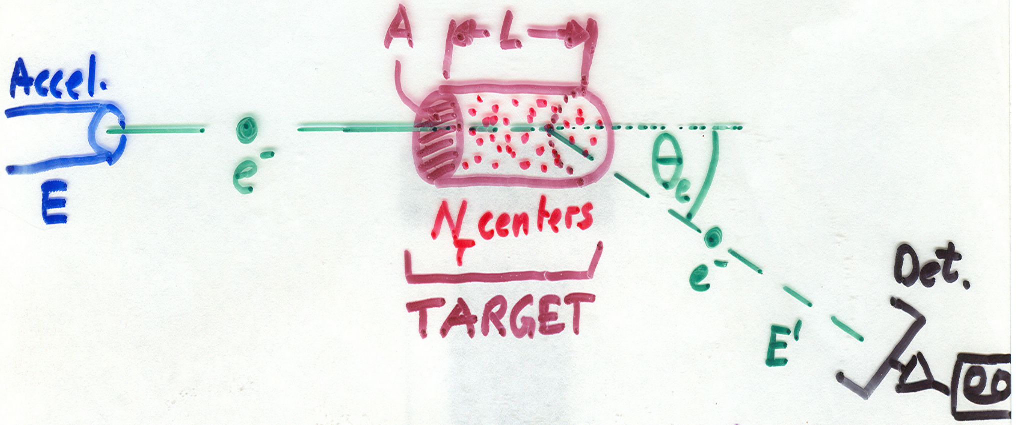
ES, IS, DIS, SIDIS
and Hadron Structure

Electron Scattering – what can we measure?



What is the likelihood to find the electron scattered into the detector?

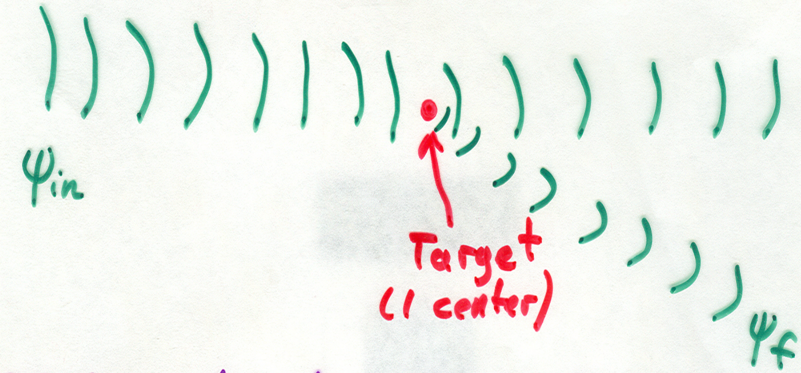
$$P \sim n_T \cdot L = \frac{N_T}{A\,L} \cdot L = \frac{N_T}{A}\ ^{*}$$

$\Rightarrow$ call $\Delta\sigma = P / \left(\frac{N_T}{A}\right)$ (cross section)

$\Delta\sigma$ DEPENDS on the kinematics (E, E', θ_e) and is $\approx$ proportional to SIZE of kinematic bin spanned by the detector

*Note: $\frac{N_T}{A} = \rho\left[\frac{g}{cm^3}\right] \cdot L\,[cm] \cdot \frac{\text{Avogadro}}{\text{Atomic Weight}\,[u]}$

Electron Scattering – Theorist's View



What is the transition rate $W_{i\to f}$?

$$\dot{N}_{e,f} = \dot{N}_{e,in} \cdot P(i\to f) = I_{e,in} \cdot \frac{N_T}{A} \cdot \Delta\sigma$$

$$= \frac{I_{e,in}}{A} \cdot N_T \cdot \Delta\sigma = |\vec{j}_{e,in}|_z \cdot N_T \cdot \Delta\sigma$$

$$\Rightarrow W_{i\to f} = j_{in} \cdot \Delta\sigma$$

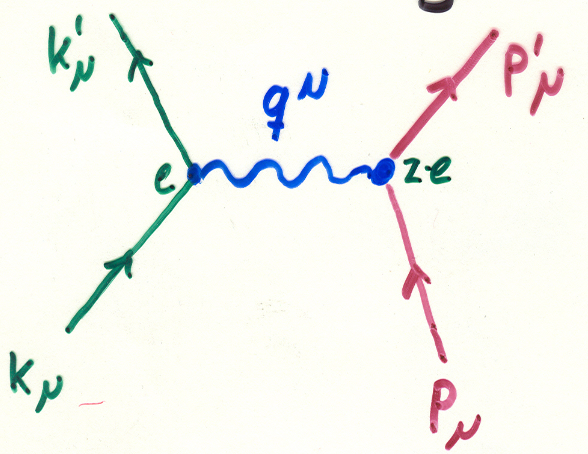
Fermi's GOLDEN Rule:

$$W_{i\to f} = \frac{2\pi}{\hbar} |\mathcal{M}_{fi}|^2 \Delta\phi$$

← Phase space spanned by detector/kinematic bin

$$\mathcal{M}_{fi} = \langle \psi_f | H_{int} | \psi_{in} \rangle$$

Elastic Scattering – Feynman diagram



$$\mathcal{M}_{fi} = e\, j_\nu \left(-\frac{1}{q^\nu q_\nu}\right) z j^\nu \qquad Q^2 := -q^\nu q_\nu$$

Based on $\Box^2 A_\nu = j_\nu \Rightarrow A_\nu = \left(\frac{1}{iq}\right)^2 j_\nu$

and $H_{int} = A_\nu j^\nu = V\rho - \vec{A}\cdot\vec{j}$

$$\Rightarrow \Delta\sigma = \frac{4z^2\alpha^2(\hbar c)^2}{Q^4}\left(1-\beta^2\sin^2\frac{\theta}{2}\right)^{*)}\left(1+2\frac{\nu^2}{Q^2}\tan^2\frac{\theta}{2}\right)^{*)}\cdot$$

