PHYSICS 722/822 - Fall Semester 2018 - ODU

Nuclear Physics - FINAL - Solution

This is a take-home Final - your solution is due back to me by 3:30 on Thursday, December 13 (either as hardcopy delivered to my office or as email with attachments). You may use any sources you can find (including our text books) but you must cite any sources used.

Problem 1)

Both the lambda baryon (Λ^o) and the neutral delta (Δ^o) decay into a nucleon and a pion. However, the lambda has a life time of about 0.26 ns, while the delta lives for less than 10^{-23} s. Can you explain this difference? (2-3 sentences)

SOLUTION

There are two differences between the Λ^o and the neutral delta (Δ^o): the former has spin 1/2 and the latter spin 3/2, and, more importantly, the former contains a strange quark whereas the latter doesn't. Since in both cases only decay into a nucleon and a pion is kinematically possible, the final state does not contain any strange quarks. The only interaction that can convert a strange quark into a different flavor is the weak interaction; therefore, the Λ^o must decay weakly, which explains its comparatively long life time. (The delta decays via the strong interaction, i.e. in no time flat).

Problem 2

Consider semi-exclusive electron scattering on a nucleus, A(e,e'p). If the residual nucleus (A-1) is left in its ground state, the energy transfer ν from the electron is uniquely determined by the momentum transfer **q** from the scattered electron and the momentum **p'** of the observed final state proton. Derive the corresponding equations, using relativistic kinematics for the proton throughout. What is the interpretation of p' in a Fermi-gas or shell model picture of the reaction (in the Impulse Approximation)?

SOLUTION

This is purely a relativistic kinematics problem. The process can be thought of as a two-body reaction: a virtual photon of energy ν and momentum **q** is absorbed by a nucleus of mass m_A and momentum **0**. The final state contains a proton of momentum **p'** and energy $E' = (m_p^2 + \mathbf{p'}^2)^{1/2}$, and a recoiling nucleus of mass m_{A-1} and momentum
$\mathbf{p}_{A-1} = \mathbf{q} - \mathbf{p'}$. Relativistic four-momentum conservation requires

$$m_A + \nu = \sqrt{m_p^2 + \vec{p}^2} + \sqrt{m_{A-1}^2 + (\vec{q} - \vec{p})^2} \quad \Rightarrow$$

$$\nu = \sqrt{m_p^2 + \vec{p}^2} + \sqrt{m_{A-1}^2 + (\vec{q} - \vec{p})^2} - m_A \approx E_p + (m_{A-1} - m_A) + \frac{(\vec{q} - \vec{p})^2}{2m_{A-1}}$$

The last expression can be interpreted as the sum of the kinetic energy of the knocked-out proton, plus its binding energy, plus the recoil energy of the nucleus A-1. In the Impulse Approximation, we assume that only the observed proton took part in the interaction, and therefore the missing momentum $\mathbf{p}_{miss} = \mathbf{p}_{A-1} = \mathbf{q} - \mathbf{p'}$ is equal (but opposite) to the initial proton momentum. By measuring the distribution of events as a function of **p'** , we can therefore determine the initial momentum distribution of protons inside the nucleus and compare the result with the Fermi gas model or the shell model wave function for the specific shell model state that the proton was in before being knocked out.

SOLVE (ONLY!) ONE OF THE TWO FOLLOWING PROBLEMS:

Problem 3a)

In a certain experiment, a 1 mm thick target of Carbon-13 (density: 2 g/cm^3) is irradiated with a proton beam of 100 nA. Among other reactions, Nitrogen-13 is produced in the reaction ^{13}C(p,n)^{13}N. Assume the reaction cross section for this is 1 mb. ^{13}N decays via electron capture with a half life of 9.97 minutes.

a) Classify this transition (degree of allowed-ness, Gamov-Teller vs. Fermi).

b) After a long time (many hours) of continuous irradiation, an equilibrium is reached, where as many new ^{13}N nuclei are produced per unit time as decay in the same time interval. How many ^{13}N nuclei will be present in the target in this equilibrium state?

SOLUTION

a) Both nuclei (initial and final) are spin 1/2, negative parity (with a lonely proton or neutron in the $1p_{1/2}$ shell, outside a full ^{12}C core). The transition is therefore allowed and can proceed both through the Fermi and the Gamov-Teller matrix element. Since the 2 nuclei form an isospin doublet, the transition is even super-allowed (same initial and final state wave function).
b) At the steady state, Nitrogen-13 nuclei are **produced** with the rate $\frac{dN}{dt_+} = L\sigma$, where L is the luminosity and σ is the production cross section.
Meanwhile, they **decay** with the rate

$$\frac{dN}{dt_-} = -\lambda N(t) = -\frac{1}{\tau}N(t) = -\frac{\ln 2}{t_{1/2}}N(t).$$

Equilibrium means that both rates add up to zero, yielding a constant number of nuclei in the target. This gives

$$N(t \to \infty) = \frac{t_{1/2}}{\ln 2} L\sigma$$

All that remains to do is to calculate the luminosity:

$$L = \frac{2gm/cm^3 \cdot 0.1cm}{13mol/gm} 6.022\,10^{23} \frac{1 \cdot 10^{-7}\,C/s}{1.6 \cdot 10^{-19}C} = 5.8 \cdot 10^{33} cm^{-2} s^{-1}$$

The cross section is $1mb=10^{-27}cm^2$. Plugging it all in, I get a total number of 5 billion Nitrogen-13 nuclei present in the target at all times.

