Nuclear Physics - Problem Set 1 - Solution

Problem 1)

^{6}Li is the only stable A=6 isobar, with Z=3 protons and N=3 neutrons (therefore with charge Q = 3e). Its natural abundance is 7.5% (the remainder is made up of ^{7}Li, the only other stable Li isotope). Its total angular momentum (nuclear spin) is J=1 and its ground state parity is positive (π=+1). Its "mass excess" (e.g. from the resources on my Web page) is Δ = 14.086 MeV, which means that the neutral atom is heavier than 6u by this amount. This gives a total mass of 5603.05 MeV/c^2 for the neutral atom, and 5601.52 for the nucleus alone (after subtracting the electrons). This is 31.995 MeV/c^2 lighter than the combined mass of 3 protons and 3 neutrons. This means that the total binding energy is 31.995 MeV, while the average binding energy per nucleon is 5.333 MeV. Comparing the mass excess numbers with the neighboring nuclei ^{5}Li and ^{5}He, we find that the removal energy for a neutron is $\Delta(^5\text{Li}) + \Delta(n) - \Delta(^6\text{Li})$ = 5.665 MeV and that of a proton is $\Delta(^5\text{He}) + \Delta(^1\text{H}) - \Delta(^6\text{Li})$ = 4.593 MeV, not much different from the average binding energy of a nucleon. (Clearly, ^{6}Li is stable against proton or neutron decay).
Additional information can be found in the Nuclear Data tables, for instance "Energy Levels of light Nuclei A=6" by the group at TUNL/Duke University. Here I find that the magnetic moment of ^{6}Li is about 0.822 nuclear magnetons, that it has a small quadrupole deformation of -0.818 mb, and that its nuclear radius is about 2 fm.
Its first excited states are a $J^\pi = 3^+$ state with 2.186 MeV excitation energy and a $J^\pi = 0^+$ state at 3.563 MeV. There are many more excited states, all of which are rather broad. From the level diagram, I can also infer that all excited states of ^{6}Li are above nuclear decay thresholds, i.e. ^{6}Li -> d + α.

See also https://www-nds.iaea.org

Problem 2)

a) Using the equation and constants from Povh et al., I get the following atomic masses for the 3 nuclides in question:
   m(^{4}He) = M(Z=2,N=2) = 3733.633 MeV;
   m(^{197}Au) = M(Z=79,N=118) = 183,474.578 MeV;
   m(^{193}Ir) = M(Z=77,N=116) = 179,743.070 MeV.
b) From a table of mass excesses (see Problem 1) I calculate the actual masses as
   m(^{4}He) = 3728.401 MeV (5.232 MeV lighter, i.e. more tightly bound);
   m(^{197}Au) = 183,473.161 MeV (only 1.417 MeV lighter);
   m(^{193}Ir) = 179,743.806 MeV (only 0.723 MeV heavier!).
   The agreement for the two heavier nuclei is astonishingly good - 1 MeV out of nearly 200,000 (even the binding energy is 1500 times larger than this small "error"). This small discrepancy is readily explained by the fact that the mass formula has only 5 fit parameters to describe a large number of nuclei, so individual values will always scatter a little around that fit. The deviation is a bit larger for ^{4}He; this happens in part because the liquid drop model is not very good for very light nuclei, and in particular

shell closure effects play a big role (^{4}He is a "double magic" nucleus which tend to be especially tightly bound - see next week).

c) The combined mass of ^{193}Ir plus ^{4}He is 183,476.7 MeV according to the mass formula. This is slightly more than the mass formula value for ^{197}Au, so one would expect that nucleus to be stable against α-decay. However, using the **actual** masses, one finds a combined mass of ^{193}Ir plus ^{4}He of 183,472.21 MeV, which is nearly 1 MeV **lighter** than the mass of ^{197}Au. Clearly, even very small changes in the masses play a big role here. From this result, one would expect that Gold is actually unstable against α-decay, a frightening prospect for people who collect that precious metal. However, the mass difference is so small that the "Coulomb barrier" discussed in the lecture is formidable, so the decay probability is incredibly small, yielding a lifetime much longer than that of the Universe.

d) In SI units, the energy of a charged sphere is $(3/20\pi\varepsilon_o)q^2/R$. Using the Ansatz $\rho_o = A/(4\pi R^3/3)$ for the density of nucleons in the nucleus, I conclude $R=(3/4\pi \rho_o)^{1/3}A^{1/3} = 1.22$ fm $A^{1/3}$, or I can directly use $R_o = 1.2$ fm. Plugging it all in, I get $a_C = 0.708$ MeV/c^2. Pretty close to the value quoted in Povh et al.!

Problem 3)

The uranium is decaying with a half life $t_{1/2}=4.5 \cdot 10^9$ years. This corresponds to a decay constant $\lambda_U = \ln 2 / t_{1/2} = 4.88 \cdot 10^{-18}$/s. On the other hand, the Thorium has a decay constant of $\lambda_{Th} = 3.329 \cdot 10^{-7}$/s. The differential equation describing the creation and decay of Thorium is given by $dN_{Th}/dt = \lambda_U N_U - \lambda_{Th} N_{Th}$.
Since the ore was undisturbed, it is safe to assume that everything is in an equilibrium, i.e.
$dN_{Th}/dt = 0$. Also, given its extremely long life time, we can assume N_U is roughly constant. This yields $N_{Th} = \lambda_U N_U / \lambda_{Th} = 1.46 \cdot 10^{-11} N_U$ .Using the mass number, I find that 1 kg of ^{238}U contains 4.2 mols, i.e. $2.53 \cdot 10^{24}$ atoms. Therefore, there must be $3.71 \cdot 10^{13}$ atoms of Thorium, or 0.0144μg.

Problem 4)

The total rest mass of ^{40}K is 39.963998 *u*, that of ^{40}Ar is 39.962383 *u*, and that of ^{40}Ca is 39.962590 *u*. Hence the mass of potassium is indeed larger than that of the corresponding argon and calcium isotopes. Since these masses include electrons, that means that any decay of Potassium that keeps the total number of electrons + positrons constant is energetically allowed. This means that beta- decay to ^{40}Ca is possible, and so is electron capture to ^{40}Ar. Finally, the energy difference to the latter, 0.001615 *u* = 1.5 MeV, is just big enough to also allow beta+ decay (with nearly 0.5 MeV to spare after accounting for the 1.022 MeV it takes to produce the positron and to count the extra electron left over from the decay.) This is a famous example for the effect of nucleon pairing, which favors both ^{40}Ar and ^{40}Ca because both have even numbers of protons and neutrons. (It is famous because ^{40}K is the lightest long-lived radioactive nuclide and responsible for a large fraction of the internal radiation dose received by all living things).