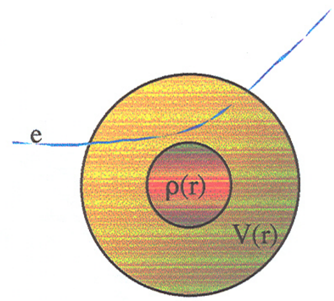
$$\cdot \int d^3\vec{k}'\,\delta(E-E'_{el})\left(= E'^2\cdot\Delta\Omega\right)$$

*) magnetic interaction due to electron spin

*) magnetic interaction due to target spin

Form Factors

Low-medium energy: Distribution of charge and magnetism inside the hadron



$$\vec{\nabla}^2 V(\mathbf{r}) = -4\pi\rho(\mathbf{r}) \quad \Rightarrow$$

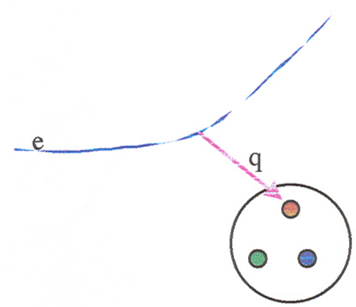
$$q^2 V \propto \int e^{i\mathbf{qr}}\rho(\mathbf{r})d^3\mathbf{r} = F(q)$$

$$\mathcal{H} \approx -eV \propto \frac{F(q)}{q^2}$$

$$\frac{d\sigma}{d\Omega} \propto |\langle f|\mathcal{H}|i\rangle|^2 \propto \frac{F^2(q)}{q^4}$$

Ex: $\rho(r) = a\cdot e^{-\alpha r} \Rightarrow F(q^2) = (1+ q^2/\alpha^2)^{-2}$ (Dipole Form)

High energy: "Stability" of internal structure against hard "blows"



Can the hadron absorb a high momentum virtual photon without breaking apart?

Some kinematics – in the lab system

$k^\mu = (E, 0, 0, E) = (E, \vec{k})$

$k'^\mu = (E', E'\sin\theta, 0, E'\cos\theta) = (E', \vec{k}')$

$q^\mu = (E-E', -E'\sin\theta, 0, E-E'\cos\theta) = (\nu, \vec{q})$

$Q^2 = -q^\mu q_\mu = -\nu^2 + \vec{q}^{\,2} = (E-E'\cos\theta)^2 + E'^2\sin^2\theta - (E-E')$

$= E^2 - 2EE'\cos\theta + E'^2 - E^2 - E'^2 + 2EE'$

$= 2EE'(1-\cos\theta) = 4EE'\sin^2\frac{\theta}{2}$

$p^\mu = (M, 0, 0, 0) = (M, \vec{0})$

$p'^\mu = p^\mu + q^\mu = (M+\nu, \vec{q})$

$p'^\mu p'_\mu =: W^2 = M^2 + 2M\nu + \nu^2 - \vec{q}^{\,2}$

$= M^2 + 2M\nu - Q^2$

Elastic Scattering: $W^2 \stackrel{!}{=} M^2$

$\Rightarrow \nu_{el} \stackrel{!}{=} \frac{Q^2}{2M}$ or $x_{el} = \frac{Q^2}{2M\nu_{el}} \stackrel{!}{=} 1$

Elastic Cross section – final form

$$\Delta\sigma = \frac{4z^2\alpha^2(\hbar c)^2}{Q^4}\cos^2\frac{\theta}{2}\left[\frac{G_E^2 + \tau G_M^2}{1+\tau} + 2\tau\tan^2\frac{\theta}{2}\,G_M^2\right] \cdot E'^2\,\Delta\Omega\,\frac{E'}{E}$$

LONGITUDINAL (charge) — TRANSVERSE (magnetic) — Recoil

$\tau = \frac{\nu^2}{Q^2}$, $G_E(Q^2)$, $G_M(Q^2)$: Form Factors

Dirac Particle: $G_E = G_M = 1$ (const.)

Anomalous magnetic moment: $G_M \approx (1+\kappa)G_E$

Extended Charge distribution:

$G_E(Q^2) \approx$ Fourier transform of $\rho(r)$

Ex: $\rho(r) \approx \frac{a^3}{8\pi}e^{-ar} \Rightarrow G_E(Q^2) \approx \left(\frac{1}{1+\frac{Q^2}{a^2}}\right)^2$

(Dipole Form). p: $a^2 = 0.71\ \text{GeV}^2$

Inelastic Cross Section – What's different

Phase space factor $d^3k' = k'^2 d\Omega_{k'} dk'$

(δ-function drops since E' can have any value)

— conversion —

$k'^2 d\Omega dk' = E'^2 2\pi \sin\theta d\theta dE'$

$= -2E'^2 d\cos\theta \, \pi dE'$

$= \frac{\pi E'}{E} \underbrace{(-2EE' d\cos\theta)}_{+dQ^2} \underbrace{dE'}_{|d\nu|}$ since $\nu = E - E'$

Replace $\frac{G_E^2 + \tau G_M^2}{1+\tau}$ with $W_2(Q^2, \nu)$ *)

and τG_M^2 with $W_1(Q^2, \nu) \Rightarrow$

$$\Delta\sigma = \frac{4\pi\alpha^2(\hbar c)^2}{Q^4} \frac{E'}{E} \cos^2\theta/2 \left[W_2 + 2\tan^2\frac{\theta}{2} W_1\right] \Delta Q^2 \Delta\nu$$

*) $\Rightarrow W_2^{el}(Q^2, \nu) = \frac{G_E^2 + \tau G_M^2}{1+\tau} \cdot \delta(\nu - \nu_{el})$

$W_1^{el}(Q^2, \nu) = \tau G_M^2 \cdot \delta(\nu - \nu_{el})$

Elastic Cross section – final form

$$\Delta\sigma = \frac{4z^2\alpha^2(\hbar c)^2}{Q^4} \cos^2\frac{\theta}{2} \left[\underbrace{\frac{G_E^2 + \tau G_M^2}{1+\tau}}_{\text{LONGITUDINAL (charge)}} + \underbrace{2\tau\tan^2\frac{\theta}{2} G_M^2}_{\text{TRANSVERSE (magnetic)}}\right] \cdot E'^2 \Delta\Omega \underbrace{\frac{E'}{E}}_{\text{Recoil}}$$

