Physics 722/822
9/25/2018

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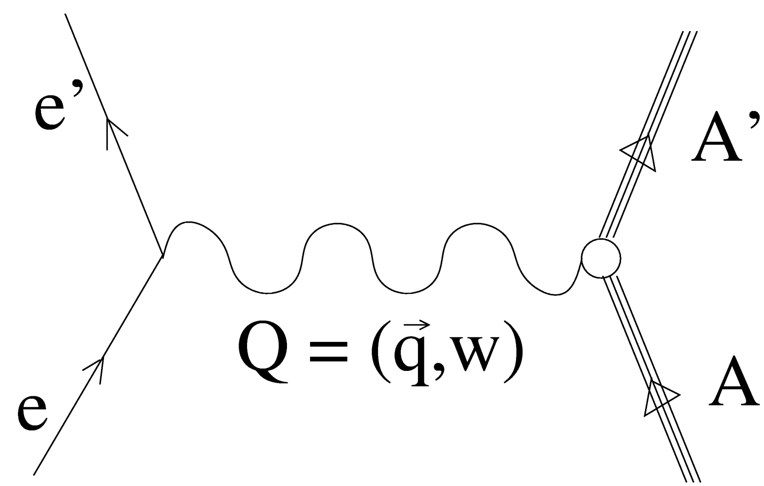
Why use electrons and photons?

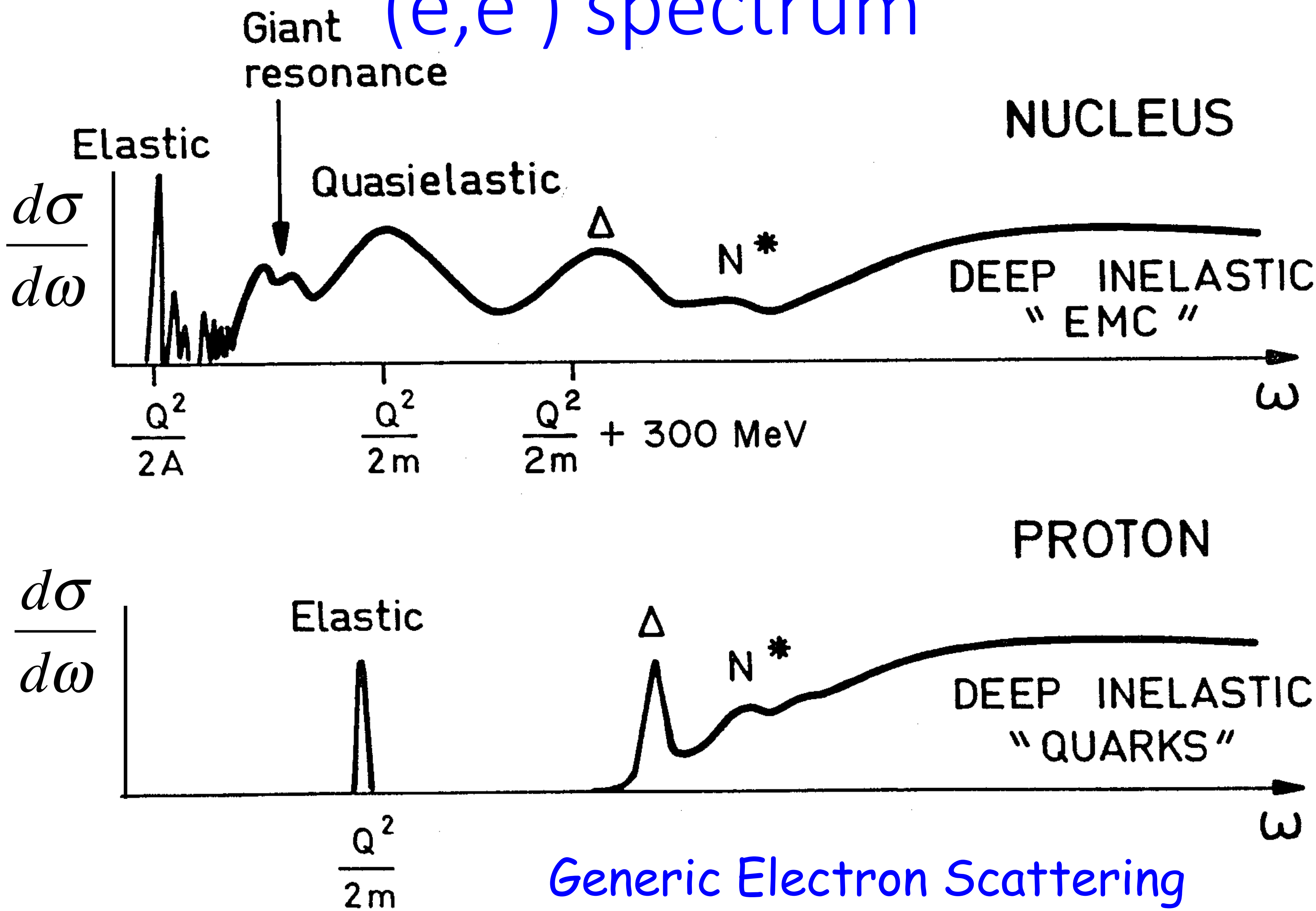
- Probe structure understood (point particles)
- Electromagnetic interaction understood (QED)
- Interaction is weak (α = 1/137)
  - Perturbation theory works!
    - First Born Approx / one photon exchange
  - Probe interacts only once
  - Study the entire nuclear volume

BUT:

- Cross sections are small
- Electrons radiate

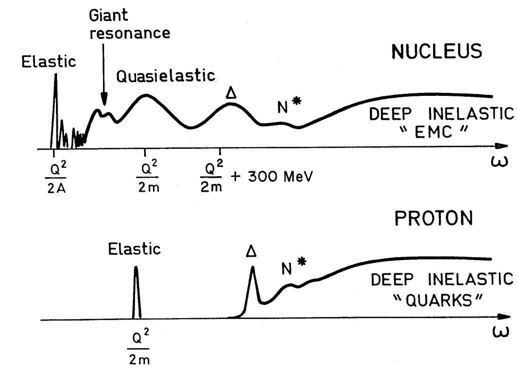


(e,e') spectrum



Generic Electron Scattering
at fixed momentum transfer

Experimental goals:



- Elastic scattering
  - structure of the nucleus
    - Form factors, charge distributions, spin dependent FF
- Quasielastic (QE) scattering
  - Shell structure
    - Momentum distributions
    - Occupancies
  - Short Range Correlated nucleon pairs
  - Nuclear transparency and color transparency
- Deep Inelastic Scattering (DIS)
  - The EMC Effect and Nucleon modification in nuclei
  - Quark hadronization in nuclei

Energy vs length

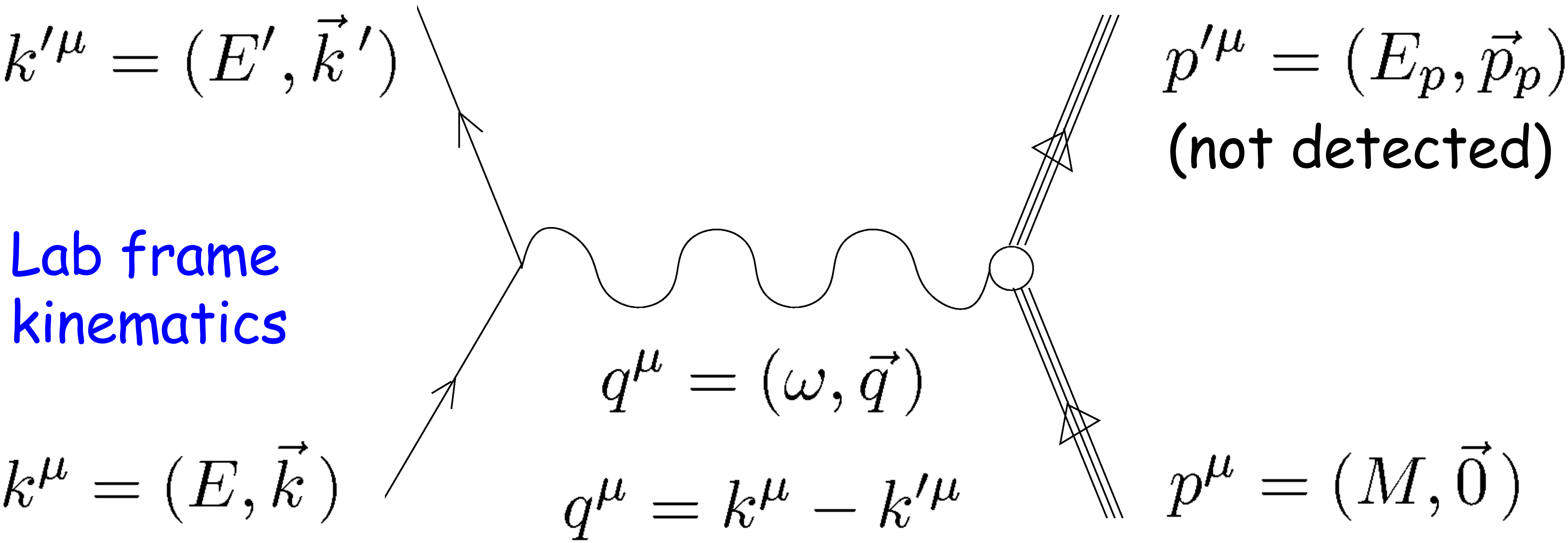
Select spatial resolution and excitation energy independently

- Photon energy ν determines excitation energy
- Photon momentum q determines spatial resolution: $\lambda \approx \dfrac{\hbar}{q}$

Three cases:

- Low q
  - Photon wavelength λ larger than the nucleon size (R_p)
- Medium q: $0.2 < q < 1$ GeV/c
  - $\lambda \sim R_p$
  - Nucleons resolvable
- High *q*: $q > 1$ GeV/c
  - $\lambda < R_p$
  - Nucleon structure resolvable

Inclusive electron scattering (e,e’)



Invariants:

$$p^{\mu} p_{\mu} = M^2 \qquad p_{\mu} q^{\mu} = M\omega$$

$$Q^2 = -q^{\mu} q_{\mu} = |\vec{q}|^2 - \omega^2 \qquad W^2 = (q^{\mu} + p^{\mu})^2 = p'_{\mu} p'^{\mu}$$

Elastic cross section ($p'^2 = m^2$)

Recoil factor   Form factors

$$\frac{d\sigma}{d\Omega} = \sigma_M \frac{E'}{E}\left\{\left[F_1^2(Q^2) + \frac{Q^2}{4M^2}\kappa^2 F_2^2(Q^2)\right] + \frac{Q^2}{2M^2}[F_1(Q^2) + \kappa F_2(Q^2)]^2 \tan^2\frac{\theta}{2}\right\}$$

$$= \sigma_M \frac{E'}{E}\left[\frac{G_E^2(Q^2) + \tau G_M^2(Q^2)}{1+\tau} + 2\tau \tan^2\frac{\theta}{2} G_M^2(Q^2)\right]$$

$$= \sigma_M \frac{E'}{E}\left[\frac{Q^4}{\vec{q}^4} R_L(Q^2) + \left(\frac{Q^2}{2\vec{q}^2} + \tan^2\frac{\theta}{2}\right) R_T(Q^2)\right]$$

Mott cross section

$$\sigma_M = \frac{\alpha^2 \cos^2\left(\frac{\theta_e}{2}\right)}{4E^2 \sin^4\left(\frac{\theta_e}{2}\right)}$$

F_1, F_2: Dirac and Pauli form factors

G_E, G_M: Sachs form factors (electric and magnetic)

$G_E(Q^2) = F_1(Q^2) - \tau\varkappa F_2(Q^2)$ $\tau = Q^2/4M^2$

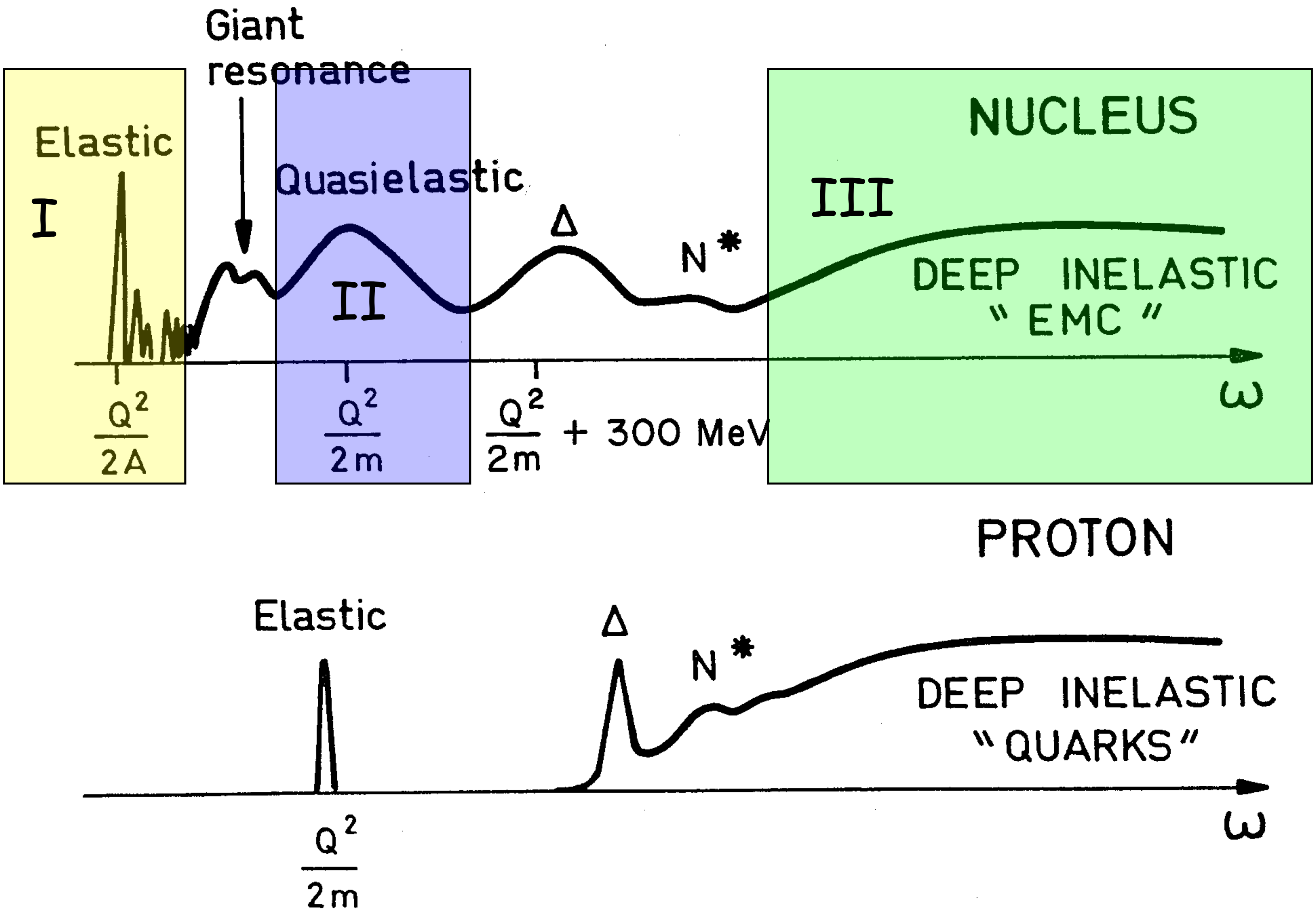
$G_M(Q^2) = F_1(Q^2) + \varkappa F_2(Q^2)$ $\varkappa$ = anomalous magnetic moment

R_L, R_T: Longitudinal and transverse response fn

Notes on form factors

- G_E, G_M, F_1 and F_2 refer to nucleons
  - $F_1^p(0) = 1$, $F_2^p(0) = \kappa_p = 1.79$
  - $F_1^n(0) = 0$, $F_2^n(0) = \kappa_n = -1.91$
  - $G_E^p(0) = 1$, $G_M^p(0) = 1 + \kappa_p = 2.79$
  - $G_E^n(0) = 0$, $G_M^n(0) = \kappa_n = -1.91$
- R_L and R_T refer to nuclei

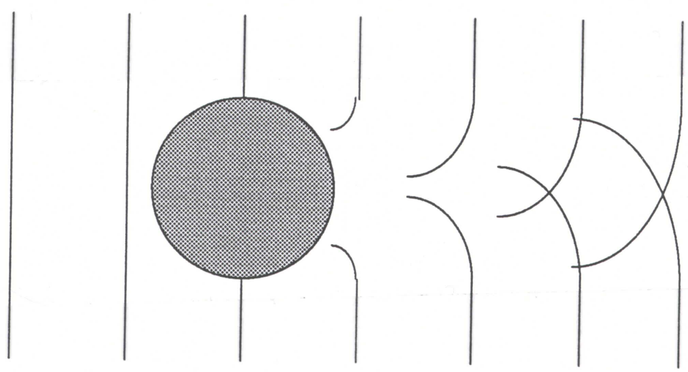
Electron-nucleus interactions



Electrons as Waves

Scattering process is quantum mechanical

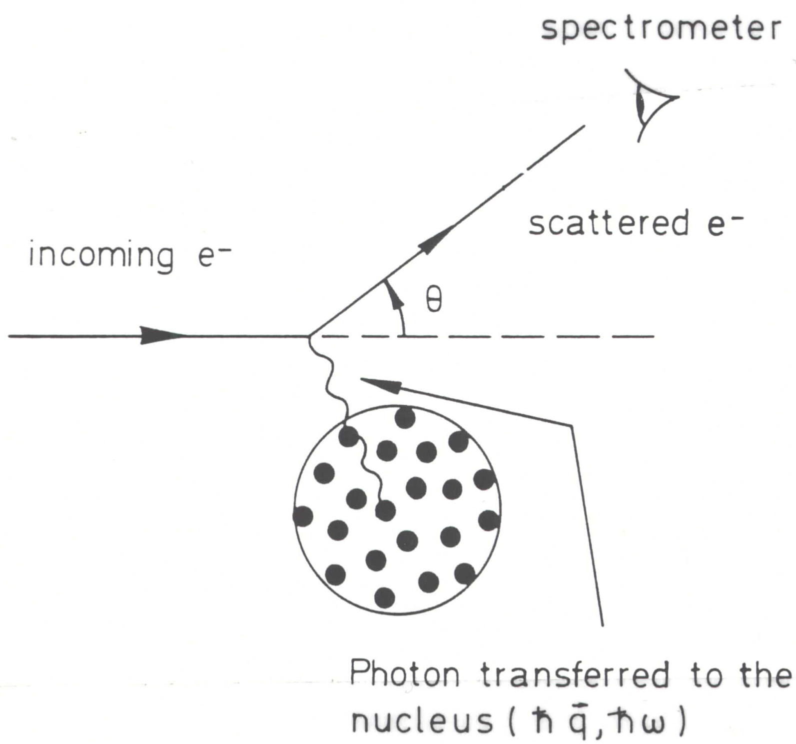
De broglie wavelength: $$\lambda = \frac{h}{p}$$

