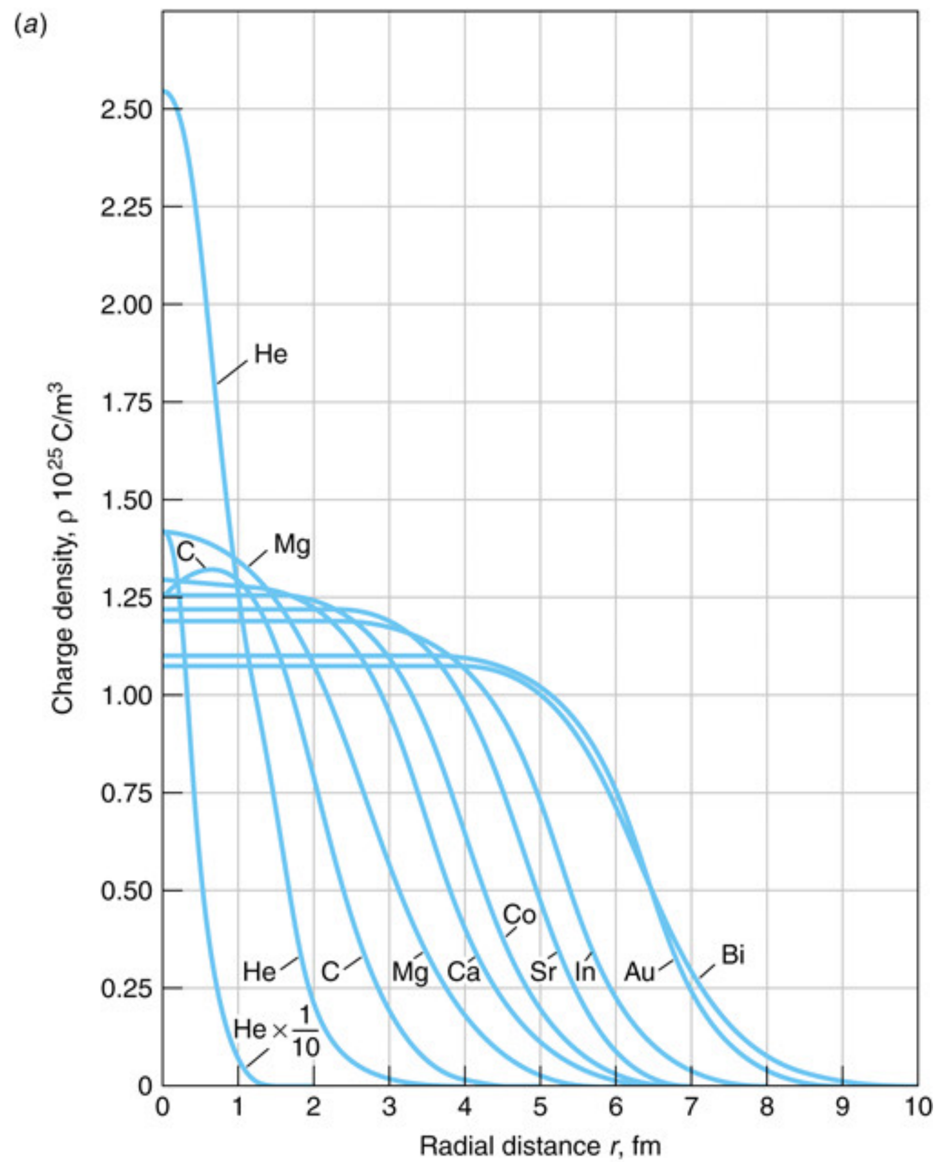
Electron Scattering -> Nuclear Density





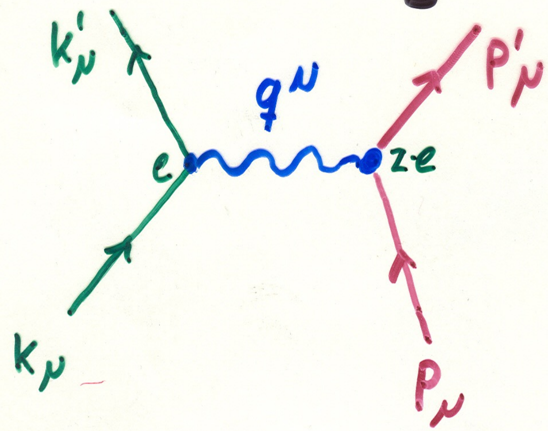
Elastic Scattering off Nuclei





-> Nuclear radii, density distributions

Elastic Scattering – Feynman diagram



$\mathcal{M}_{fi} = e\, j_\nu \left(-\frac{1}{q^\nu q_\nu}\right) z\, j^\nu$ $\qquad Q^2 := -q^\nu q_\nu$

Based on $\Box^2 A_\nu = j_\nu \Rightarrow A_\nu = \left(\frac{1}{iq}\right)^2 j_\nu$

and $H_{int} = A_\nu j^\nu = V\rho - \vec{A}\cdot\vec{j}$

$$\Rightarrow \Delta\sigma = \frac{4z^2\alpha^2(\hbar c)^2}{Q^4}\left(1-\beta^2\sin^2\frac{\theta}{2}\right)^{\#)}\left(1+2\frac{\nu^2}{Q^2}\tan^2\frac{\theta}{2}\right)^{*)}\cdot$$

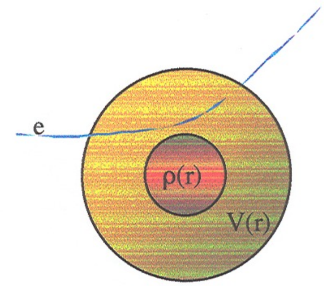
$$\cdot \int d^3\vec{k}'\,\delta(E'-E'_{el}) \left(= E'^2\cdot\Delta\Omega\right)$$

#) magnetic interaction due to electron spin

*) magnetic interaction due to target spin

Form Factors

Low-medium energy: Distribution of charge and magnetism inside the hadron



$$\vec{\nabla}^2 V(\mathbf{r}) = -4\pi\rho(\mathbf{r}) \quad \Rightarrow$$

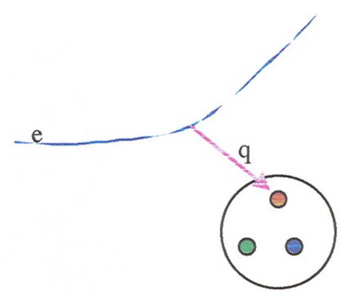
$$q^2 V \propto \int e^{i\mathbf{qr}}\rho(\mathbf{r})d^3\mathbf{r} = F(q)$$

$$\mathcal{H} \approx -eV \propto \frac{F(q)}{q^2}$$

$$\frac{d\sigma}{d\Omega} \propto \left|\langle f|\mathcal{H}|i\rangle\right|^2 \propto \frac{F^2(q)}{q^4}$$

Ex: $\rho(r) = a\cdot e^{-\alpha r} \Rightarrow F(q^2) = (1+ q^2/\alpha^2)^{-2}$ (Dipole Form)

High energy: "Stability" of internal structure against hard "blows"



Can the hadron absorb a high momentum virtual photon without breaking apart?