$\tau = \frac{\nu^2}{Q^2}$, $G_E(Q^2)$, $G_M(Q^2)$: Form Factors

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Ex: $\rho(r) \simeq \frac{a^3}{8\pi} e^{-ar} \Rightarrow G_E(Q^2) \simeq \left(\frac{1}{1 + \frac{Q^2}{a^2}}\right)^2$

(Dipole Form). p: $a^2 = 0.71\ \text{GeV}^2$

Elastic scattering

$$\frac{\Delta\sigma}{\Delta\Omega} = \frac{4\alpha^2(\hbar c)^2 E'^2 cos^2\frac{\theta}{2}}{Q^4}\frac{E'}{E}\left(\frac{G_E^2(Q^2)+\tau G_M^2(Q^2)}{1+\tau} + 2\tau\, tan^2\frac{\theta}{2}\, G_M^2(Q^2)\right)$$

(

where $\tau = \nu^2/Q^2$.

Inelastic Scattering

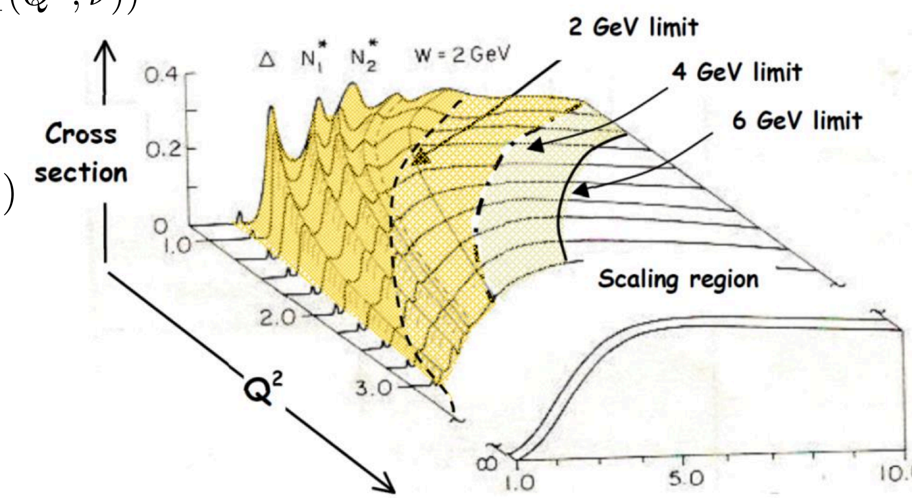
$x = \frac{p}{P}$

W: γ^*-p invariant mass

$$\frac{\Delta\sigma}{\Delta Q^2 \Delta\nu} = \frac{4\pi\alpha^2(\hbar c)^2 E' cos^2(\theta/2)}{Q^4 E}(W_2(Q^2,\nu) + 2\, tan^2(\theta/2)\, W_1(Q^2,\nu))$$

$$\frac{\Delta\sigma}{\Delta Q^2 \Delta\nu} = \frac{4\pi\alpha^2(\hbar c)^2 E' cos^2(\theta/2)}{Q^4 E}\frac{W_1(Q^2,\nu)}{\epsilon(1+\tau)}(1+\epsilon R(Q^2,\nu))$$

with $\epsilon = (1 + 2(1+\tau)tan^2(\theta/2))^{-1}$



$$\frac{\Delta\sigma}{\Delta Q^2 \Delta\nu} = \frac{4\pi\alpha^2(\hbar c)^2 E' cos^2(\theta/2)}{Q^4 E}(\frac{1}{\nu}F_2(x) + 2\, tan^2(\theta/2)\frac{1}{M}F_1(x))$$

$$\omega' = \frac{1}{x} + \frac{m^2}{Q^2}$$

$= W^2/Q^2 + 1$

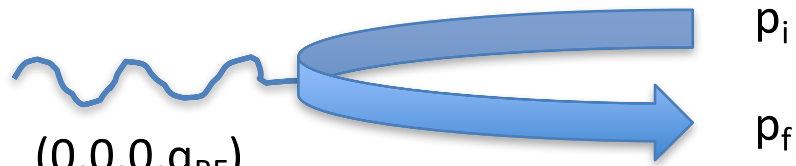
$$F_1(x) = MW_1(Q^2,\nu) \qquad F_2(x) = \nu W_2(Q^2,\nu)$$

Only "mildly" dependent on Q^2

Picture from F. Gross,
«Making the case for Jefferson Lab»
The first decade of Science at Jefferson Lab
JoP, Conf. Series 299 (2011) 012001

Deep Inelastic Scattering (DIS)

Breit ("Brickwall") Frame



$$\begin{pmatrix} \Gamma & 0 & 0 & \Gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \Gamma\beta & 0 & 0 & \Gamma \end{pmatrix}\begin{pmatrix} \nu \\ 0 \\ 0 \\ -q \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ q_{BF} \end{pmatrix} \Rightarrow$$

$$p_i = p_f = \frac{q_{BF}}{2} = \frac{\sqrt{Q^2}}{2}$$

$$\Gamma = \frac{q}{\sqrt{Q^2}}, \Gamma\beta = \frac{\nu}{\sqrt{Q^2}}, q_{BF} = \sqrt{Q^2}$$

$$\begin{pmatrix} \Gamma & 0 & 0 & \Gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \Gamma\beta & 0 & 0 & \Gamma \end{pmatrix}\begin{pmatrix} M \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \Gamma M \\ 0 \\ 0 \\ \Gamma\beta M \end{pmatrix} = \begin{pmatrix} \frac{qM}{\sqrt{Q^2}} \\ 0 \\ 0 \\ \frac{\nu M}{\sqrt{Q^2}} \end{pmatrix} \Rightarrow$$

$$x = \frac{p_i^{BF}}{P_i^{BF}} = \frac{\frac{\sqrt{Q^2}}{2}}{\frac{\nu M}{\sqrt{Q^2}}} = \frac{Q^2}{2M\nu}$$

$$Q^2 \to \infty, \nu \to \infty, x = \frac{Q^2}{2M\nu} \text{ fixed: } q = \nu\sqrt{1+\frac{Q^2}{\nu^2}} = \nu\sqrt{1+\frac{4M^2x^2}{Q^2}} \approx \nu$$