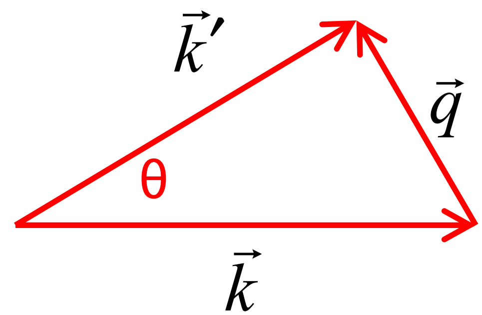


Electron energy: $$E_e \approx pc$$

λ resolving "scale": $$\lambda = \frac{2\pi(197\ \mathrm{MeV}\cdot\mathrm{fm})}{E_e}$$

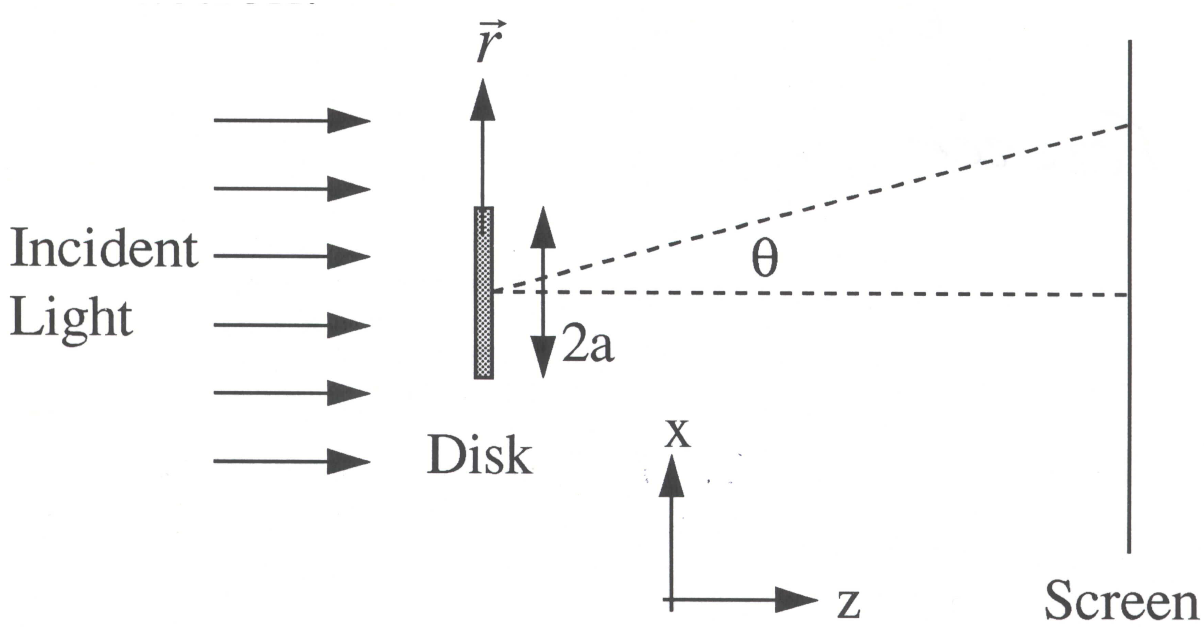
Analogy between elastic electron scattering and diffraction





Simple analogy for elastic electron scattering….

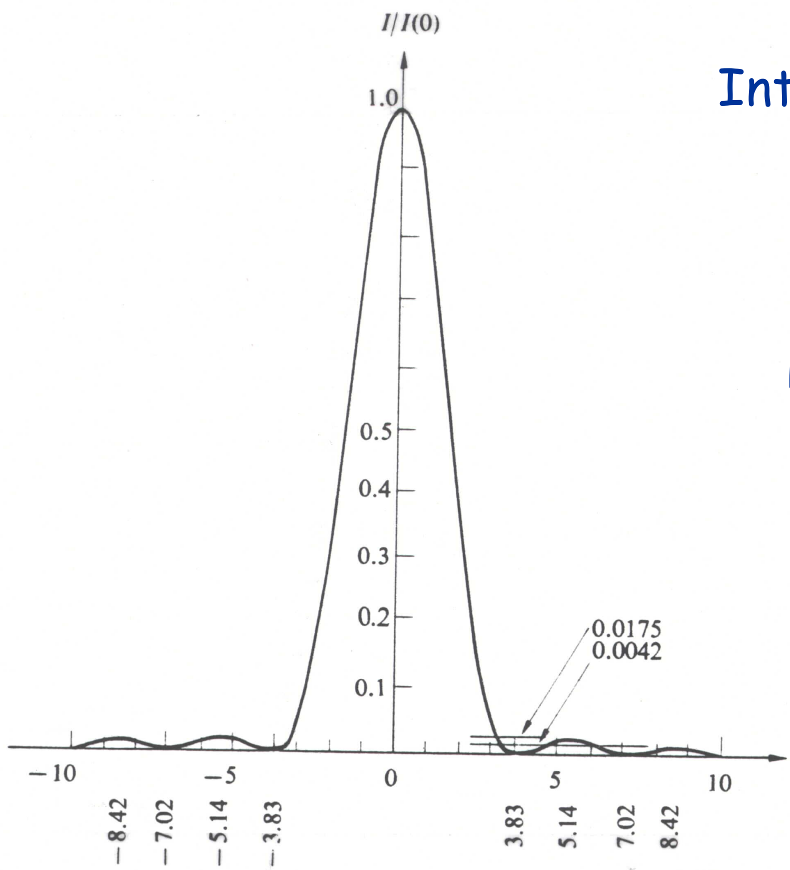
Classical Fraunhofer Diffraction



Amplitude of wave at screen:

$$\Phi \propto \int_0^a \int_0^{2\pi} \exp(ibr\cos\phi)\, r\, d\phi\, dr$$

Classical Fraunhofer Diffraction



Intensity: $$\Phi^2 \propto \left(\frac{J_1((2\pi a/\lambda)\sin\theta)}{\sin\theta} \right)^2$$

Minima occur at zeroes of
Bessel function. 1st zero: x = 3.8317

...some algebra...

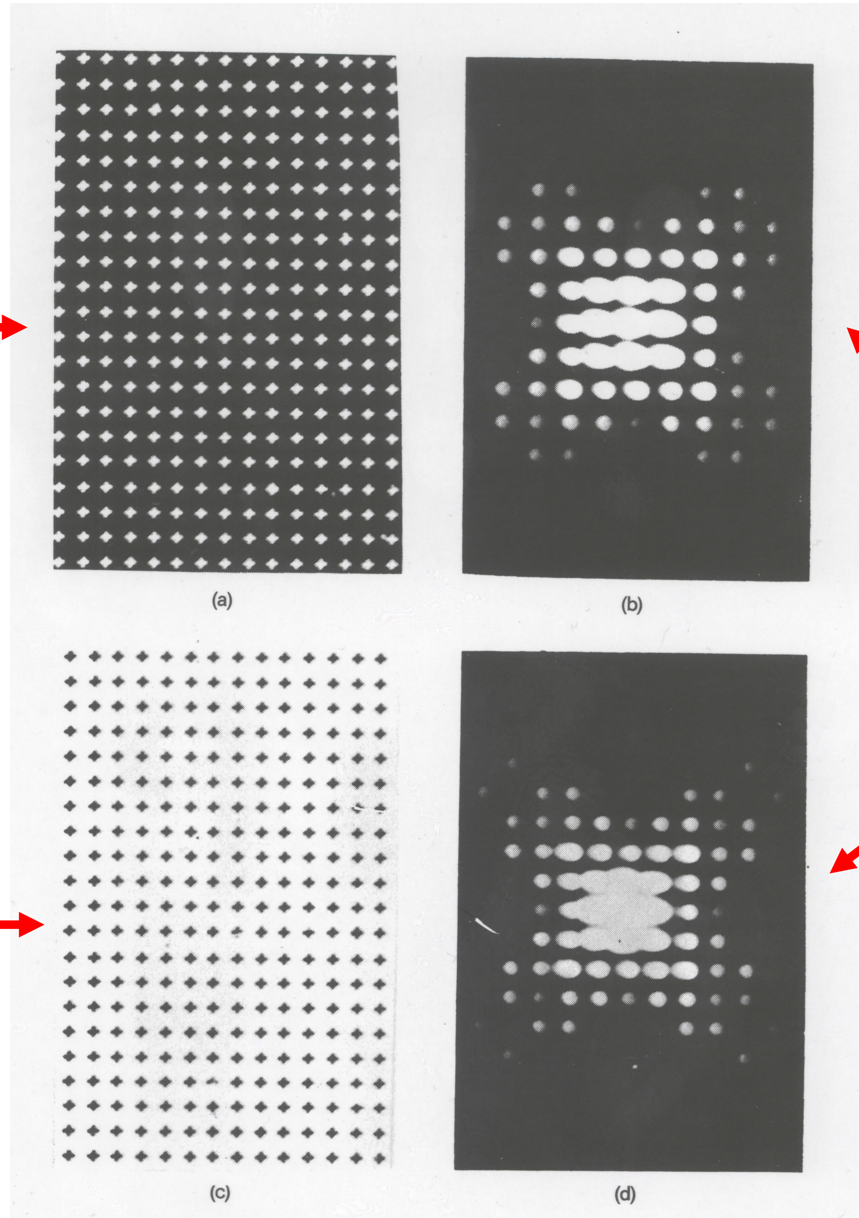
Hence $$2a \approx \frac{1.22\lambda}{\sin\theta}$$

Excursion: Babinet's principle

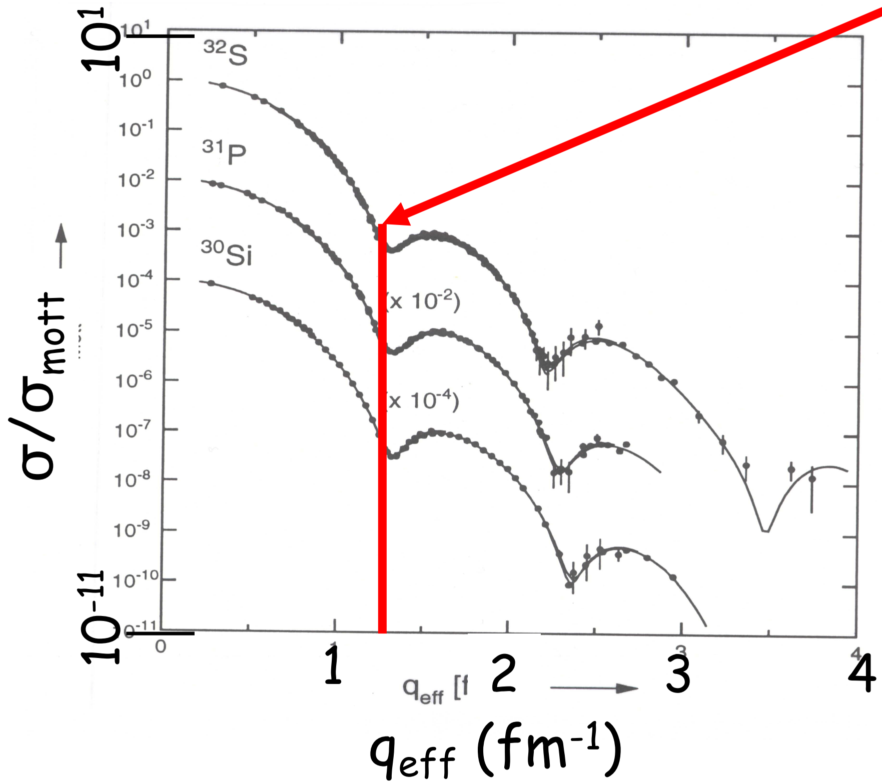
Screen with apertures

Patterns appear the same

Complementary screen



Example: $^{30}Si(e,e')$



1st minimum = 1.3 fm^{-1}

➔ $\theta = 32.8^o$

Electron energy = 454.3 MeV

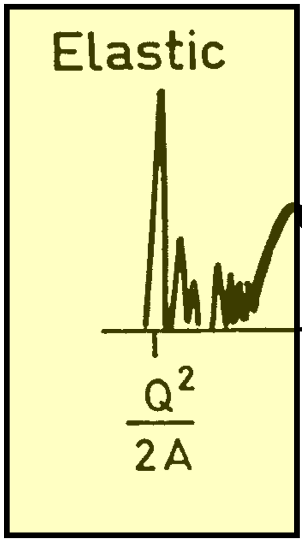
➔ $\lambda = 2.73$ fm

Calculated radius = 3.07 fm

Measured rms radius = 3.19 fm
(from fit to entire curve)

(1 fm^{-1} = 197 MeV/c)

I. Elastic Electron Scattering from Nuclei (done formally)

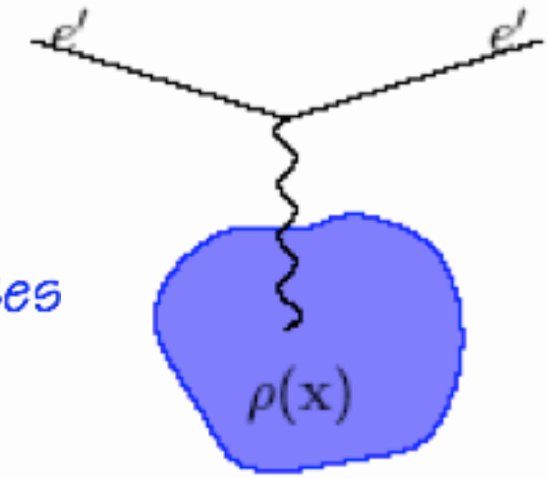


Fermi's Golden Rule

$$\frac{d\sigma}{d\Omega} = \frac{2\pi}{\hbar} \mid M_{fi} \mid^2 D_f$$

M_{fi}: scattering amplitude

D_f: density of the final states (or phase factor)



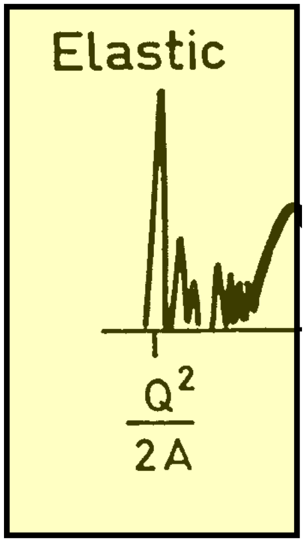
$$M_{fi} = \int \Psi_f^* V(x) \Psi_i d^3x$$

$$= \int e^{-k_f \cdot x} V(x) e^{-k_i \cdot x} d^3x$$

$$= \int e^{iq \cdot x} V(x) d^3x$$

Plane wave approximation for incoming and outgoing electrons
Born approximation (interact only once)

I. Elastic Electron Scattering from (spin-0) Nuclei



Form Factor and Charge Distribution

Using Coulomb potential from a charge distribution, ρ(x),

$$V(x) = -\frac{Ze^2}{4\pi\epsilon_0}\int \frac{\rho(x')}{|x-x'|}d^3x'$$

$$M_{fi} = -\frac{Ze^2}{4\pi\epsilon_0}\int e^{iq\cdot x}\int \frac{\rho(x')}{|x-x'|}d^3x'\,d^3x$$

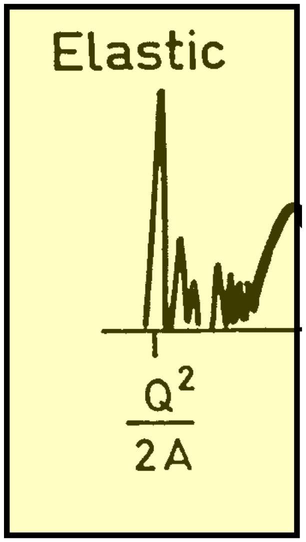
$$= -\frac{Ze^2}{4\pi\epsilon_0}\int e^{iqR}\left[\int \frac{e^{iq\cdot x'}\rho(x')}{|R|}d^3x'\right]d^3R$$

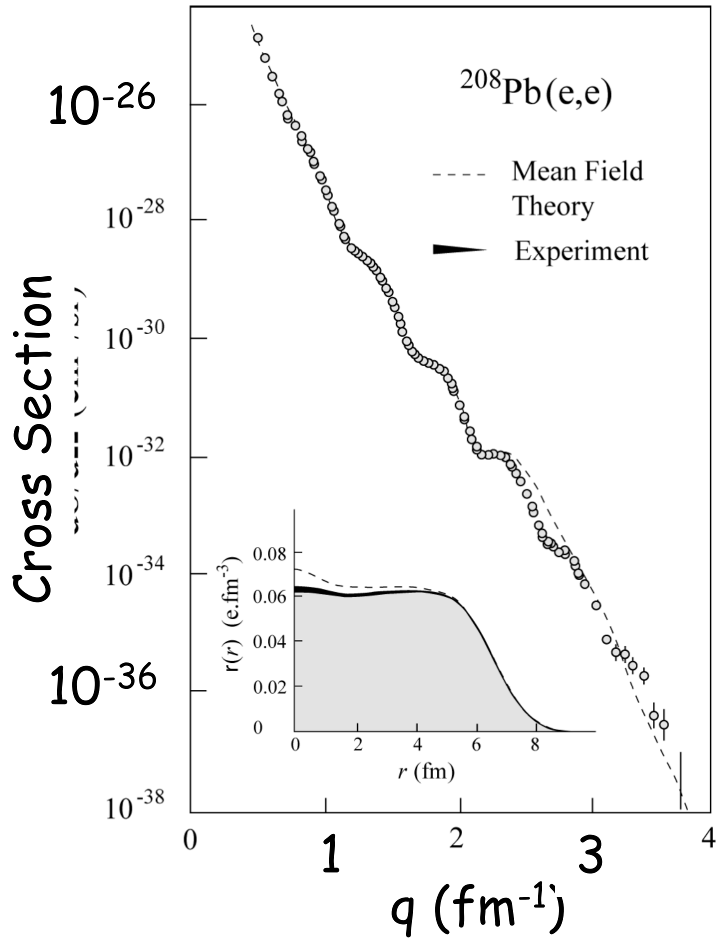
$$= -\frac{Ze^2}{4\pi\epsilon_0}\int \frac{e^{iqR}}{R}d^3R\int e^{iq\cdot x'}\rho(x')d^3x'$$

