Nuclear Physics - Problem Set 5 - Solution

Problem 1)

a) At the average energy of the scattered electron, E' = 15 GeV, we have ν = 5 GeV, Q^2=1.85 GeV^2, and $x = Q^2/2m\nu = 0.197$.
b) We are clearly in the scaling region: $Q^2 > 1$, and $W^2 = 8.4$ (way above the resonance region $W^2 \leq 4$). Therefore, we can assume that the structure functions F_1 and F_2 scale (which is an especially good assumption at intermediate x, as is the case here). Since $\tan^2\theta/2 = 0.0015$, we can ignore the contribution from W_1 (F_1), and therefore write the cross section as $\frac{d\sigma}{d\Omega dE'} = \frac{d\sigma}{d\Omega}_{Mott} \cdot W_2 = \frac{d\sigma}{d\Omega}_{Mott} \cdot \frac{F_2(x)}{\nu} = \frac{d\sigma}{d\Omega}_{Mott} \cdot \frac{0.33}{5GeV}$
(depending which source you use, $F_2(x)$ is somewhere between 0.32 and 0.35).
For the Mott cross section, I get $5.444 \cdot 10^{-30}$ cm^2/sr. The total cross section is then $3.6 \cdot 10^{-31}$ cm^2/sr/GeV.
c) The luminosity comes out to $L = 2.63 \cdot 10^{35}$ /s/cm^2. Multiplying this with the cross section and the angular (0.002 sr) and energy (0.2 GeV) acceptance, I get a count rate of 38 per second.

Problem 2)

Equation 7.16 tells you that the integral

$$F = \int_0^1 F_2(x)dx = \frac{4}{9}\int_0^1 x(u(x) + \bar{u}(x))\,dx + \frac{1}{9}\int_0^1 x(d(x) + \bar{d}(x))dx$$

gives you the momentum carried by u- and d-quarks separately, weighed with their respective charge squared (again, I ignore s-quarks etc.). Looking at Fig. 7.4, p. 84, in Povh et al., I can estimate that integral "by eye" as being roughly equal to 0.17 .
Now all I have to do is to assume that the first integral is twice the second one (since there are twice as many up-quarks than down-quarks). That means that F is actually equal to the second integral ($2 \cdot 4/9 + 1/9$). Therefore, to get the searched-for integral X, I simply have to multiply F with 3 (1 for the second integral + 2 for the first integral), which yields 0.51. Therefore, we conclude that roughly 50% of the proton momentum is carried by quarks (the remaining half is carried by gluons).

Problem 3)

a) Before the electron hits the proton, the total energy available is that of the electron (4 GeV) plus the mass of the proton (0.9382 GeV). The sum of these two is more than the energy observed in the final state (2.33 GeV + 1.617 GeV = 3.947 GeV). Therefore, the excess energy (4 GeV + 0.9382 GeV - 3.947 GeV = .9912 GeV) must be carried away by another, unobserved particle.

b) The energy of the missing particle is obviously .9912 GeV. The momentum can be inferred by using momentum conservation:

$$\vec{p}_x = \vec{p}_e - \vec{p}_e' - \vec{p}_p' \Rightarrow |\vec{p}_x| = \sqrt{\vec{p}_e^{\,2} + \vec{p}_e'^{\,2} + \vec{p}_p'^{\,2} - 2\vec{p}_e \cdot \vec{p}_e' - 2\vec{p}_e \cdot \vec{p}_p' + 2\vec{p}_e' \cdot \vec{p}_p'}$$

Plugging all the momenta and angles in (note that for the electron, E=p, and for the angle between the outgoing particles, $\theta_{p'e'}$=60.44°), I get p_x = 0.9822 GeV.

c) From b), I can calculate the mass of the missing particle to 133 MeV. This is suspiciously close to the mass of a neutral pion (π^o). Charge conservation requires that the particle is neutral, so a π^o it is! That also explains how it could be missed - it decays very quickly into two photons, which must have escaped the detector.

d) I could add the 4-momenta of both the pion and the proton in the final state to get the 4-momentum of the originally produced object. The problem is that I haven't actually figured out what the relative angle between the momenta of these two particles is, and it would be messy to do so. Instead, I can invoke energy-momentum conservation and use the electron variables:

$$\vec{p}_y = \vec{p}_p' + \vec{p}_\pi' = \vec{p}_e - \vec{p}_e' = \vec{q}$$

$$E_y = E_p' + E_\pi' = m_p + E_e - E_e' = m_p + \nu \Rightarrow$$

$$M_y = \sqrt{W^2} = \sqrt{(m_p + \nu)^2 - \vec{q}^{\,2}} = \sqrt{m_p^2 + 2m_p\nu - Q^2} =$$

$$\sqrt{m_p^2 + 2m_p(E_e - E_e') - 4E_eE_e'\sin^2\left(\frac{\theta_e}{2}\right)} = 1{,}232\,\text{GeV}$$

This is the mass of a Δ resonance. Charge conservation requires that it is a Δ^+ in this case.

e) Obviously, I DID find out without using the measured proton momentum. However, seeing the proton in the final state determines the decay mode ($\Delta^+ \to p + \pi^o$; it COULD have decayed into $n + \pi^+$), and I can measure the relative angle between the decay products and the momentum transfer vector **q**. This information allows me to learn something about the angular momenta involved (with many events to look at, I could do an angular momentum decomposition).