Deep Inelastic Scattering (DIS)

Reminder: Elastic scattering

$$\frac{\Delta\sigma}{\Delta\cancel{\Omega}\,Q^2} = \frac{4\alpha^2(\hbar c)^2 \cancel{E'^2}\,\pi\, cos^2\frac{\theta}{2}}{Q^4}\frac{E'}{E}\left(\frac{G_E^2(Q^2)+\tau G_M^2(Q^2)}{1+\tau} + 2\tau\, tan^2\frac{\theta}{2}\, G_M^2(Q^2)\right)$$

Elastic scattering from quarks:

$$\Delta\sigma = \frac{4\pi z_q^2\alpha^2(\hbar c)^2 E' cos^2(\theta/2)}{Q^4 E}(q(x)\Delta x + 2\nu^2/Q^2 tan^2(\theta/2)q(x)\Delta x)\Delta Q^2. \quad (12)$$

We can use the relation $\Delta x = -Q^2/(2M\nu^2)\Delta\nu = -x\Delta\nu/\nu$ to rewrite this as

$$\frac{\Delta\sigma}{\Delta Q^2\Delta\nu} = \frac{4\pi\alpha^2(\hbar c)^2 E' cos^2(\theta/2)}{Q^4 E}(\frac{x}{\nu}z_q^2 q(x) + \frac{1}{M}tan^2(\theta/2)z_q^2 q(x)). \quad (13)$$

Reminder: IN-Elastic scattering

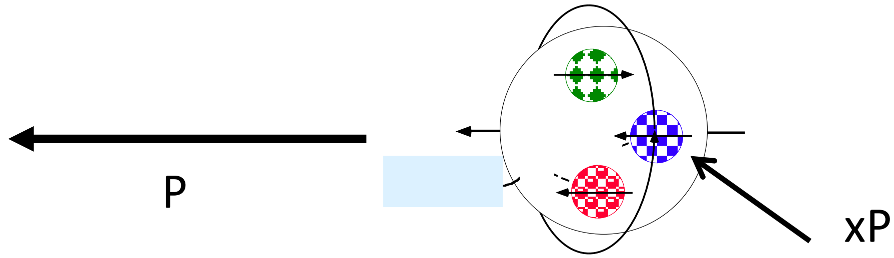
$$\frac{\Delta\sigma}{\Delta Q^2\Delta\nu} = \frac{4\pi\alpha^2(\hbar c)^2 E' cos^2(\theta/2)}{Q^4 E}(\frac{1}{\nu}F_2(x) + 2\, tan^2(\theta/2)\frac{1}{M}F_1(x))$$

NOTE: $\Rightarrow$
$F_2 = 2xF_1$
Callan-Gross

$$\Rightarrow \quad F_1(x) = \frac{1}{2}\left(\frac{4}{9}\left[u(x)+\bar{u}(x)\right] + \frac{1}{9}\left[d(x)+\bar{d}(x)+s(x)+\bar{s}(x)\right] + \ldots\right)$$

No Q^2!

Quark-Parton Structure of the Proton



q(x) $\sim \left\langle P,s \middle| \bar{q}\gamma^{\mu} q \middle| P,s \right\rangle$

Δq(x) $= q\Uparrow\uparrow(x) - q\Uparrow\downarrow(x) + \bar{q}\Uparrow\uparrow(x) - \bar{q}\Uparrow\downarrow(x) \sim \left\langle P,s \middle| \bar{q}\gamma^{\mu}\gamma^{5} q \middle| P,s \right\rangle$

"axial charge", similarly G(x) and ΔG(x) for gluons

Spin Sum Rule:

$$S_p = \frac{1}{2} = \frac{1}{2}\underbrace{\sum_q \Delta q}_{\Delta\Sigma} + \Delta G + L_q + L_G$$

Structure Functions

$$\frac{\Delta\sigma}{\Delta Q^2 \Delta\nu} = \frac{4\pi\alpha^2(\hbar c)^2 E' cos^2(\theta/2)}{Q^4 E}(\frac{1}{\nu}F_2(x) + 2\,tan^2(\theta/2)\frac{1}{M}F_1(x))$$

$$\frac{\Delta\sigma}{\Delta Q^2 \Delta\nu}\downarrow\Uparrow - \frac{\Delta\sigma}{\Delta Q^2 \Delta\nu}\uparrow\Uparrow = \frac{4\pi\alpha^2}{M\nu Q^2 E^2}\left[(E + E'\cos\theta)\mathbf{g_1} - 2xM\mathbf{g_2}\right]$$

Unpolarized: $F_1(x,Q^2)$ and $F_2(x, Q^2)$

Polarized: $g_1(x,Q^2)$ and $g_2(x, Q^2)$

Parton model:

$$F_1(x) = \frac{1}{2}\sum_i e_i^2 q_i(x) \text{ and } F_2(x) = 2xF_1(x)$$

$$g_1(x) = \frac{1}{2}\sum_i e_i^2 \Delta q_i(x) \text{ and } g_2(x) = 0$$

$i =$ quark flavor

$e_i =$ quark charge

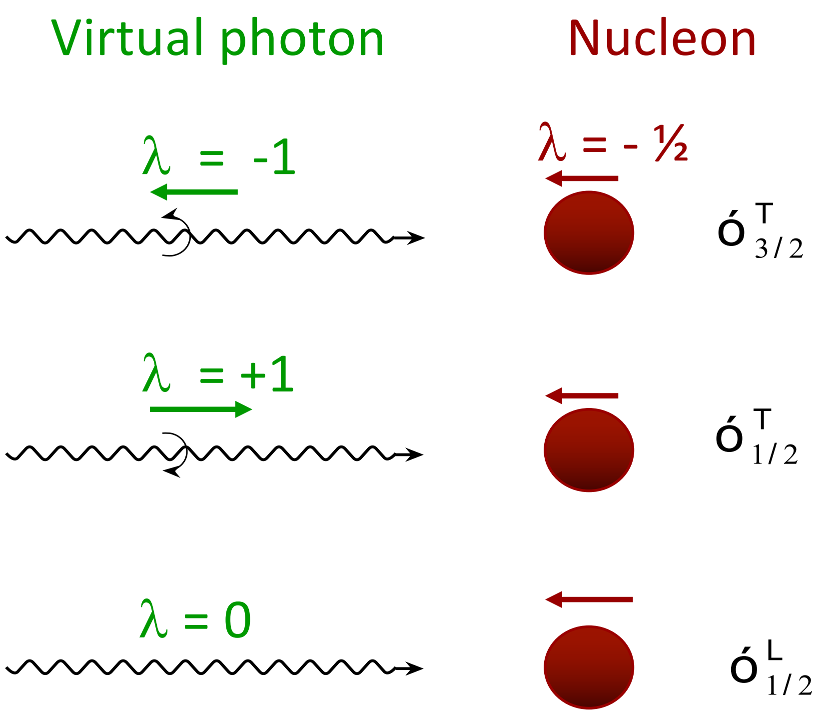
the structure functions $\mathbf{g_1}$ and $\mathbf{g_2}$ are linear combinations of $\mathbf{A_1}$ and $\mathbf{A_2}$

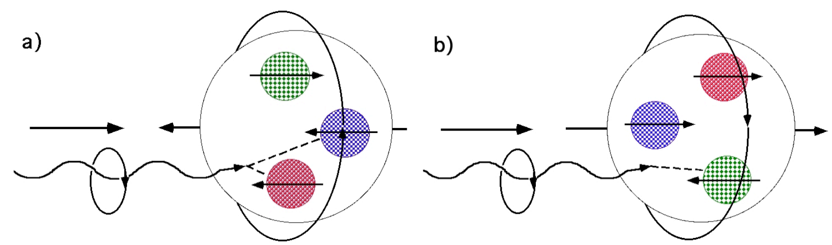
$$g_1(x,Q^2) = \frac{\tau}{1+\tau}(A_1 + \frac{1}{\sqrt{\tau}}A_2)F_1$$

$$g_2(x,Q^2) = \frac{\tau}{1+\tau}(\sqrt{\tau}A_2 - A_1)F_1$$

$$\tau = \frac{\nu^2}{Q^2}$$

Virtual Photon Asymmetries

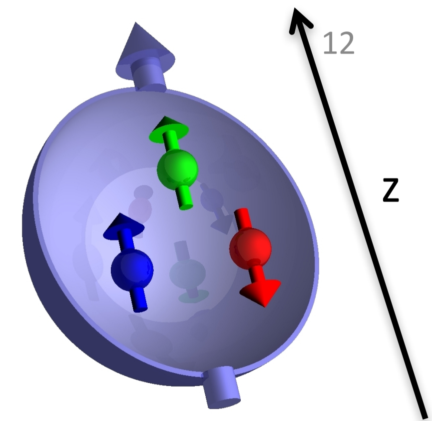




$$A_1 = \frac{\sigma_{1/2} - \sigma_{3/2}}{\sigma_T} \qquad A_2 = \frac{\sigma_{LT'}}{\sigma_T}$$

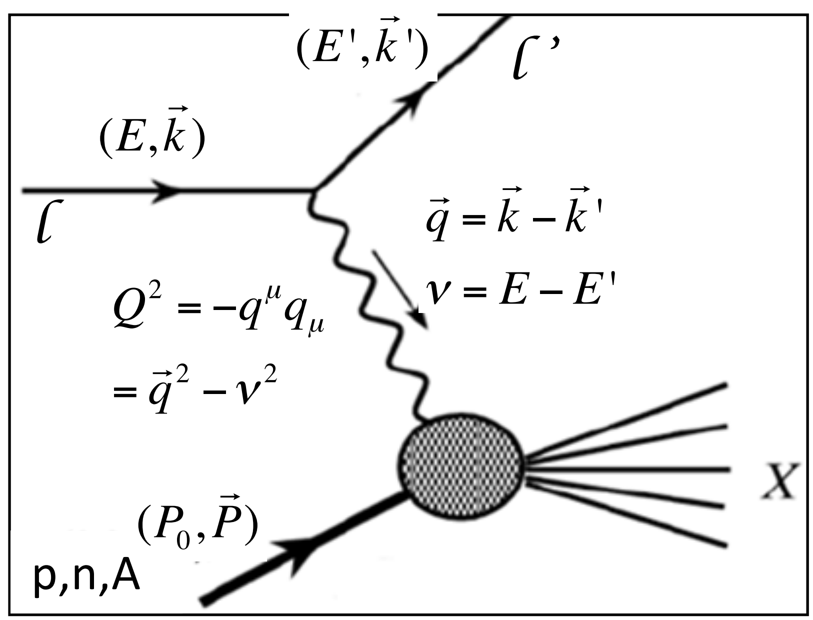
related to quark polarizations
Δq/q

Parton Distribution Functions



- The 1D world of nucleon/nuclear collinear structure:
  - Take a nucleon/nucleus
  - Move it real fast along z
    $\Rightarrow$ light cone momentum
    $P_+ = P_0 + P_z$ (>>M)
  - Select a "parton" (quark, gluon) inside
  - Measure **its** l.c. momentum
    $p_+ = p_0 + p_z$ (m≈0)
  - $\Rightarrow$ Momentum Fraction $x = p_+/P_+$ *)
  - In DIS **): $p_+/P_+ \approx \xi = (q_z - \nu)/M$
    $\approx x_{Bj} = Q^2/2M\nu$
  - Probability: $f_1^i(x), i = u, d, s, ..., G$