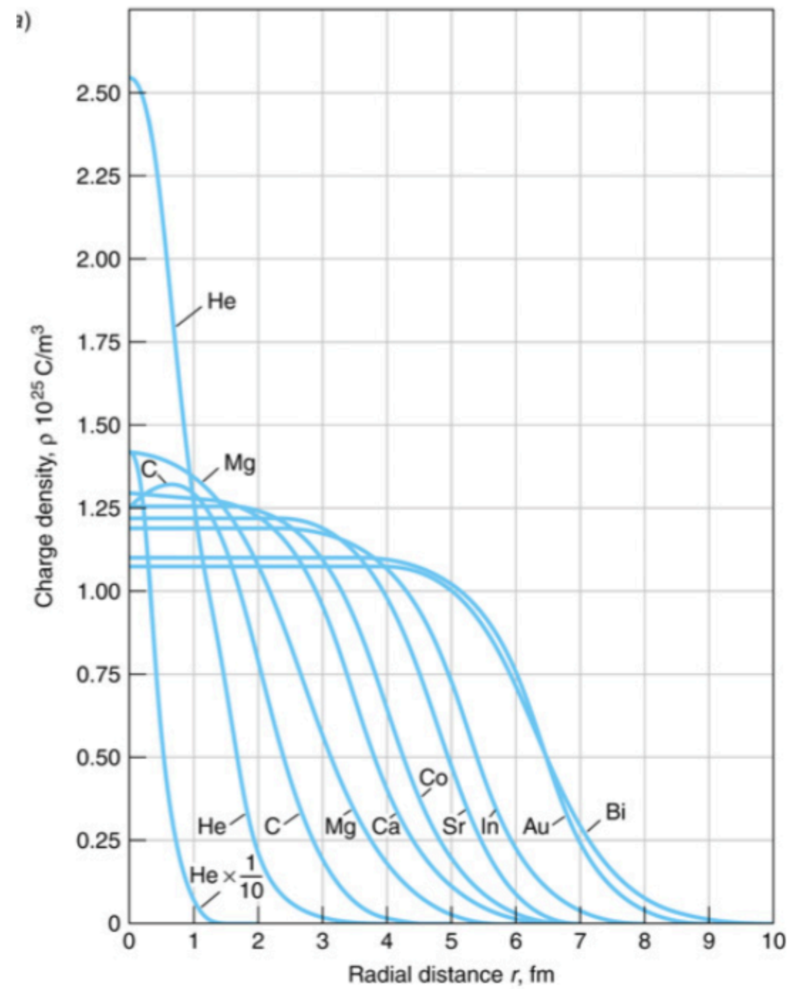
$$F(q) = \int e^{iq\cdot x'}\rho(x')d^3x'$$

Charge form factor F(q)
is the Fourrier transform
of the charge distribution ρ(x)

I. Elastic (e,e') Scattering ⇒ charge distributions



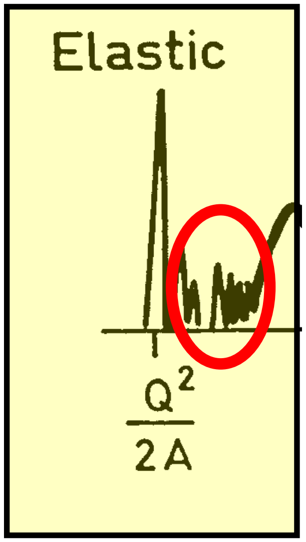


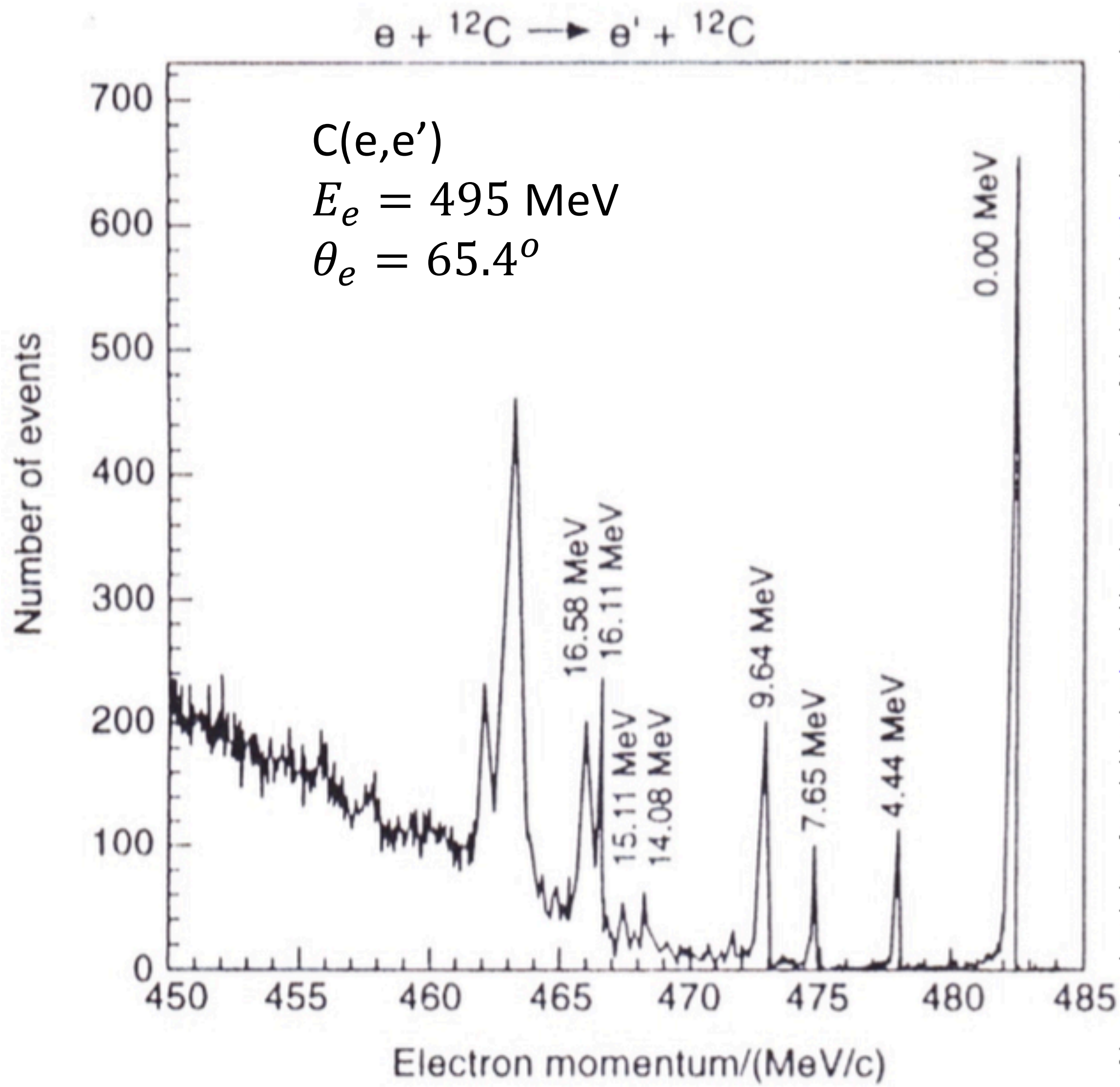


Elastic electron scattering measured for many nuclei over a wide range of Q2(mainly at Saclay in the 1970s)

Measured charge distributions agree well with mean field theory calculations.

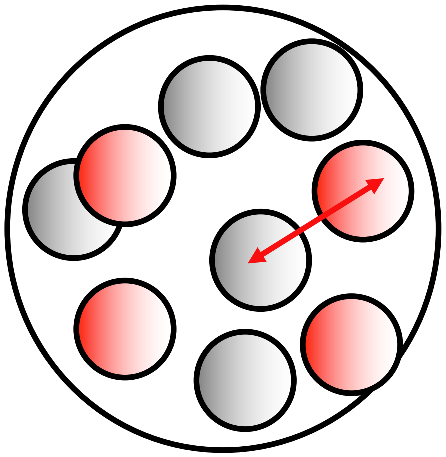
Inelastic Scattering from Nuclei





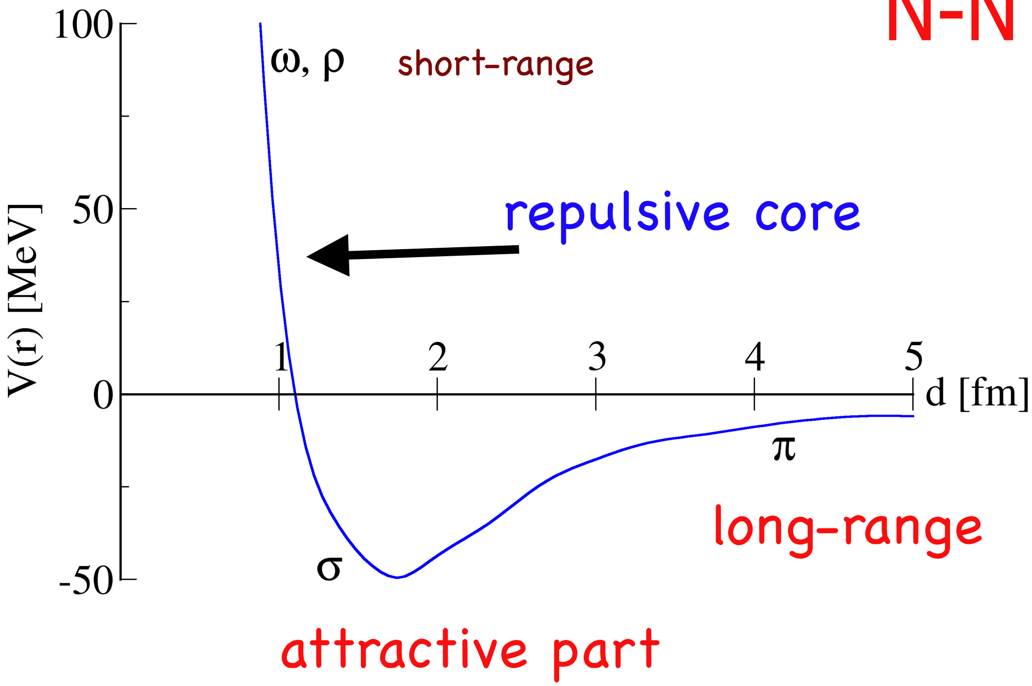
Measure spin/parity of excited states and transition matrix elements

Structure of the nucleus

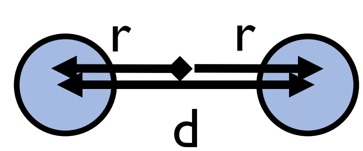


- nucleons are bound
  - energy (E) distribution
  - shell structure
- nucleons are not static
  - momentum (k) distribution

determined by the N-N potential

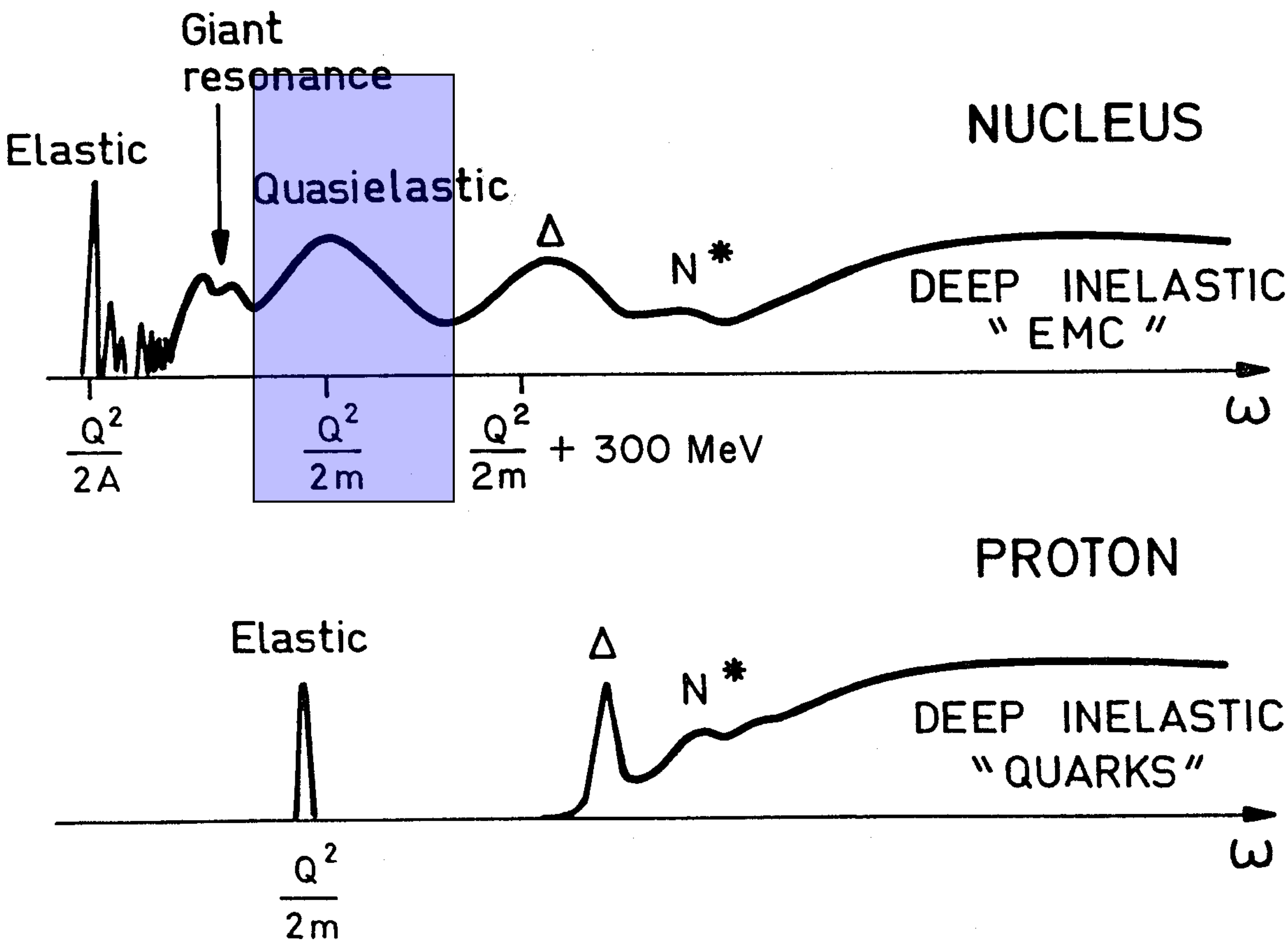


on average:
Net binding energy: ≈ 8 MeV
distance: ≈ 2 fm



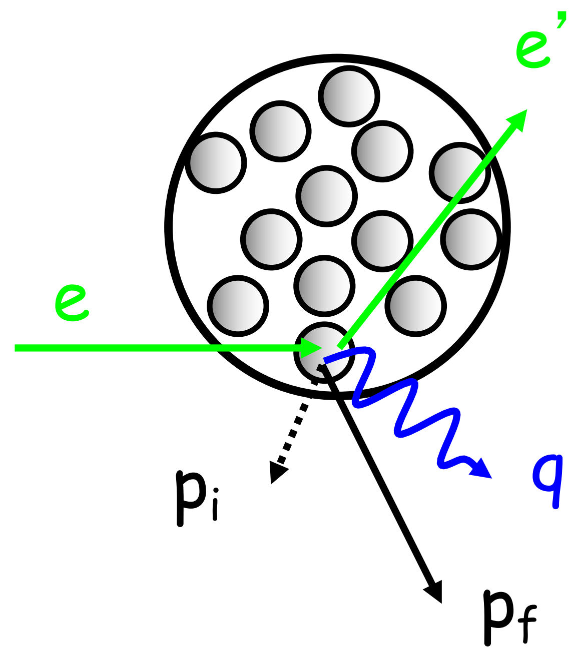
Strong repulsion ⟹ NN correlations

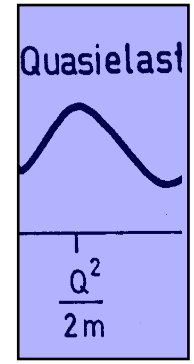
II. Quasielastic scattering



Fermi gas model:

how simple a model can you make ?





