

Nuclear Physics - Problem Set 6 – Due Tuesday, 10/30

Problem 1)

Start with the wave function for a proton given in Povh et al., Eq. 16.4 page 262 or in Wong, Eq. 2-46 p. 48 or my HUGS writeup. Calculate the magnetic moment of protons and neutrons. (Neutrons have the same wave function as protons, just exchange u with d and vice versa). To do this, you have to know the magnetic moments of the quark. Assume they are given by $\mu_q = \frac{z_q}{m_q} m_p \cdot \mu_N$, where $m_q = 330 \text{ MeV}/c^2$. Express your result in units of the nuclear magneton μ_N . Does your result agree with the accepted values?

Problem 2)

Using the same wave function as in Problem 1, answer the following: Analog to the (“unpolarized”) structure function F_1 , one can define a polarized structure function

$$g_1(x) = \frac{1}{2} \left(z_u^2 [u(x) \uparrow - u(x) \downarrow] + z_d^2 [d(x) \uparrow - d(x) \downarrow] \right) \text{ (ignoring all sea quarks}$$

completely), where the arrows indicated spin up and spin down ($u(x) \uparrow$ is the number of u-quarks with momentum fraction x that have their spin up, etc.). Integrating this expression over all x, one gets the total net spin fraction carried by u and d quarks, weighted with their respective charge squared (and divided by 2). If you assume that this is simply given by the proton wave function, what result would you predict for the proton? (You don’t have to do the integration yourself - the wave function given in the book is assumed to BE already integrated over all x). What is the answer for the neutron? Compare your answer with modern values for those integrals, e.g.

Table 1

Results for the first moments of the spin structure functions g_1 from different experiments. Each experiment evolved its data to a fixed value of Q^2 which is indicated. The results for the deuteron are *not* corrected for its D-state, but are “per nucleon”.

Q^2 (GeV ²)	Γ_1^p	Γ_1^n	Γ_1^d	$\Gamma_1^p - \Gamma_1^n$	Ref.
10	0.120 ± 0.016	–	0.019 ± 0.015	0.198 ± 0.023	SMC [28]
3	–	-0.033 ± 0.011	–	–	E142 [31]
3	0.133 ± 0.010	-0.032 ± 0.018	0.047 ± 0.007	0.164 ± 0.023	E143 [34]
5	–	-0.056 ± 0.009	–	0.168 ± 0.010	E154 ^a [38]
5	0.118 ± 0.008	-0.058 ± 0.009	–	0.176 ± 0.008	E155 ^a [36]
2.5	0.120 ± 0.009	-0.028 ± 0.009	0.043 ± 0.004	0.148 ± 0.017	HERMES ^b [44]
3	–	–	0.046 ± 0.006	–	COMPASS [49]
5	Bjorken sum rule			0.182 ± 0.002	

^a From an NLO analysis.

^b Over measured region $x > 0.021$ only.