In the following, will often write "$q_i(x)$" for $f_1^i(x)$



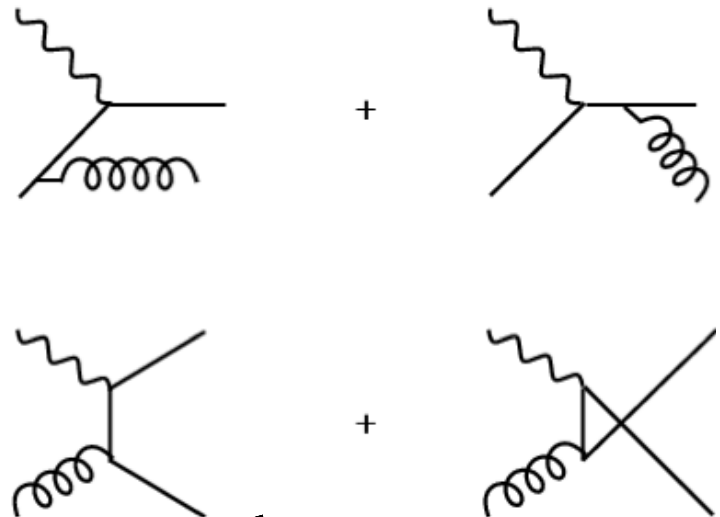
*) Advantage: Boost-independent along z

**) DIS = "Deep Inelastic (Lepton) Scattering"

Parton Distribution Functions and NLO pQCD

Two effects modify simple parton picture:

1) (Gluon) radiative corrections change elementary cross section

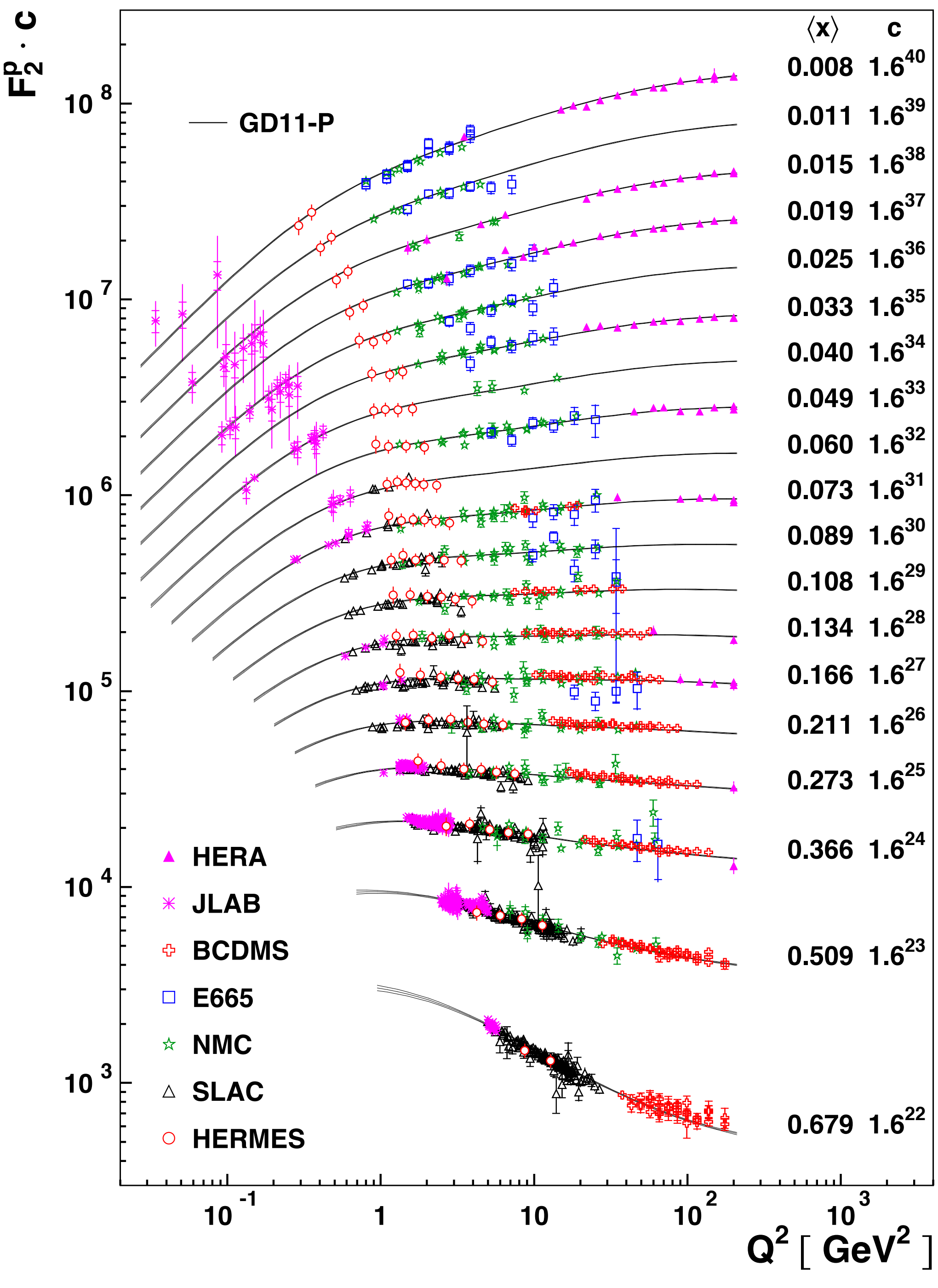


$$g_1(x,Q^2)_{pQCD} = \frac{1}{2}\sum_q^{N_f} e_q^2 [(\Delta q + \Delta\bar{q}) \otimes (1 + \frac{\alpha_s(Q^2)}{2\pi}\delta C_q) + \frac{\alpha_s(Q^2)}{2\pi}\Delta G \otimes \frac{\delta C_G}{N_f}]$$