Initial nucleon energy: $KE_i = p_i^2 / 2m_p$

Final nucleon energy: $KE_f = p_f^2 / 2m_p = (\vec{q} + \vec{p}_i)^2 / 2m_p$

Energy transfer: $\nu = KE_f - KE_i = \dfrac{\vec{q}^{\,2}}{2m_p} + \dfrac{\vec{q} \cdot \vec{p}_i}{m_p}$

Expect:

- Peak centroid at $\nu = q^2/2m_\mathrm{p} + \varepsilon$
- Peak width $2qp_\mathrm{fermi}/m_\mathrm{p}$
- Total peak cross section = $Z\sigma_\mathrm{ep} + N\sigma_\mathrm{en}$

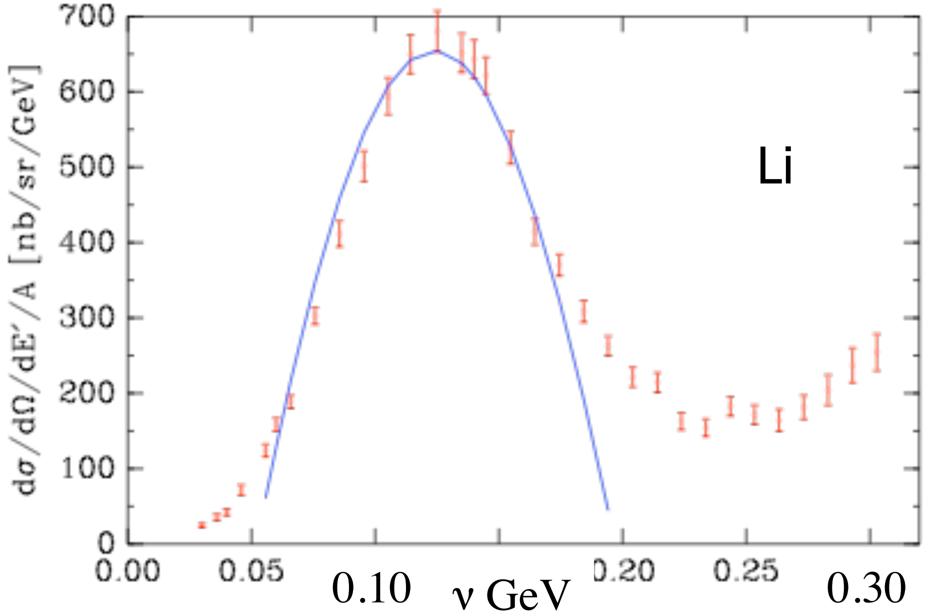
Early 1970's Quasielastic Data

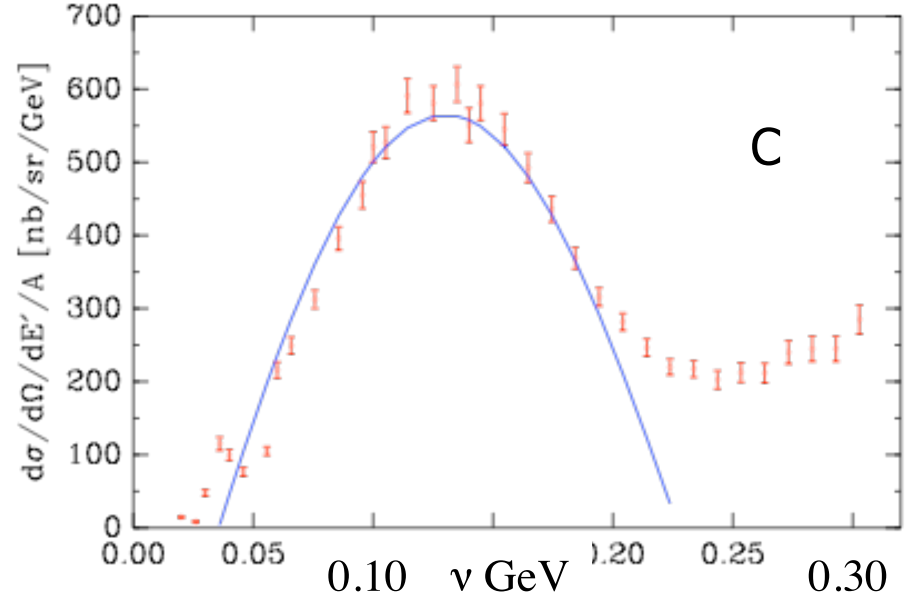
-> getting the bulk features

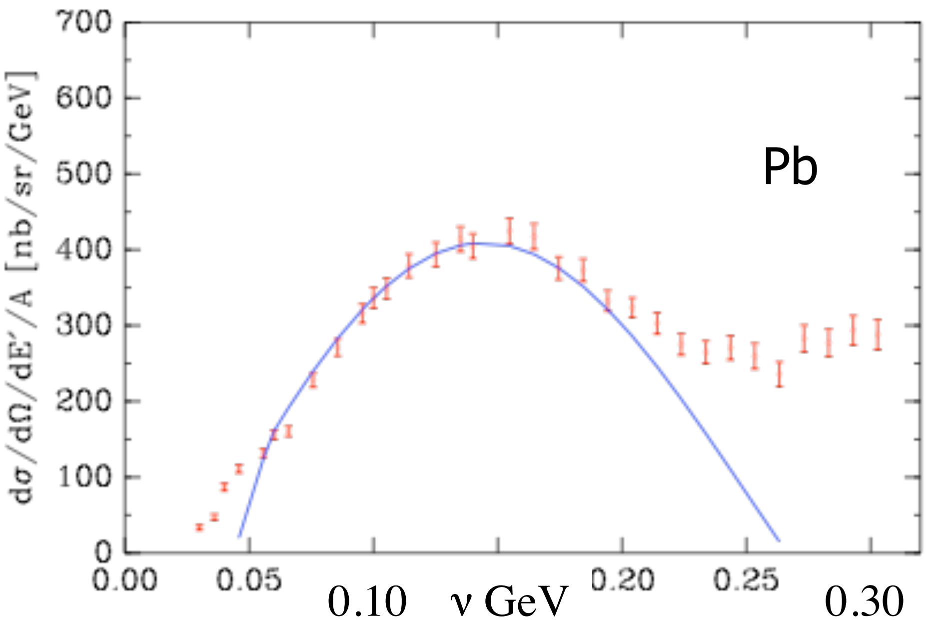
500 MeV, 60 degrees

$\vec{q} \simeq 500 MeV/c$

R.R. Whitney et al., PRC 9, 2230 (1974).







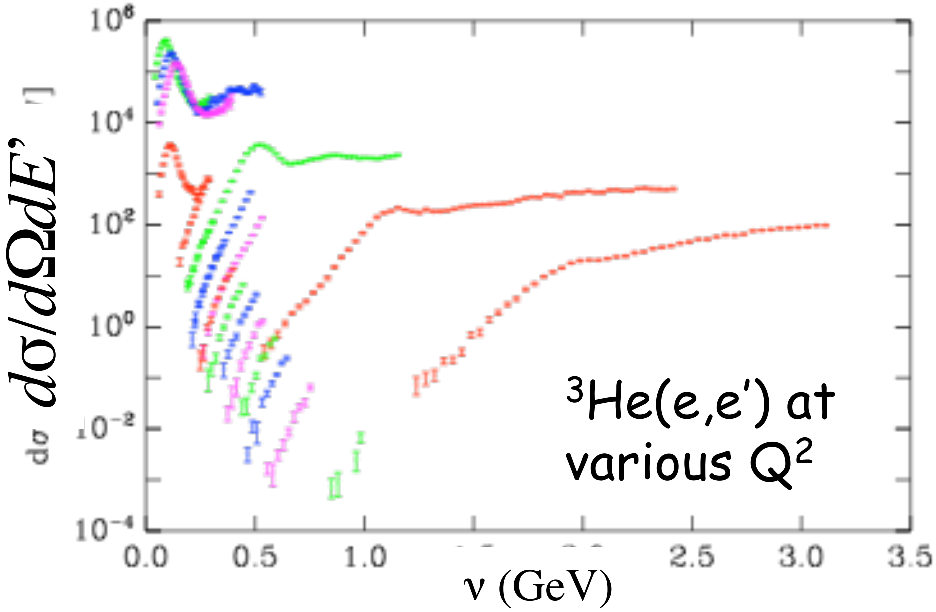
Nucleus	k_F MeV/c	$\bar{\epsilon}$ MeV
^{6}Li	169	17
^{12}C	221	25
^{24}Mg	235	32
^{40}Ca	251	28
^{nat}Ni	260	36
^{89}Y	254	39
^{nat}Sn	260	42
^{181}Ta	265	42
^{208}Pb	265	44

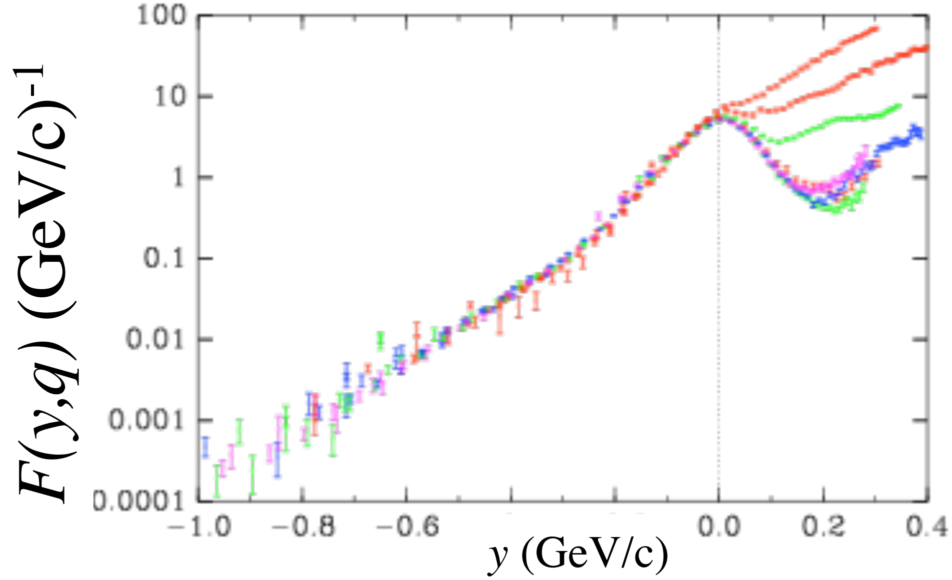
compared to Fermi model:fit parameter k_F and ε

Scaling

- The dependence of a cross section, in certain kinematic regions, on a single variable.
  - If the data scales, it validates the scaling assumption
  - Scale-breaking indicates new physics
- At moderate Q^2 and $x>1$ we expect to see evidence for y-scaling, indicating that the electrons are scattering from quasifree nucleons
  - y = minimum momentum of struck nucleon
- At high Q^2 we expect to see evidence for x-scaling, indicating that the electrons are scattering from quarks. (next lecture)
  - $x = Q^2/2m\nu$ = fraction of nucleon momentum carried by struck quark (in infinite momentum frame)

y-scaling in inclusive electron scattering from ^{3}He





$$F(y) = \frac{\sigma^{\exp}}{(Z\tilde{\sigma}_p + N\tilde{\sigma}_n)} \cdot K$$

$$n(k) = -\frac{1}{2\pi y}\frac{dF(y)}{dy}$$

Assumption: scattering takes place from a quasi-free proton or neutron in the nucleus.

y is the momentum of the struck nucleon parallel to the momentum transfer:

$y \approx -q/2 + m\nu/q$ (nonrelativistically)

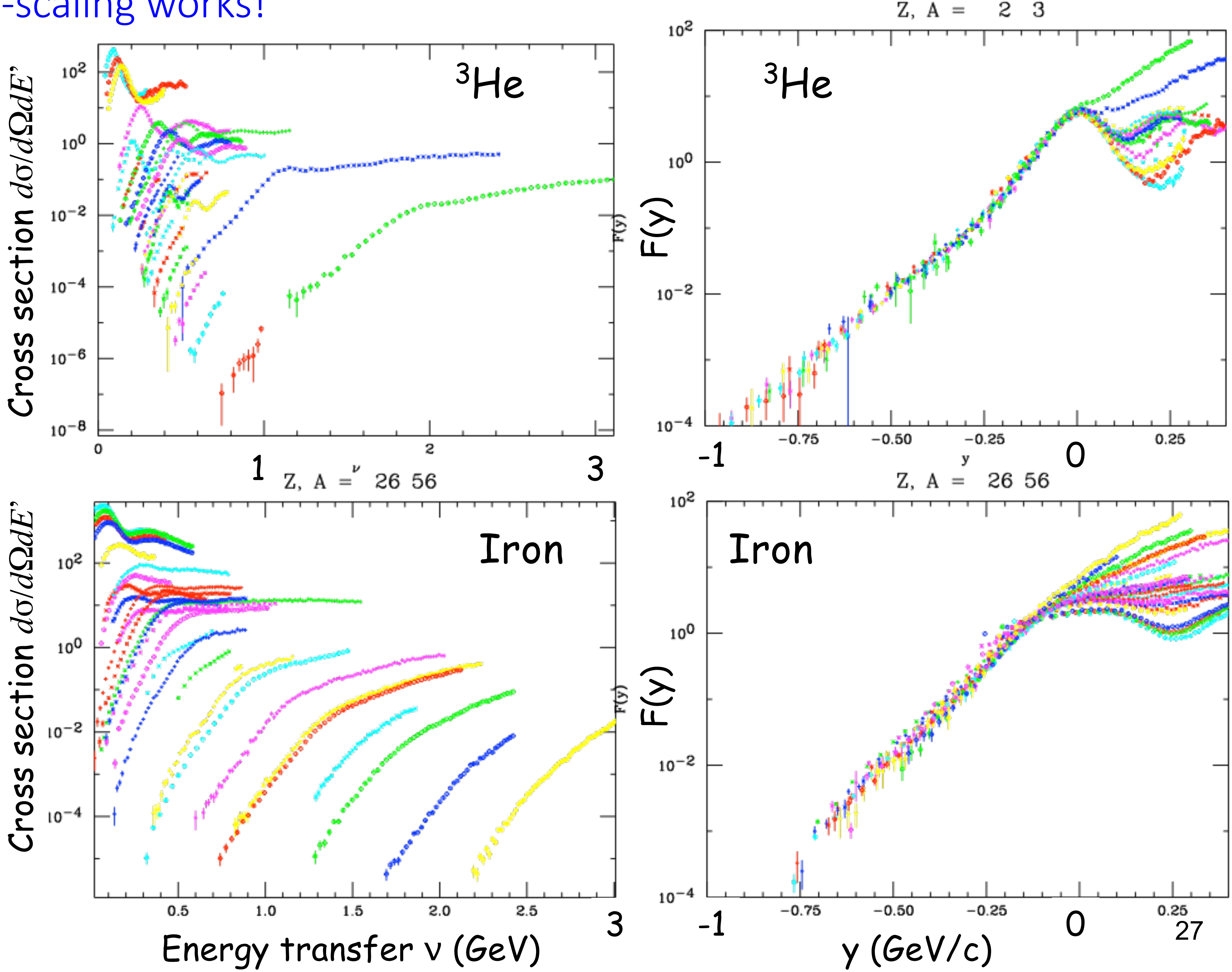
IF the scattering is quasifree, then $F(y)$ is the integral over all perpendicular nucleon momenta (nonrelativistically).

Goal: extract the momentum distribution $n(k)$ from $F(y)$.

Assumptions & Potential Scale Breaking Mechanisms

- No Final State Interactions (FSI)
- No internal excitation of (A-1)
- Full strength of Spectral function can be integrated over at finite *q*
- No inelastic processes (choose $y<0$)
- No medium modifications (discussed later)

Y-scaling works!

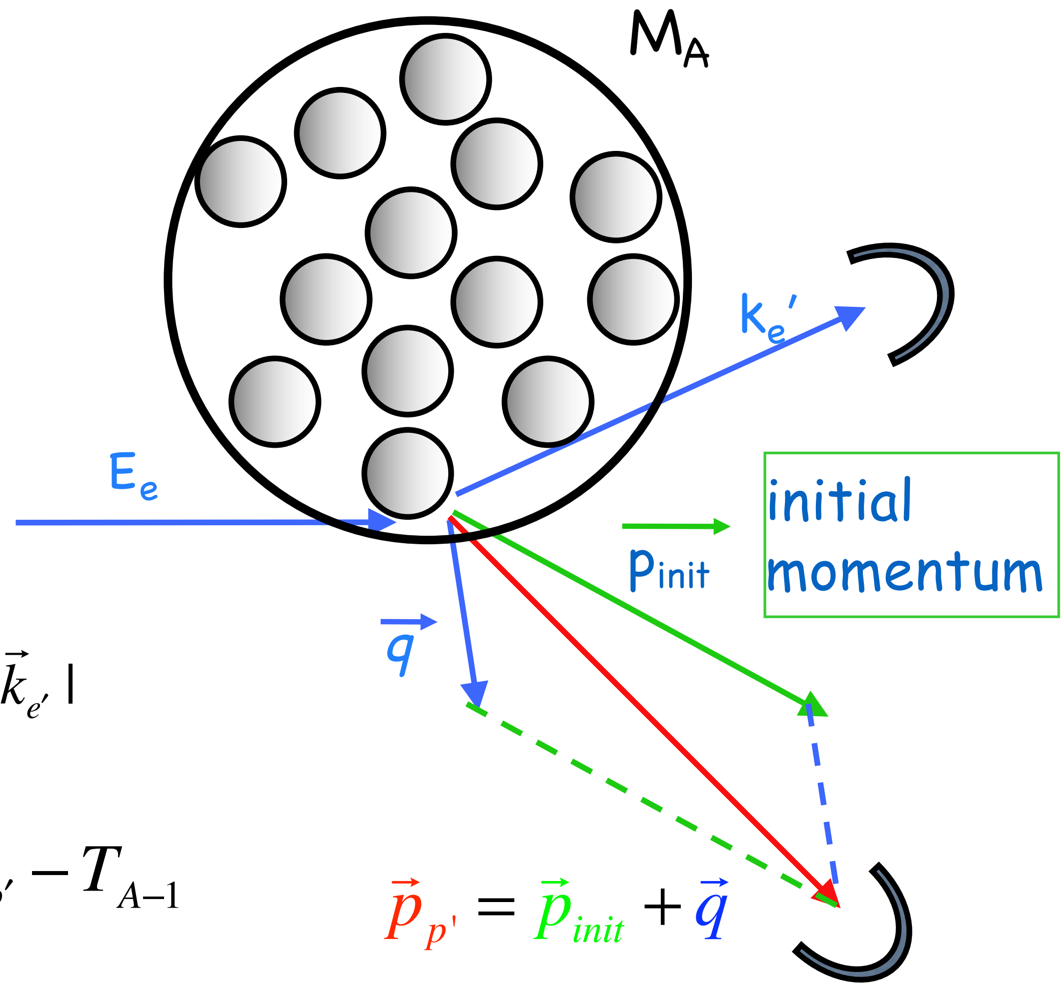


Get more information:
Detect the knocked out nucleon (e,e'p)

coincidence experiment

measure: momentum, angles

electron energy: E_e
proton: $\vec{p}_{p'}$
scattered electron: $\vec{k}_{e'}$ $\quad E_{e'} = |\vec{k}_{e'}|$