Problem 3b)

Consider the stripping reaction ${}^{12}_{6}C(d,p){}^{13}_{6}C$ with incident deuterons of 19.4 MeV kinetic energy. The stripped-off neutron gets put in the lowest possible shell model state (the ground state of ^{13}C); what would be its quantum numbers? How much energy (and momentum) would the proton (have to) carry away? (Assume it continues in the same direction as the deuteron was travelling). How much momentum would get transferred to the residual nucleus? Does this momentum transfer make sense, given the angular momentum that also must be transferred?

You can find more resources on the Web, for instance at http://www.tunl.duke.edu/~datacomp/ , http://www.shef.ac.uk/chemistry/web-elements

SOLUTION:
The neutron would be bound in the empty $1p_{1/2}$ shell, leading to the ground state of ^{13}C. The mass excess of that nucleus is 4.95 MeV lower than that of a free neutron plus Carbon, so an additional 4.95 MeV becomes available. On the other hand, the deuteron has to be broken up, which costs about 2.22 MeV, leaving 2.72 MeV extra for the proton to carry away (for a total of 22.1 MeV). This corresponds to a momentum of 204 MeV/c for the proton. The deuteron was carrying a momentum of about 270 MeV/c, leaving 66 MeV/c to be transferred (by the neutron) to the nucleus. Given that the radius of ^{12}C is 2.75 fm, that yields an angular momentum transfer of about 180 MeVfm or one $\hbar$. This makes perfect sense, given that the neutron is supposed to end up in a p-shell.

FOR THE FOLLOWING 2 PROBLEMS, READ THE ARTICLE BY HANS BETHE (posted on our web page).

Problem 4)

In Bethe's article, Eq. (1) shows the well-known relationship between nuclear radius R and baryon number A. Describe a typical electron scattering experiment and what information one extracts from it to arrive at the size of a given nucleus. As an example, what is the expected magnitude of the derivative $dF(Q^2)/dQ^2$ of the charge form factor of an ^{208}Pb nucleus according to this equation? To measure this, what kind of incoming electron energy and electron scattering angle would you choose?

SOLUTION
The radius of a nucleus is closely related to the root-mean-square charge radius $R_{ch,rms}$ which can be measured in elastic electron scattering. In a simple model where the nucleus is a uniformly charged sphere of radius R, the rms charge radius is given by $R_{ch,rms} = (3/5)^{1/2}$ R = 0.775 R = 0.93 fm $A^{1/3}$, which yields 5.5 fm for ^{208}Pb. (The actual radius is 6.8 fm which would correspond to a rms charge radius of 5.27 fm).
This rms radius can be extracted from a measurement of the charge form factor $F(Q^2)$ at small 4-momentum transfers Q^2. Experimentally, one scatters low-to-medium energy electrons (a few 100 MeV) at small (forward) scattering angles from a target made of the nucleus under study. The ratio between the measured cross section $d\sigma/d\Omega$ and the Mott cross section $(d\sigma/d\Omega)_{Mott}$ is equal to the square of the charge form factor, $|F(Q^2)|^2$. At very low Q^2, the form factor can be expanded in a Taylor series:
$F(Q^2) = 1 - R_{ch,rms}{}^2\ Q^2 / 6\hbar^2 + \ldots$
In the case of ^{208}Pb, one expects for the magnitude of the derivative
$dF(Q^2)/dQ^2 = - R_{ch,rms}{}^2 / 6\hbar^2 = 120 - 130 / \mathrm{GeV}^2$.
In order for the low-momentum approximation to be valid, one needs to measure at Q^2 of the order of 10^{-3} GeV^2, which corresponds to e.g. elastic electron scattering of 200 MeV by 9 degrees.

Problem 5)

Referring to Part III of Bethe's article, explain in your own words why the isotopes ^{235}U and ^{239}Pu can be fissioned using slow (eV) neutrons, while one needs fast (MeV) neutrons to fission ^{238}U. Note that the explanation given in the article (at the end of section D) is a bit misleading: Since fission usually ends up liberating several neutrons and huge amounts of energy in the final state, the number of neutrons in the final fission products does **not** play an important role. Instead the excitation energy of the compound nucleus after the neutron has been captured is important. From the mass formula (in the article or our books), explain why this excitation energy is needed for fission to occur, and where it comes from in the case of ^{235}U and ^{239}Pu (which term in the mass formula is responsible for that extra “oomph”?).

SOLUTION

None of the isotopes listed in the problem can fission "spontaneously", at least not in finite time. The reason is that while the total binding energy of the final state (two fission fragments at large distance, plus a smattering of neutrons) is lower than that of the initial state, there is an energy barrier that must be overcome first (tunneling takes too long). This barrier is apparent from the mass formula: the surface energy of a heavy nucleus (A=238, for instance) is of order 680 MeV, and for the fission to occur, the surface has to increase first, raising the total energy before the Coulomb repulsion takes over and decreases the overall energy.
In the case of ^{238}U, the energy to overcome this barrier comes from the kinetic energy of the fast (MeV) neutrons. However, in the case of the other isotopes mentioned, the capture of even a slow neutron leads to a net excitation energy because of the pairing term in the mass formula. They all have an odd number of neutrons, so adding one more neutron releases an amount of 0.56 - 0.73 MeV of pairing energy (depending on whether you use Bethe's mass formula or the one in Povh et al.) (In reality, the pairing energy is higher – the fit in our book is not very good for heavy nuclei). This energy is sufficient to overcome the fission barrier.