PHYSICS 722/822 - Fall Semester 2018 - ODU

Nuclear Physics - Last Problem Set 10 – Solution

Problem 1)

Since we are not supposed to count the strong coupling constant, there are only 2 constants describing electroweak coupling. Those could be g and g' or, more usefully, e and $\sin\theta_W$.
Secondly, we have to include some information about the Higgs sector – e.g. the mass of the observed Higgs boson and the vacuum expectation value of the Higgs field at the potential minimum. These in turn determine uniquely the masses of both W's and Z's (together with the couplings above).
The remaining constants are the Yukawa couplings of all 12 fundamental fermions to the Higgs field, i.e., their masses, and the 4 parameters each of the mixing matrix for the quark sector and for the neutrino sector. This would be 20 more parameters, for a total of 24 (25 if you include the strong coupling and 26 including gravity). Hardly a "fundamental theory", but very successful. (NB: For completeness, we would also have to include a "QCD phase" which makes the total 27).

Problem 2)

Here are my answers:

^{3}H -> ^{3}He (g.s.): $1/2^+$ -> $1/2^+$ transition; therefore $\Delta l=0$. Both Gamov-Teller and Fermi can (and DO) contribute. The two nuclei are isospin partners with the same wave functions (under exchange of n's with p's), therefore this transition is super-allowed.

^{20}F -> ^{20}Ne (g.s.): 2^+ -> 0^+ transition; therefore $\Delta l=2$. This transition is twice forbidden. Both Gamov-Teller and Fermi contribute.

^{42}Sc (g.s.) -> ^{42}Ca (g.s.): 0^+ -> 0^+ transition; therefore $\Delta l=0$. This transition is allowed (and in fact even super-allowed) and pure Fermi.

^{42}Sc (7^+ isomer) -> ^{42}Ca (g.s.): 7^+ -> 0^+ transition; therefore $\Delta l=6$. This transition is six times forbidden and pure Gamov-Teller (no wonder it never happens).

^{64}Cu -> ^{64}Zn (g.s.): 1^+ -> 0^+ transition; therefore $\Delta l=0$. This transition is allowed and pure Gamov-Teller.

^{84}Br -> ^{84}Kr (g.s.) : 2^- -> 0^+ transition; therefore $\Delta l=1$. This transition is once-forbidden and pure Gamov-Teller.

<u>Problem 3)</u>

From Problem 2), we realize that the transition in question is super-allowed. Therefore, the statement on page 332 applies: the *ft* value of this transition is "roughly equal" to that of the free neutron decay. From equations 16.56, 16.59 and 16.60 we conclude that for the neutron $f\tau = 0.47\frac{\varepsilon_o^5}{30}\cdot 880\text{ s}$. Now "all" we have to do is figure out the $f(E_o)$ for the transition in question. First note that in these formulas, all energies are total energies (rest mass plus kinetic energy). Therefore, in the case of the neutron,

$E_o = m_n - m_p = 1.2903$ MeV => $\varepsilon_o = 2.525$ => $f\tau = 1415$ s. To calculate $f(E_o)$ for the transition ^3H -> ^3He I use equation (16.55) (p. 278). Using the actual masses of the Helium-3 nucleus and the triton, I get $E_o = 0.5296$ => $\varepsilon_o = 1.0364$. Obviously, all energies involved are only slightly bigger than the electron rest mass, and I can use the non-relativistic approximation $E_e = m_e c^2 + \frac{p_e^2}{2m_e} \quad \Rightarrow \quad dE_e = \frac{p_e}{m_e}dp_e$. Therefore, the integral can be approximated as

$$f(E_o) = \frac{1}{m_e^5 c^{10}} \int_{m_e c^2}^{E_o} E_e c p_e \left(E_o - E_e\right)^2 dE_e =$$

$$\frac{1}{m_e^5 c^9} \int_0^{\sqrt{2m_e\left(E_o - m_e c^2\right)}} \left(m_e c^2 + \frac{p_e^2}{2m_e}\right) \frac{p_e^2}{m_e} \left(E_o - m_e c^2 - \frac{p_e^2}{2m_e}\right)^2 dp_e =$$

$$\frac{1}{m_e^5 c^9} \int_0^{0.1379 MeV/c} \left(\begin{array}{l} \frac{p_e^8}{8m_e^4} + \left[m_e c^2 - 2\left(E_o - m_e c^2\right)\right] \frac{p_e^6}{4m_e^3} + \\ \left[\left(E_o - m_e c^2\right)^2 - 2m_e c^2 \left(E_o - m_e c^2\right)\right] \frac{p_e^4}{2m_e^2} + m_e c^2 \left(E_o - m_e c^2\right)^2 \frac{p_e^2}{m_e} \end{array} \right) dp_e$$

$$= \frac{1}{m_e^5 c^9} \left[\frac{p_e^9}{72 m_e^4} + 0.4738 MeV \frac{p_e^7}{28 m_e^3} - 0.0187 MeV^2 \frac{p_e^5}{10 m_e^2} + 0.0001768 MeV^3 \frac{p_e^3}{3m_e} \right]_0^{0.1379 MeV/c}$$

$$= 2.01 \cdot 10^{-6}$$

This leads to a lifetime of τ=22.3 years or a half-life of 15.5 years. This is not too far off the measured half-life of 12.3 years.