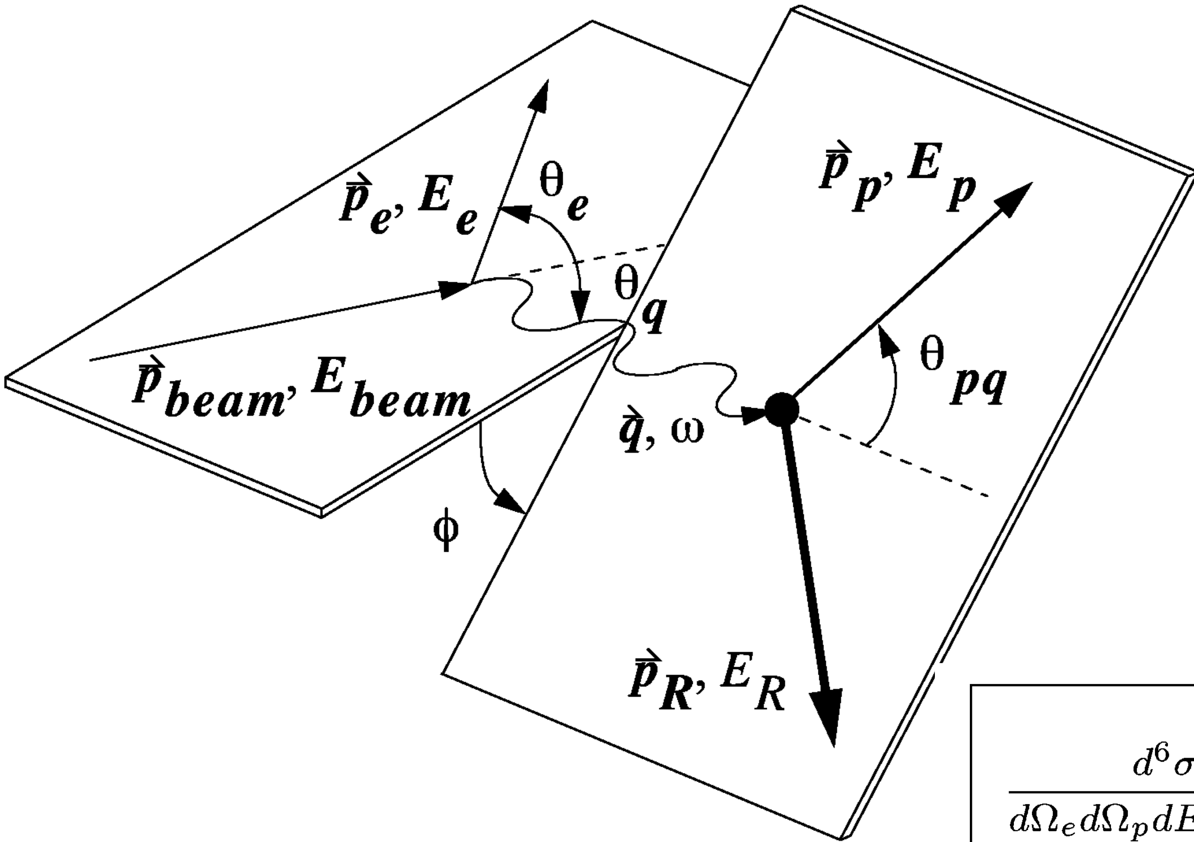
reconstructed quantites:

missing energy: $E_m = \nu - T_{p'} - T_{A-1}$

missing momentum: $\vec{p}_m = \vec{q} - \vec{p}_{p'}$

in Plane Wave Impulse Approximation (PWIA):
direct relation between measured quantities and theory:

$$|E| = E_m \qquad \vec{p}_{init} = -\vec{p}_m$$



$$\frac{d^6\sigma}{d\Omega_e d\Omega_p dE_{\rm miss} d\omega} = K\sigma_{\rm Mott}\,[\, v_L \mathbf{R_L} + v_T \mathbf{R_T} + v_{LT}\mathbf{R_{LT}}\cos(\phi) + v_{TT}\mathbf{R_{TT}}\cos(2\phi)\,]$$

where

K = (phase space)

$\sigma_{\rm Mott}$ = (relativistic Rutherford scattering)

$v = v\ (q,\omega)$ (electron kinematics)

And then there were four
(response functions, that is)

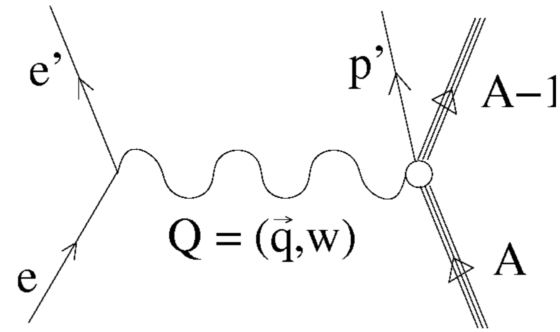
Each R now depends on more variables
$R = R(q, \omega, p_{miss}, E_{miss})$

(When you include electron and proton spin, there are 18. Yikes!)

(And if you scatter from a polarized spin-1 target, there are 41. Double Yikes!!)

(e,e'p) Plane Wave Impulse Approximation (PWIA)

1. Only one nucleon absorbs the virtual photon
2. That nucleon does not interact further
3. That nucleon is detected

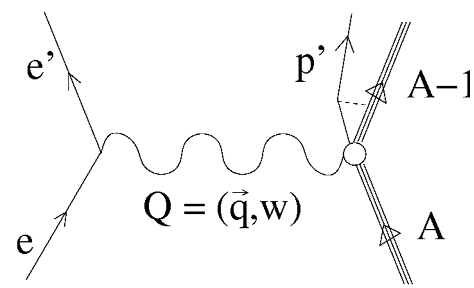


Cross section factorizes:

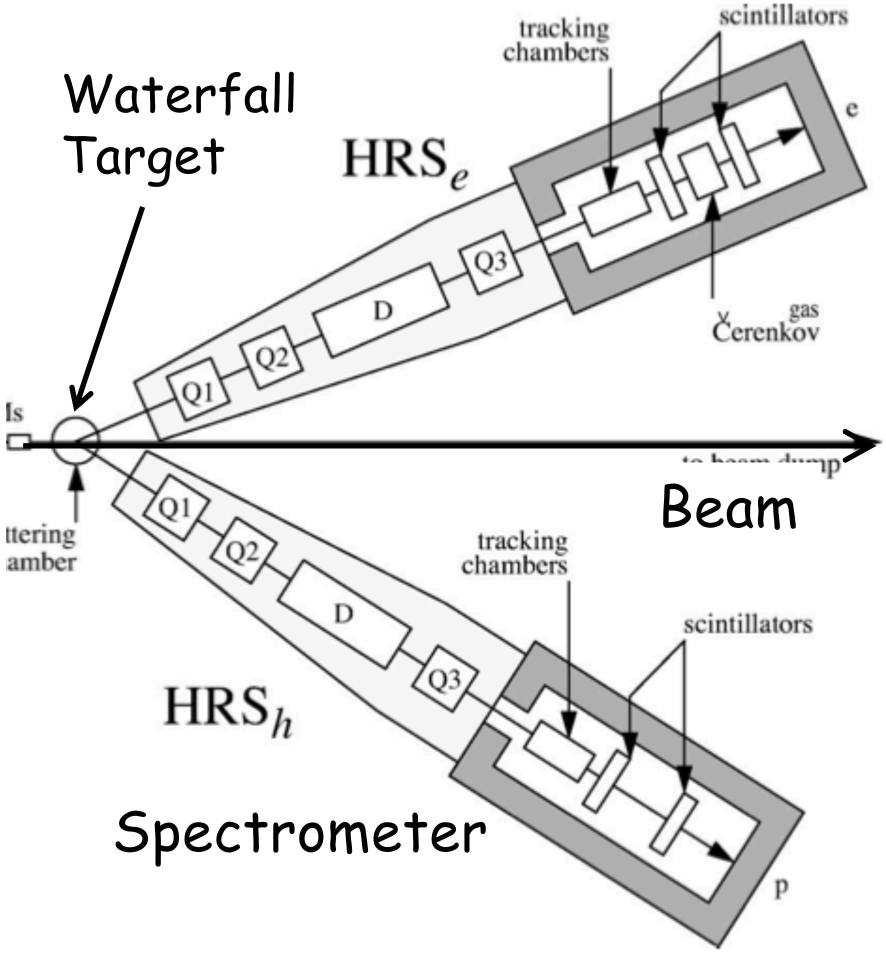
$$\frac{d\sigma^{fi}}{dE_1 d\Omega_1 dE_2 d\Omega_2} = KS(\vec{k},E)\frac{d\sigma^{free}}{d\Omega}$$

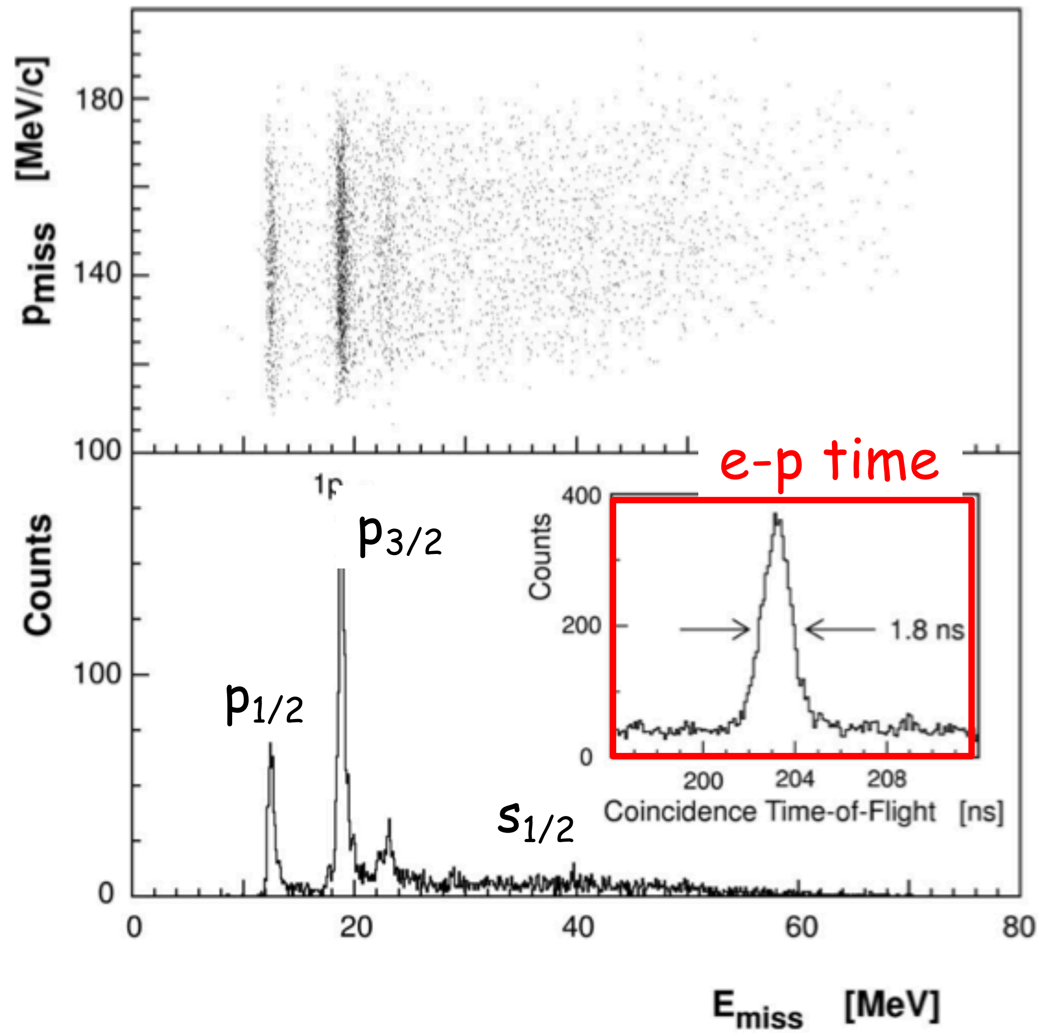
Single nucleon pickup reactions [eg: (p,d), (d,^{3}He) …] are also sensitive to S(p,E) but only sensitive to surface nucleons due to strong absorption in the nucleus

DWIA: If the struck nucleon interacts with the rest of the nucleus, then the cross section still factorizes (usually) but we measure a distorted spectral function.



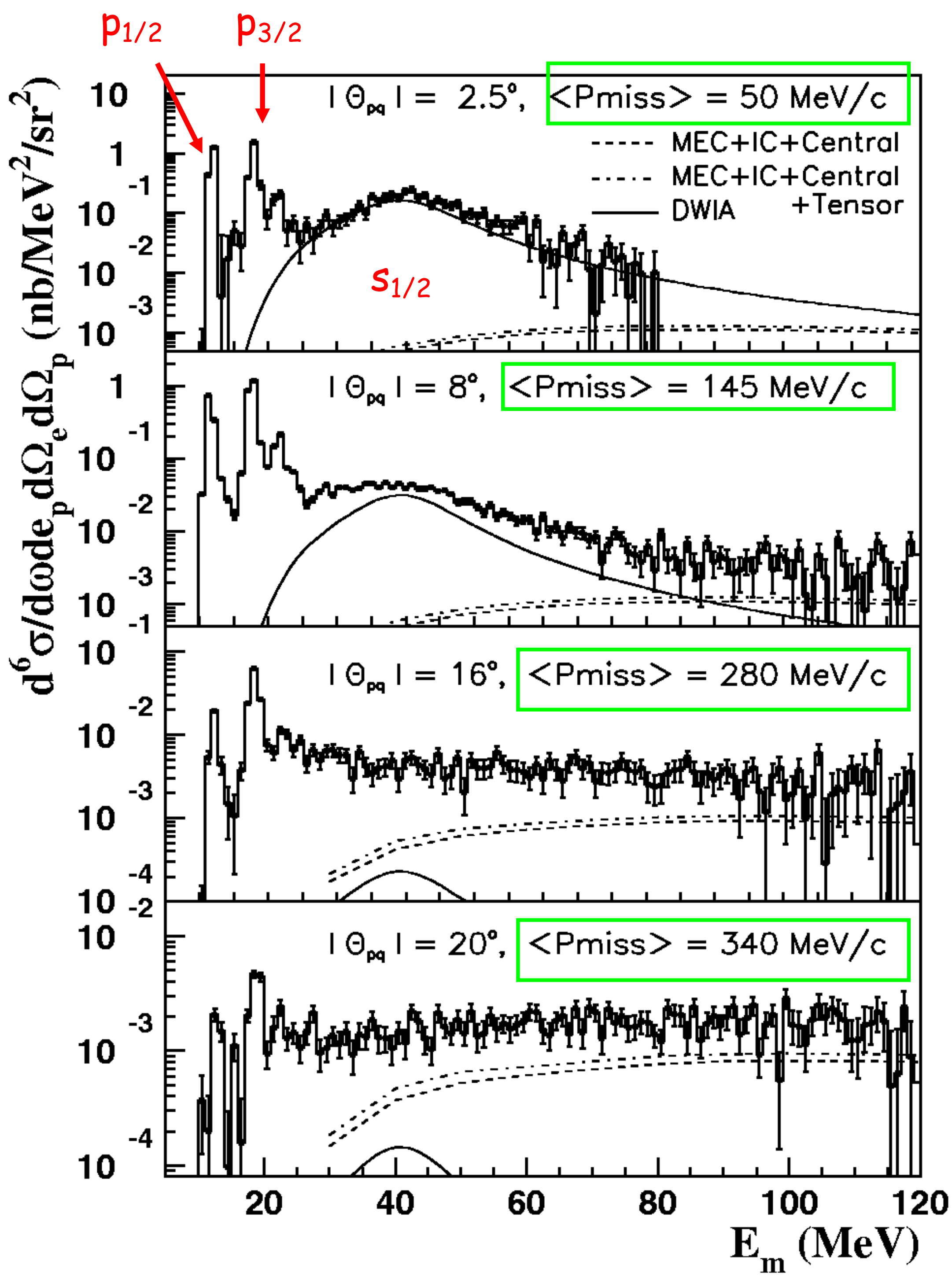
Measuring O(e,e'p) in Hall A

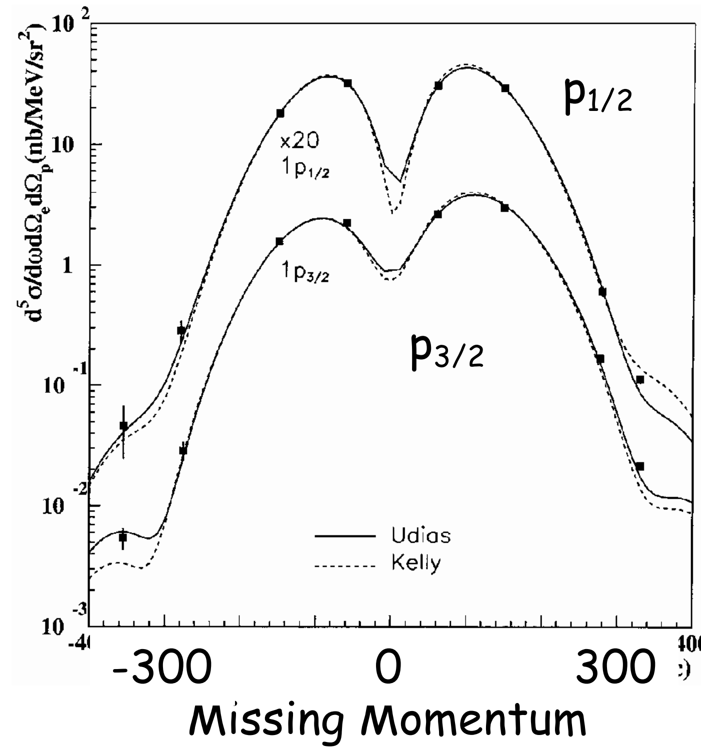




Fissum et al, PRC 70, (2004) 034606

O(e,e'p) and shell structure



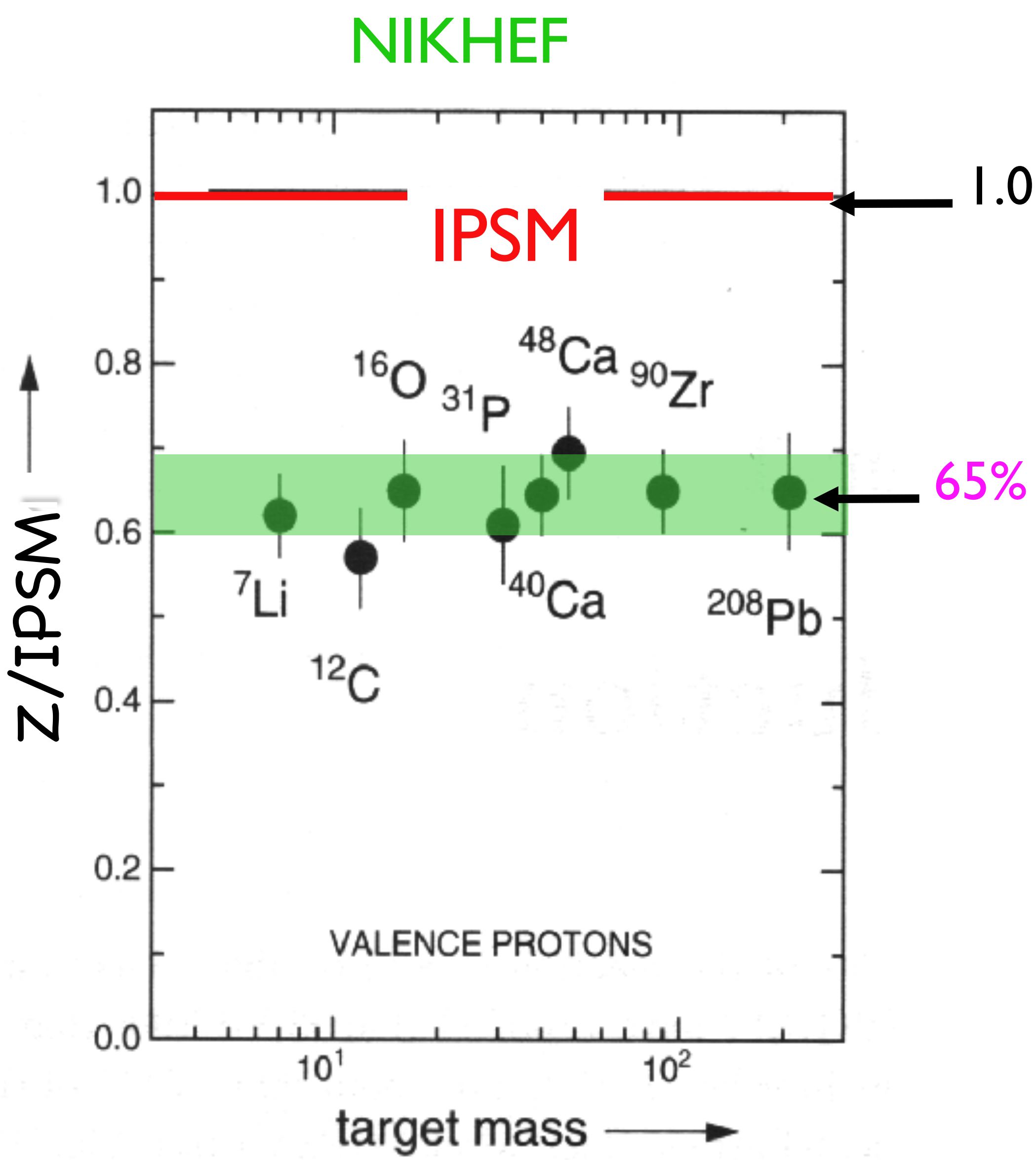


$1p_{1/2}$, $1p_{3/2}$ and $1s_{1/2}$ shells visible

Momentum distribution as expected for $l = 0, 1$

Fissum et al, PRC 70, 034606 (2003)

But we do not see enough protons!