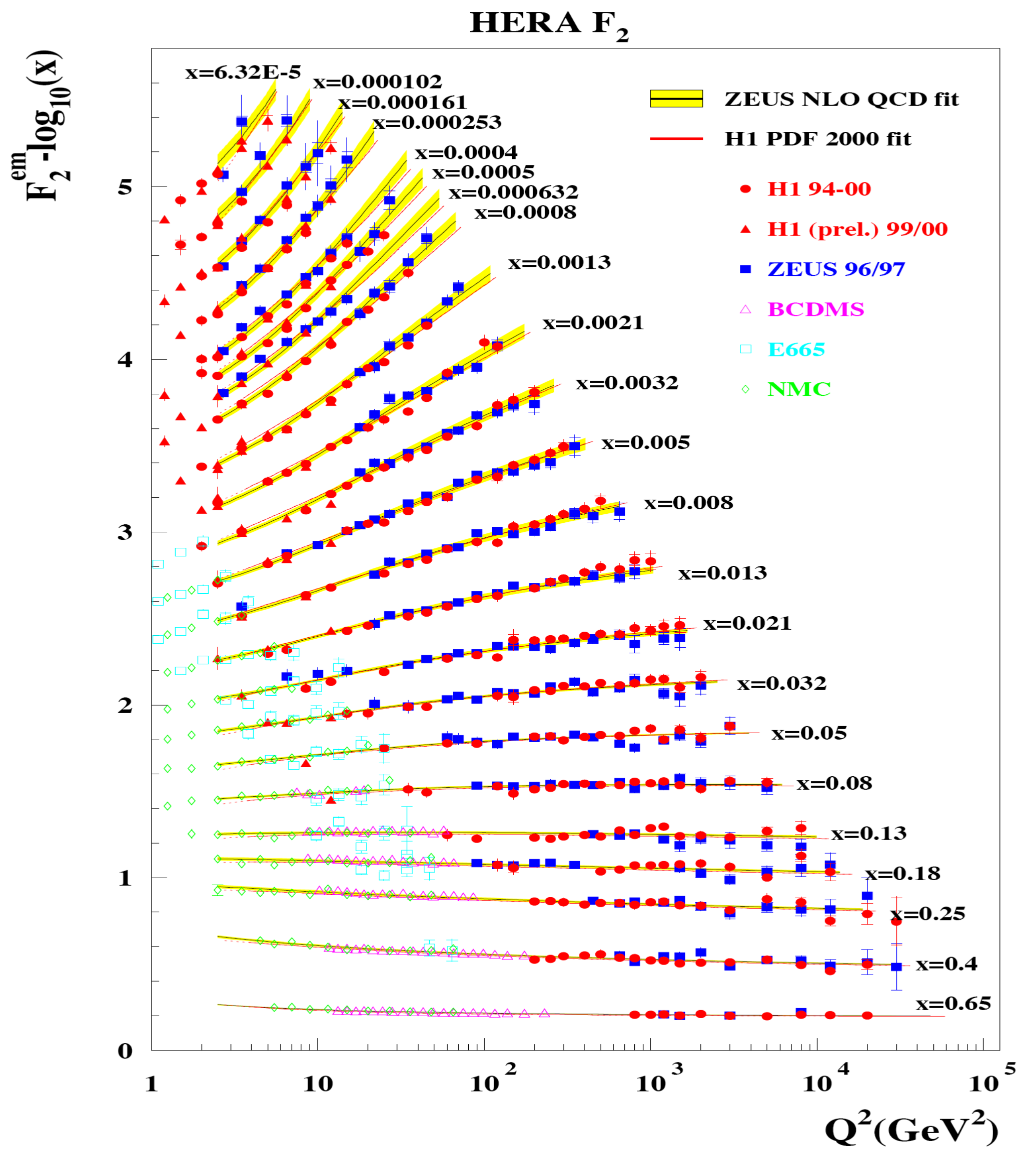
$\delta C_q, \delta C_G$ – *Wilson* coefficient functions

2) pQCD evolution makes PDFs Q^2-dependent

Results



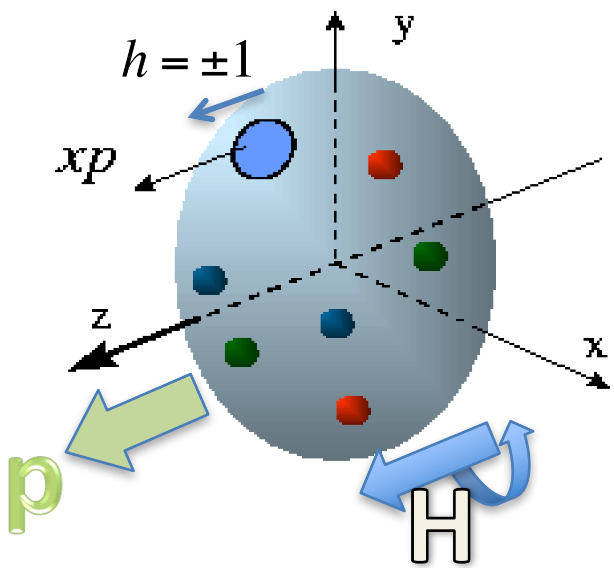
$F_2^p \cdot c$
GD11-P
⟨x⟩ c
0.008 1.6^40
0.011 1.6^39
0.015 1.6^38
0.019 1.6^37
0.025 1.6^36
0.033 1.6^35
0.040 1.6^34
0.049 1.6^33
0.060 1.6^32
0.073 1.6^31
0.089 1.6^30
0.108 1.6^29
0.134 1.6^28
0.166 1.6^27
0.211 1.6^26
0.273 1.6^25
0.366 1.6^24
0.509 1.6^23
0.679 1.6^22
HERA
JLAB
BCDMS
E665
NMC
SLAC
HERMES
Q^2 [GeV^2]



HERA F$_2$
F_2^{em}-log$_{10}$(x)
x=6.32E-5
x=0.000102
x=0.000161
x=0.000253
x=0.0004
x=0.0005
x=0.000632
x=0.0008
x=0.0013
x=0.0021
x=0.0032
x=0.005
x=0.008
x=0.013
x=0.021
x=0.032
x=0.05
x=0.08
x=0.13
x=0.18
x=0.25
x=0.4
x=0.65
ZEUS NLO QCD fit
H1 PDF 2000 fit
H1 94-00
H1 (prel.) 99/00
ZEUS 96/97
BCDMS
E665
NMC
Q^2(GeV^2)

Inclusive lepton scattering

Parton model: DIS can access



$$q(x;Q^2),\langle h\cdot H\rangle q(x;Q^2)$$

Traditional "1-D" Parton Distributions (PDFs) (integrated over many variables)