(e,e'p) summary

- Measure shell structure directly
- Measure nucleon momentum distributions
- But:
  - Not enough nucleons seen!

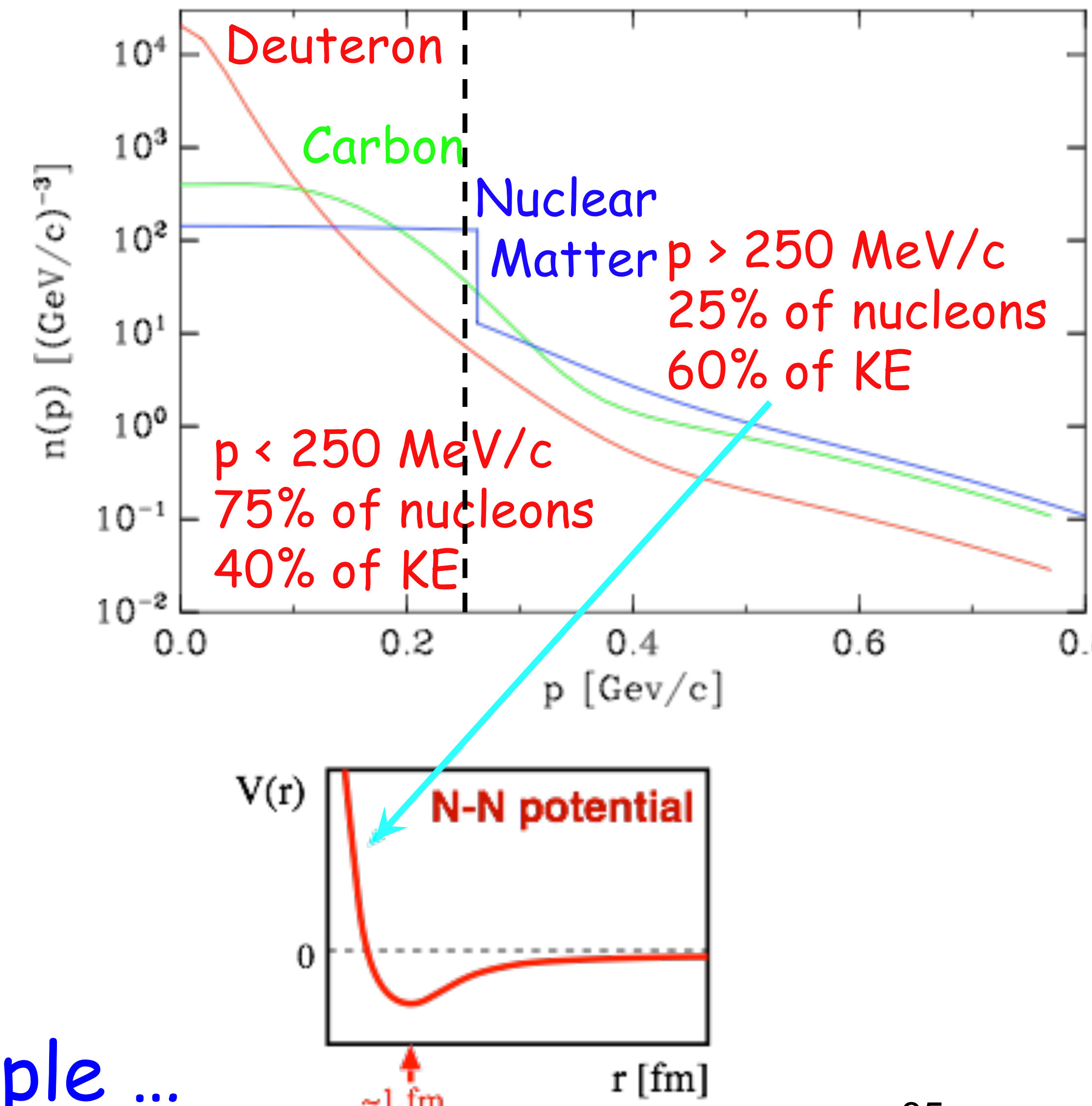
Short Range Correlations (SRCs)

Mean field contributions: $p < p_{Fermi} \approx 250$ MeV/c
Well understood, Spectroscopic Factors ≈ 0.65

High momentum tails: $p > p_{Fermi}$
Calculable for few-body nuclei, nuclear matter.
Dominated by two-nucleon short range correlations

Poorly understood part of nuclear structure
NN potential models not applicable at p > 350 MeV/c

Uncertainty in SR interaction leads to uncertainty at $p > p_{Fermi}$, even for simplest systems



Nucleons are like people …

35

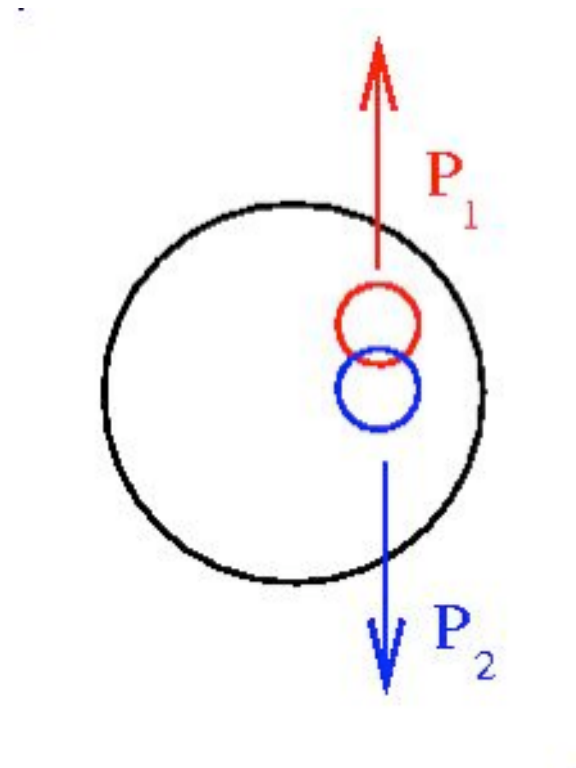
What are Correlations?

Average Two Nucleon Properties in the Nuclear Ground State

Not Two-Body Currents

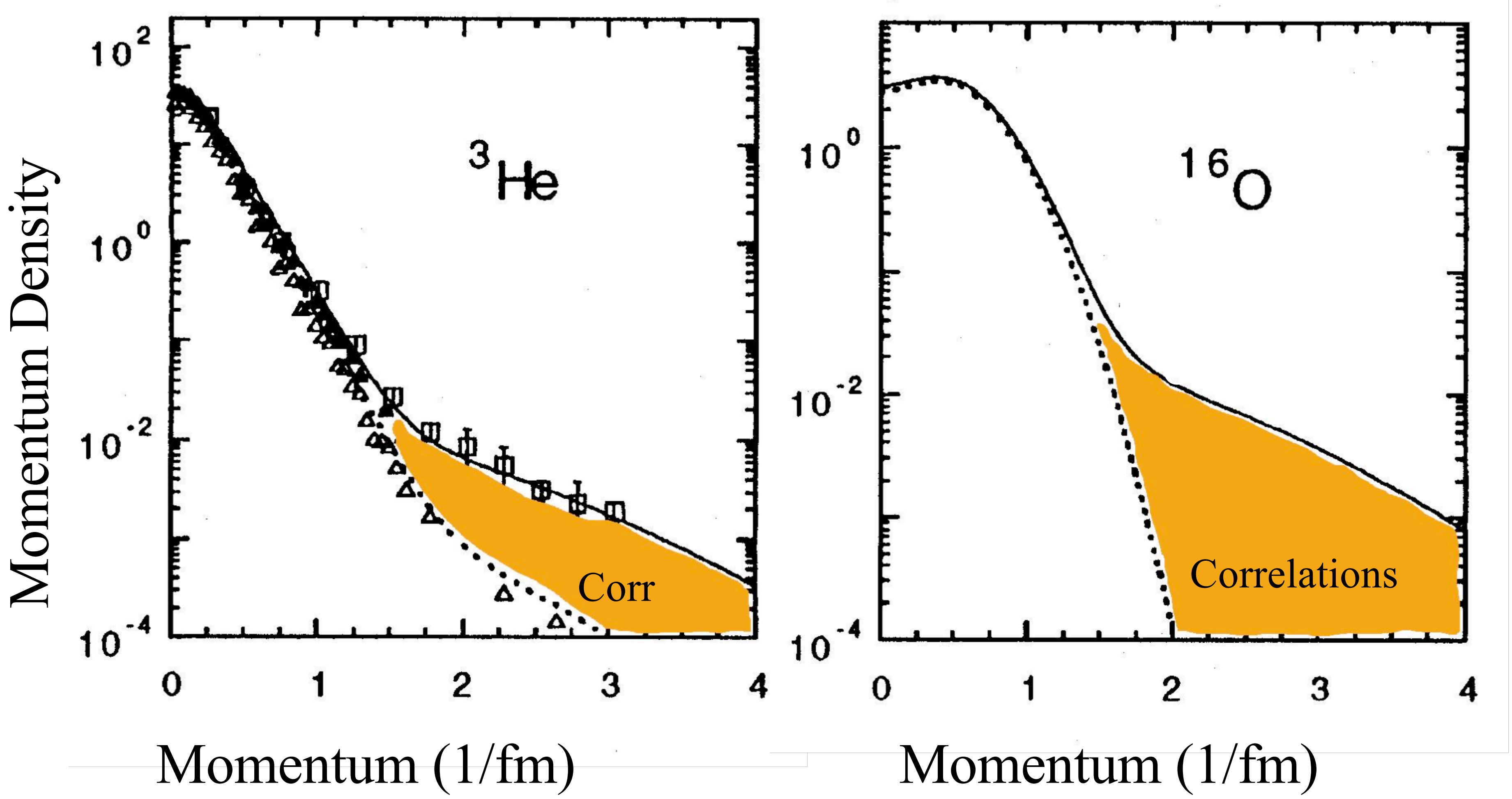
An Experimentalist's Definition:

- A high momentum nucleon whose momentum is balanced by **one** other nucleon
  - NN Pair with
    - Large Relative Momentum
    - Small Total Momentum
- Whatever a theorist says it is



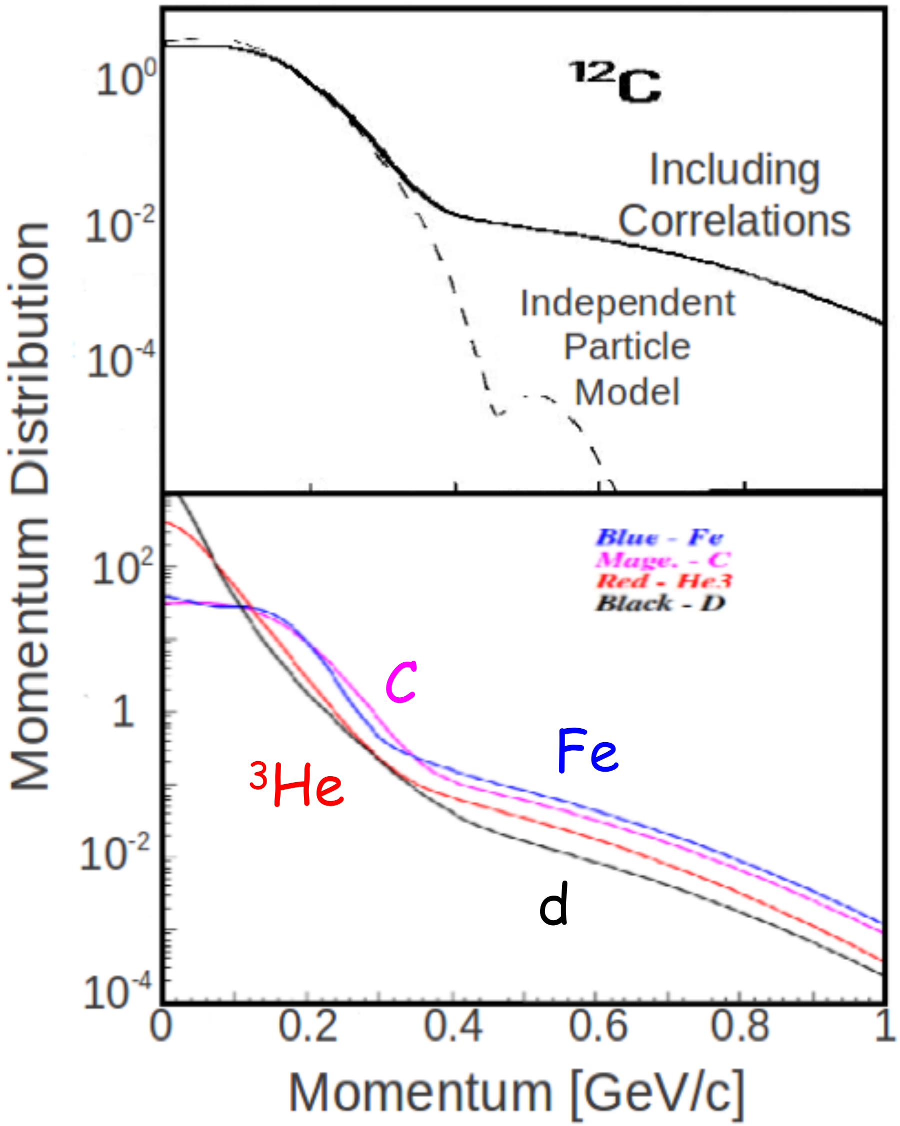
Why are Correlations Interesting?

Responsible for high momentum part of Nuclear WF



Ciofi degli Atti, PRC 53 (1996) 1689

Correlations should be universal



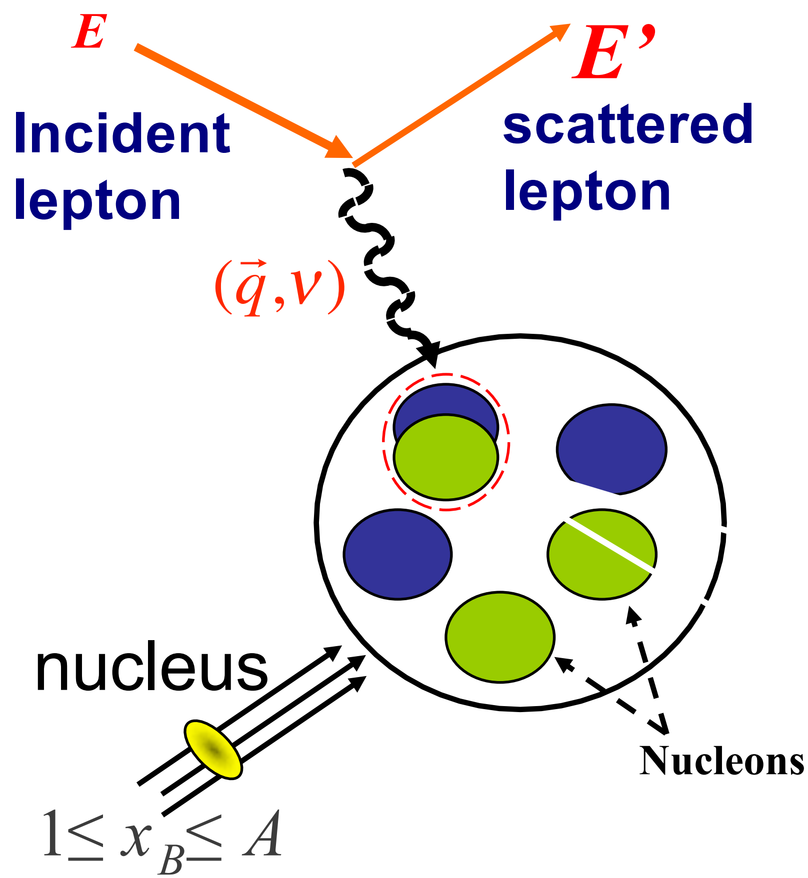
Many-body calculations predict that the high momentum distribution for all nuclei has the same shape:

$n_A(k)/n_d(k) = a_2(A/d)$

O. Benhar, Phys Lett B **177** (1986) 135
C. Ciofi degli Atti, Phys Rev C **53** (1996) 1689.

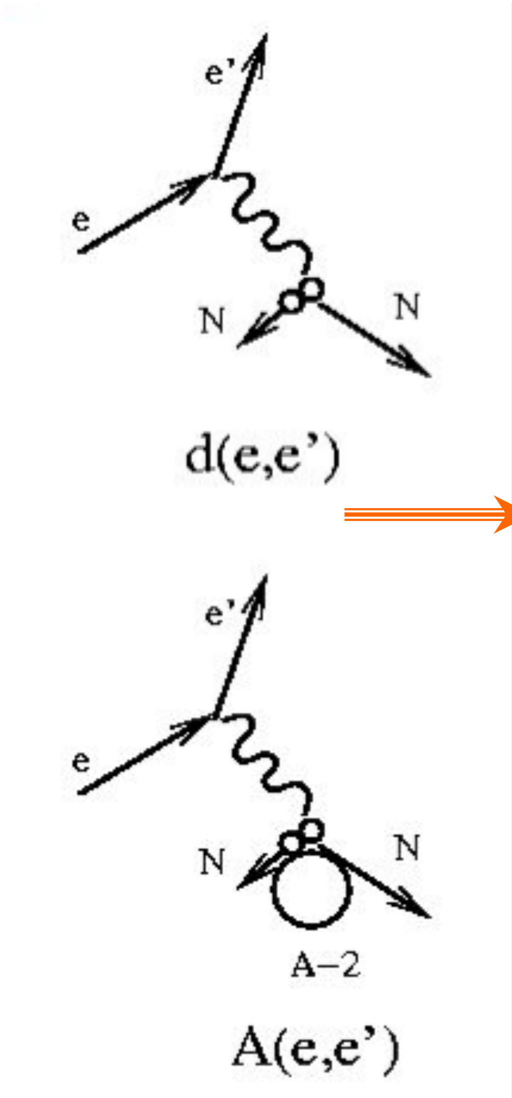
Inclusive Electron Scattering at x_B>1

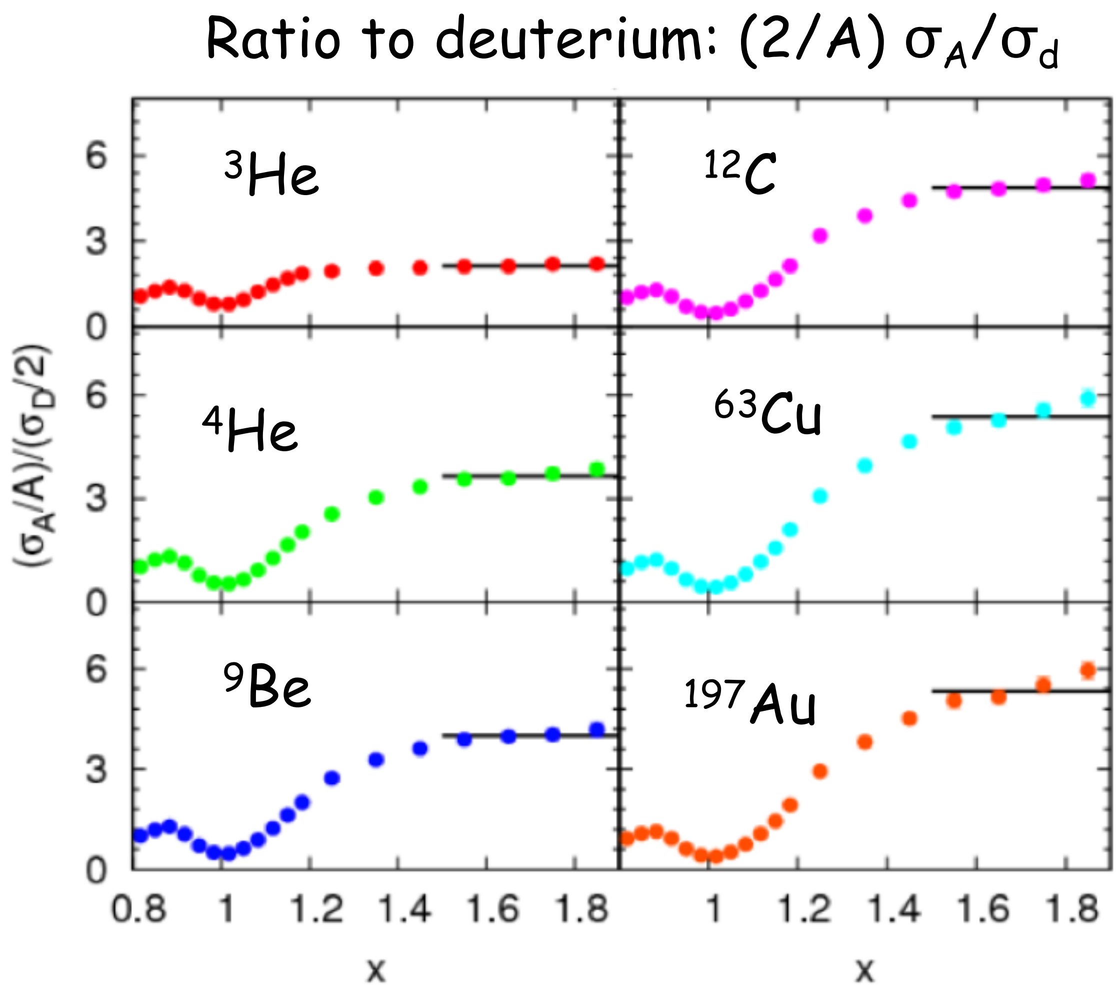
- At fixed Q^2, x_B determines a minimum initial momentum for the scattered nucleon (remember y-scaling?)
- If the momentum distributions of two nuclei have the same shape, then the ratios of their cross sections should be flat



momentum scaling ↔ x_B scaling

Correlations are Universal





$a_{2N} \approx 20\%$

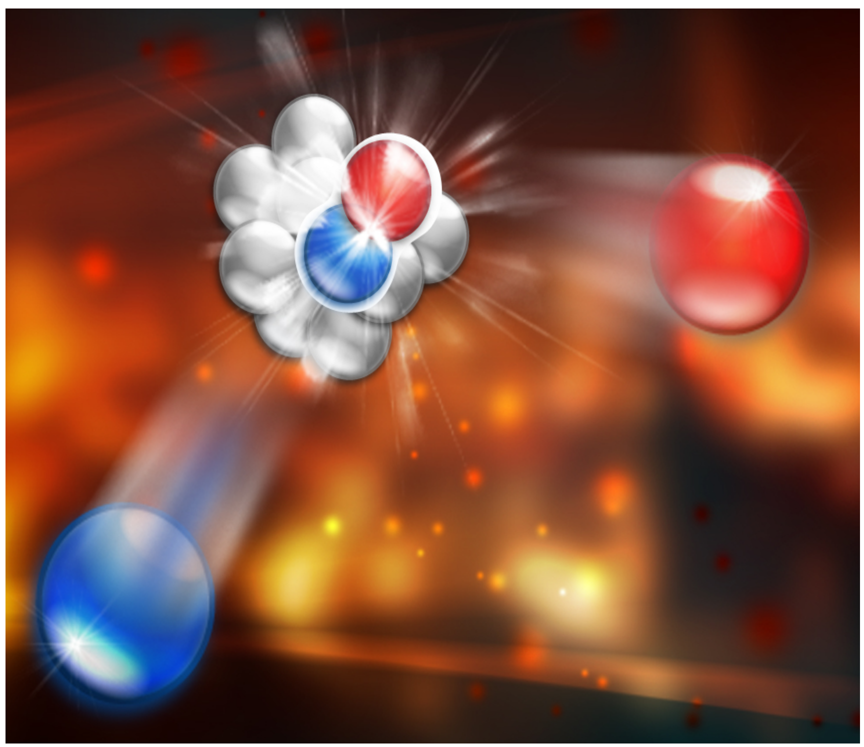
$a_{3N} \approx 1\%$

Scaling (flat ratios) indicates a common momentum distribution.
1 < x < 1.5: dominated by different mean field n(k)
1.5 < x < 2: dominated by 2N SRC n(k)

Day et al, PRL **59**, 427 (1987)
Frankfurt et al, PRC **48** 2451 (1993)
Egiyan et al., PRL **96**, 082501 (2006)
Fomin et al., PRL **108**, 092502 (2012)

Short Range Correlations (SRC)

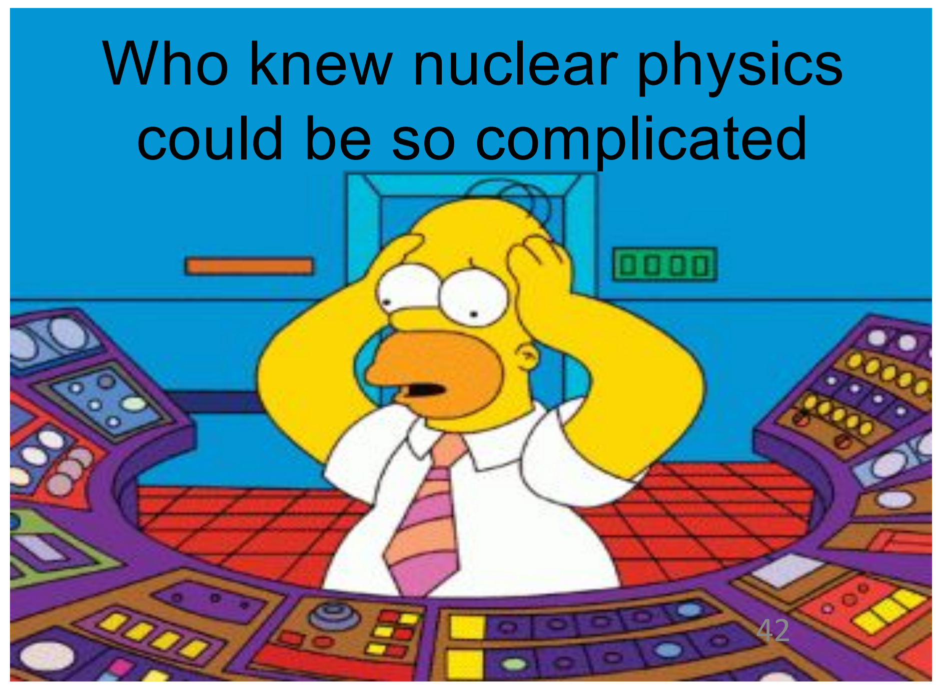
- 2N-SRC are pairs of nucleons that:
  - Are close together (overlap) in the nucleus.
  - Have high relative momentum and low center-of-mass (cm) momentum, where high and low is compared to the Fermi momentum of the nucleons (≈250 MeV/c in heavy nuclei)



Exclusive SRC Studies

A(e,e'pN): detect electron + two nucleons

- Pros: Measure the both nucleons to characterize the 2N-SRC pairs
- Cons:
  - Interpretation difficulties:
    - Competing processes,
    - Final State Interactions (FSI)
    - Transparency.
  - Experimental difficulties:
    - Large backgrounds,
    - Low rates,
    - Large installation,
    - Dedicated detectors



Exclusive SRC Studies

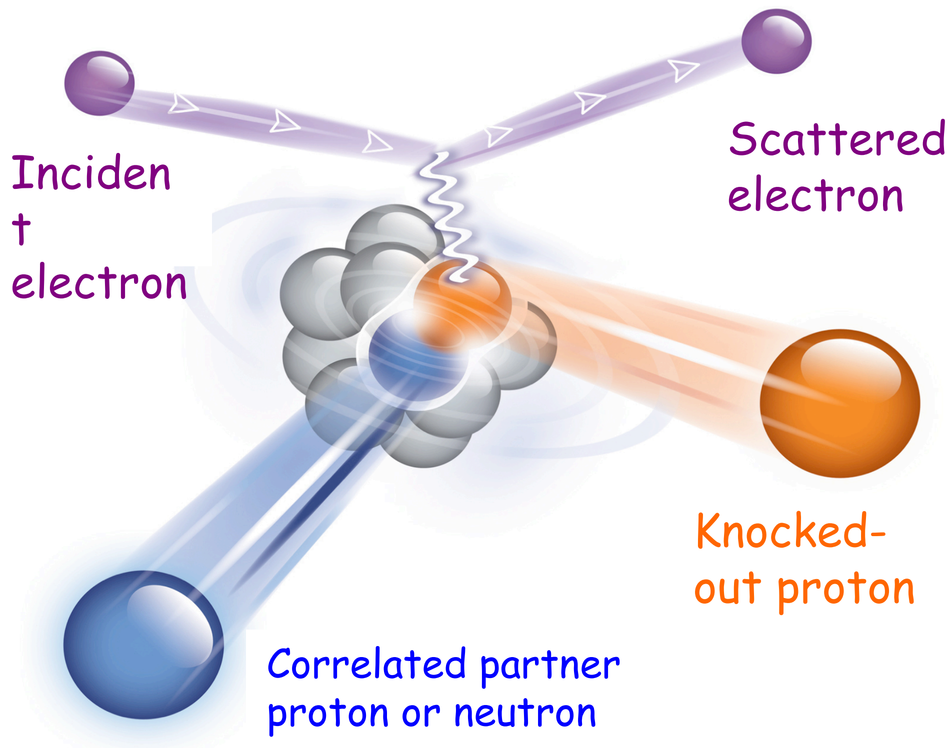
A(e,e'pN): detect electron + two nucleons

Measurement Concept:

1. Hit a high momentum proton hard ($Q^2 > 1$ GeV^2)

2. Reconstruct the initial (missing) momentum of the struck nucleon

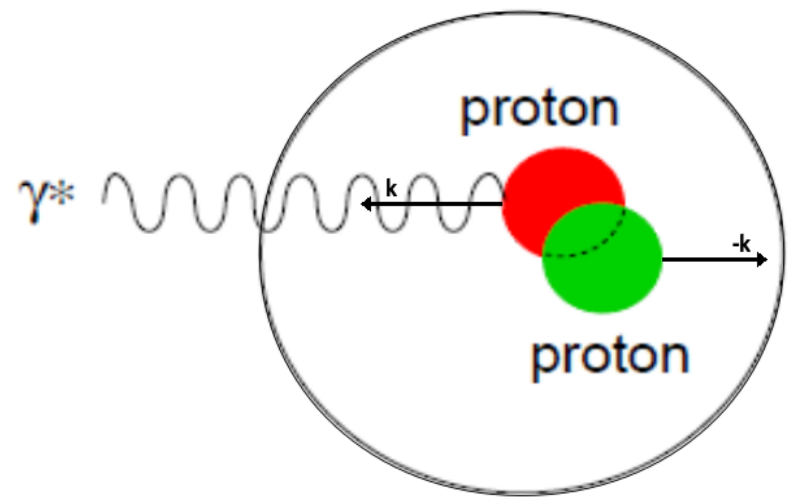
3. Look for a recoil nucleon with momentum that balanced that of the struck proton



$$\vec{p}_{miss} = \vec{q} - \vec{p}_p = -\vec{p}_{initial}$$

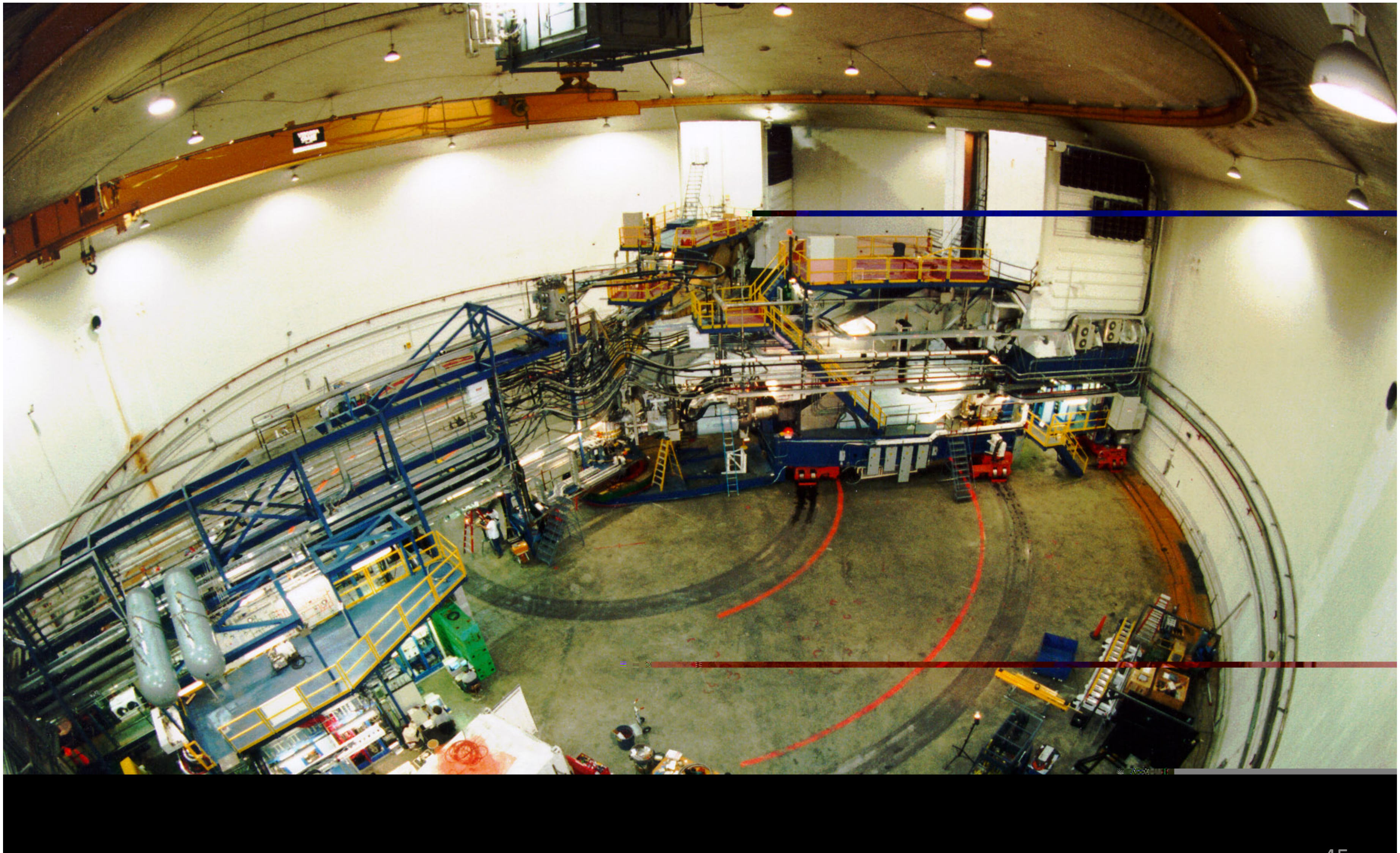
JLab Hall-A E01-015 (2004)

- Goal: Study both **pn** and **pp S**RC in ^{12}C over an (e,e'p) missing momentum range of 300-600 MeV/c