Callan-Gross

Wandzura-Wilczek

$$F_1(x)=\tfrac{1}{2}\sum_i e_i^2 q_i(x)\ \left(\text{and } F_2(x)\approx 2xF_1(x)\right)$$

$$g_1(x)=\tfrac{1}{2}\sum_i e_i^2 \Delta q_i(x)\ \left(\text{and } g_2(x)\approx -g_1(x)+\int_x^1 \frac{g_1(y)}{y}dy\right)$$

At finite Q²: pQCD evolution ($q(x,Q^2)$, $\Delta q(x,Q^2)$ ⇒ DGLAP equations), and gluon radiation

$$g_1(x,Q^2)_{pQCD}=\frac{1}{2}\sum_q^{N_f} e_q^2[(\Delta q+\Delta\bar{q})\otimes(1+\frac{\alpha_s(Q^2)}{2\pi}\delta C_q)+\frac{\alpha_s(Q^2)}{2\pi}\Delta G\otimes\frac{\delta C_G}{N_f}]$$

⇒ access to gluons. $\delta C_q,\delta C_G$ – *Wilson* coefficient functions

SIDIS: Tag the flavor of the struck quark with the leading FS hadron ⇒ separate $q_i(x,Q^2)$, $\Delta q_i(x,Q^2)$

Jefferson Lab kinematics: $Q^2\approx M^2$ ⇒ target mass effects, higher twist contributions and resonance excitations

- Non-zero $R=\frac{F_2}{2xF_1}\left(\frac{4M^2x^2}{Q^2}+1\right)-1,\ g_2^{HT}(x)=g_2(x)-g_2^{WW}(x)$
- Further Q^2-dependence (power series in $\frac{1}{Q^n}$)

⇒ Our 1D View of the Nucleon

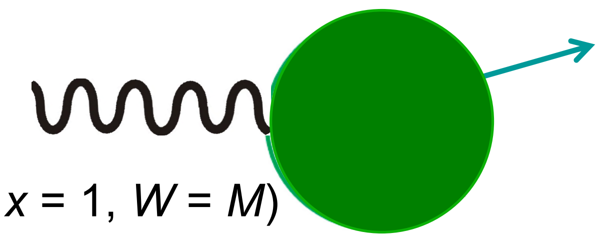
(depends on x and the resolution of the virtual photon ~ $1/Q^2$)

W = final state invariant mass = $\sqrt{M^2 + \left(\frac{1}{x} - 1\right) Q^2}$

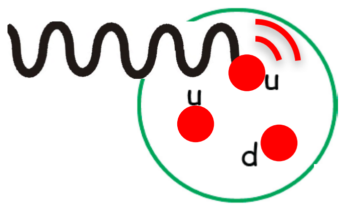
JLab

DIS

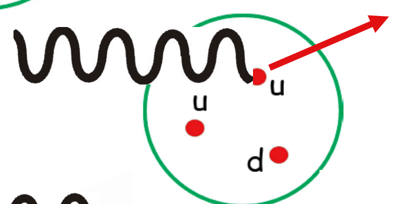
- Elastic scattering
  (Whole system recoils, $x = 1$, $W = M$)



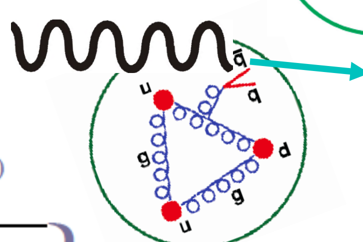
- Resonances
  ($x < 1$, $W < 2$ GeV)



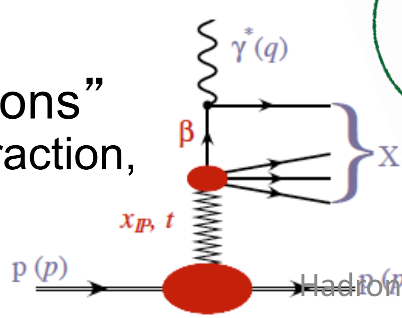
- Valence quarks
  ($x \geq 0.3$, $W > 2$ GeV)

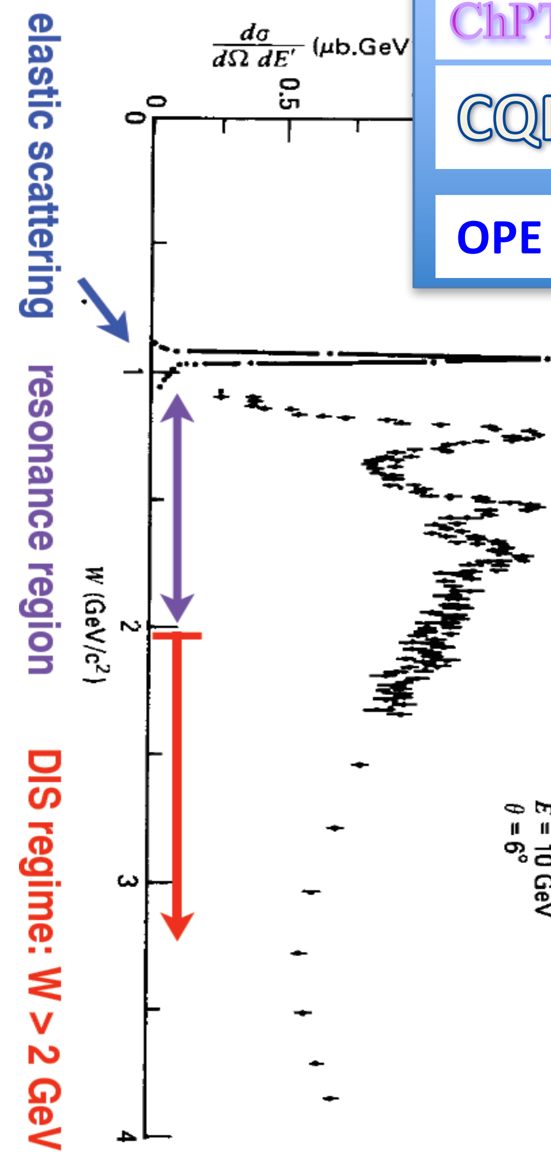


- Sea quarks, gluons
  ($x < 0.3$)



- "Wee Partons"
  ($x \to 0$, Diffraction, Pomerons)





Low Q^2:

ChPT

CQM

OPE

Experimental Facilities