JLab Hall-A

Physicists Tend To Fill Empty Space©



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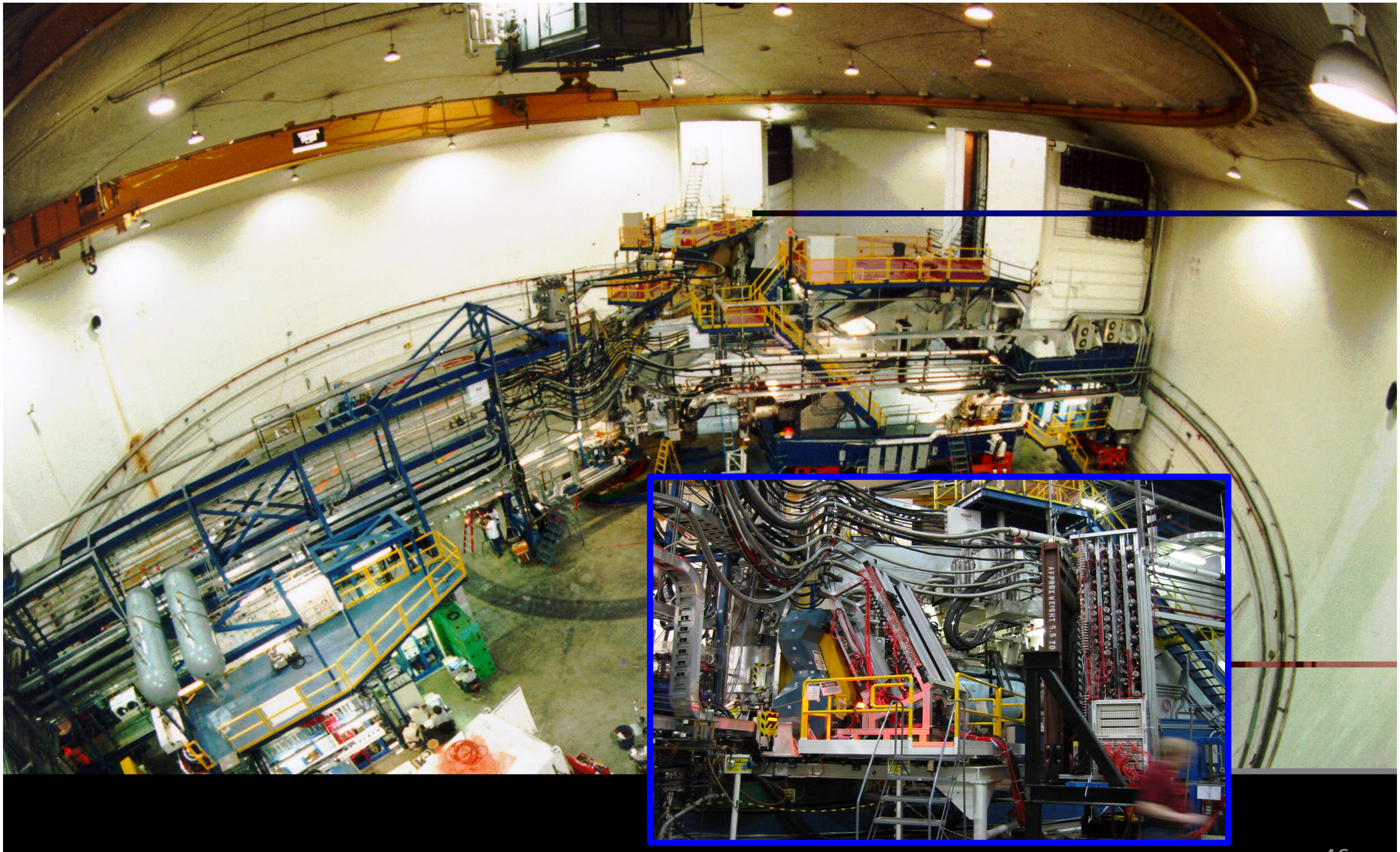
JLab Hall-A E01-015

Physicists Tend To Fill Empty Space©



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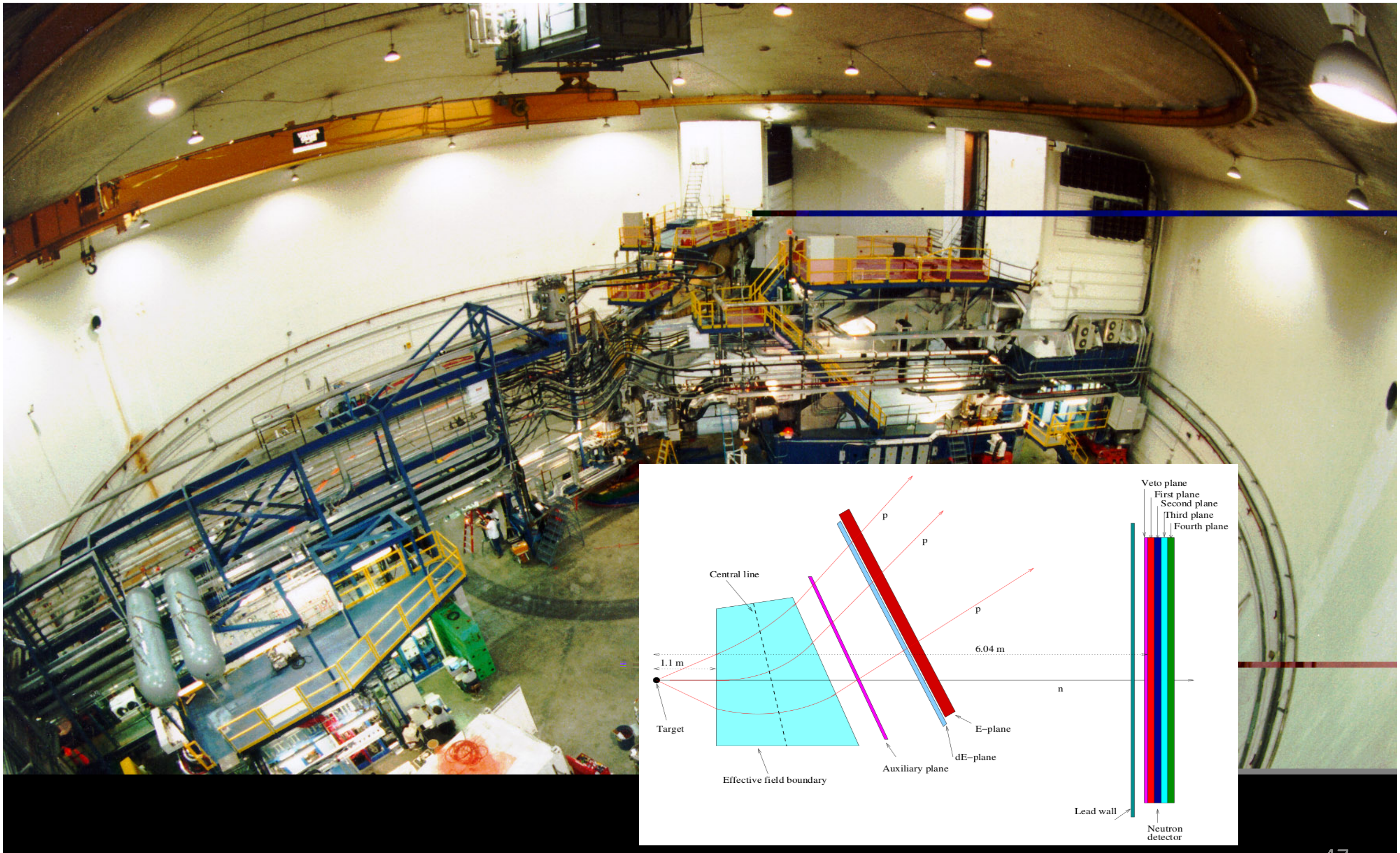
Physicists Tend To Fill Empty Space©

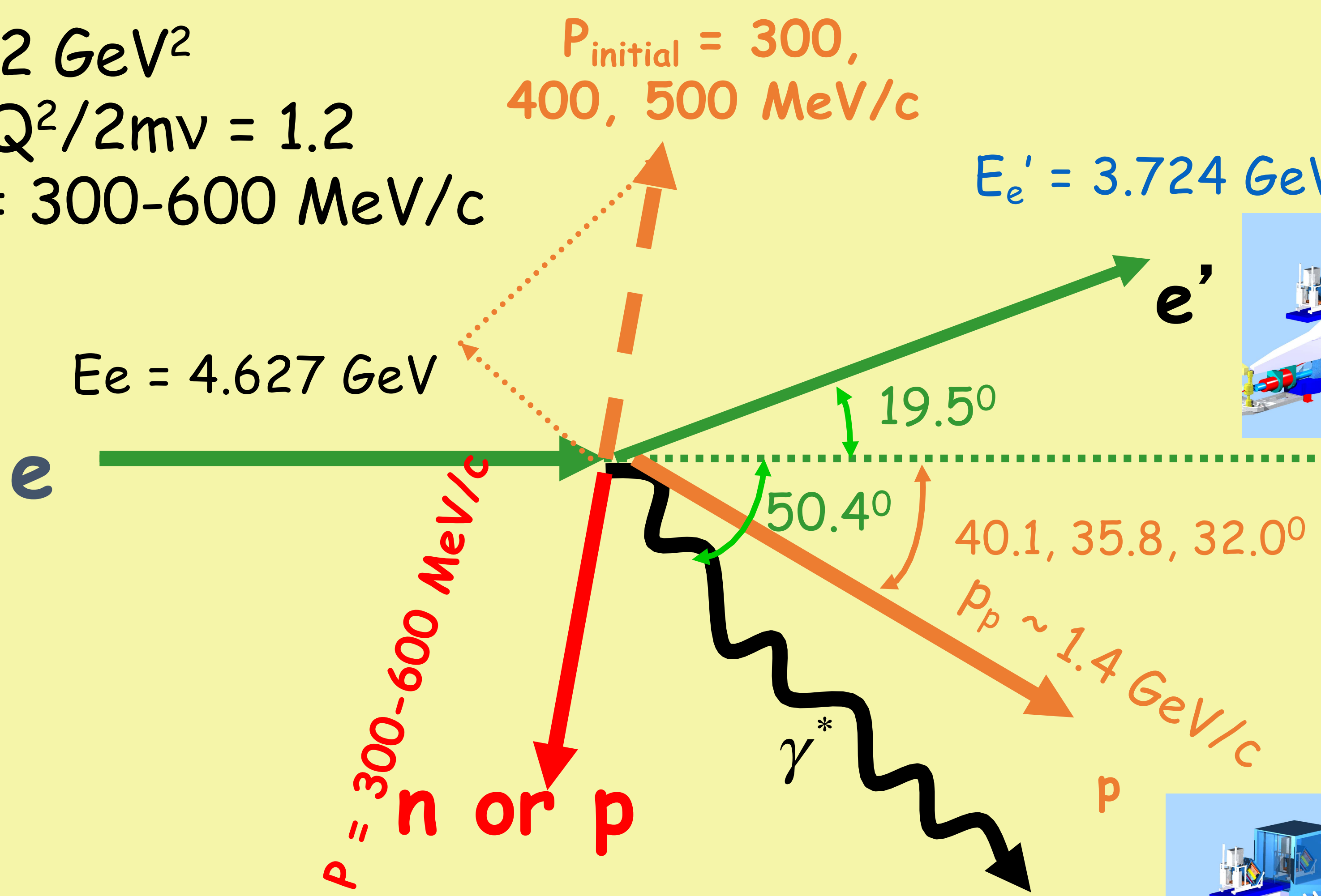


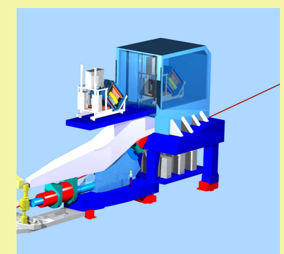
Now detect yet another nucleon
JLab Hall A C(e,e'pN) - selected kinematics

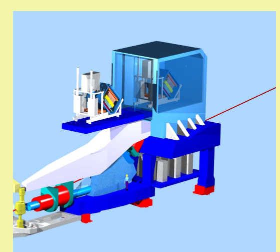
$Q^2 = 2\ GeV^2$

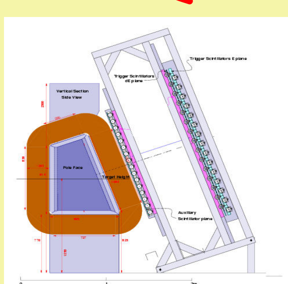
$x_B = Q^2/2m\nu = 1.2$

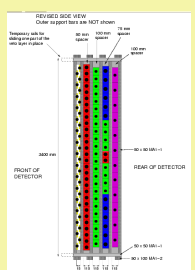
$P_{miss} = 300\text{-}600\ MeV/c$











Detect the proton,
look for its partner nucleon

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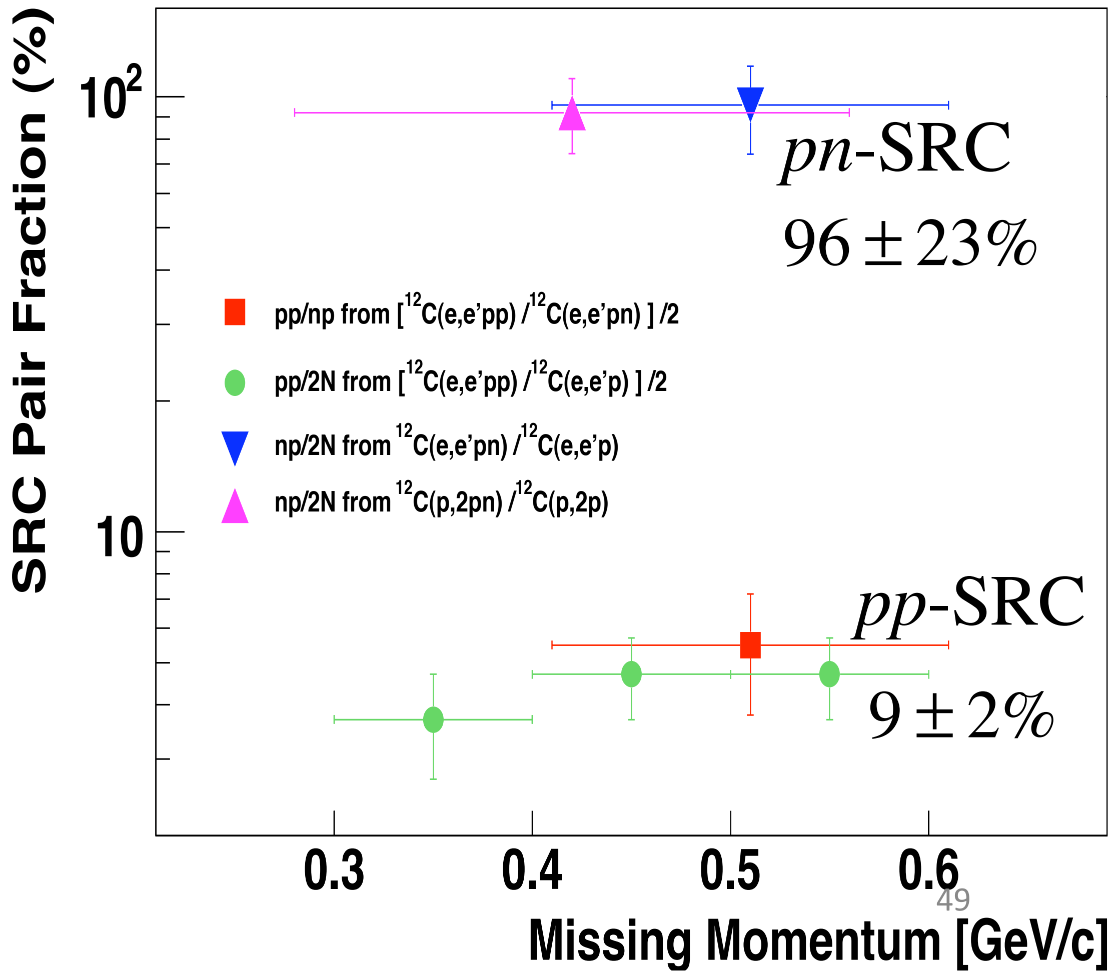
[pn and pp SRC Probabilities and the pp/np ratio]

- The (e,e'pN)/(e,e'p) ratio gives the probability for a high momentum proton to be part of a pN-SRC pair.

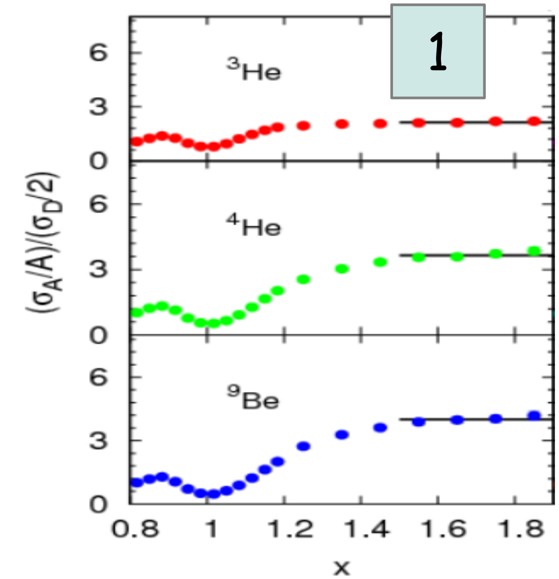
- **All** high $p_{initial}$ protons have a correlated partner
- np pairs dominate
  - Importance of tensor force at $0.3<p_{initial}<0.6$ GeV/c



R. Subedi et al., ***Science* 320**, 1476 (2008)

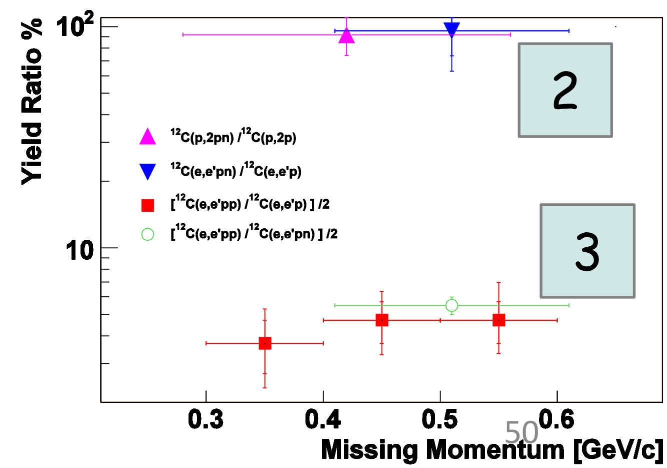
2N-SRC from inclusive and exclusive measurements

1 The probability for a nucleon to have p≥300 MeV/c in medium nuclei is 20-25%



2 More than ~90% of all nucleons with p≥300 MeV/c belong to 2N-SRC.

3 2N-SRC dominated by np pairs



1 2 → ~80% of kinetic energy of nucleon in nuclei is carried by nucleons in 2N-SRC.

Quasielastic summary: (e,e'), (e,e'p) and (e,e'pN)

- (e,e') scaling shows the electron is (mostly) scattering from single nucleons
- (e,e') ratios measure the probability of short range correlations (SRC) in nuclei
- (e,e'p) measures E and p distributions of single nucleons
- (e,e'pN) measures E and p distributions of nucleon pairs

The nucleus:
60-70% single particle – E + p dists measured
20±5% SRC - starting to measure
10-20% LRC