Nuclear Physics - Problem Set 7 – Solution

Problem 1)

a) Plug and chug:

$$g_A = \langle p\uparrow | \sum_{3q} I_+\sigma_z | n\uparrow\rangle =$$

$$\frac{1}{\sqrt{3}}\left\langle \sqrt{2}u\uparrow u\uparrow d\downarrow - \frac{u\uparrow u\downarrow + u\downarrow u\uparrow}{\sqrt{2}} d\uparrow \right| \sum_{3q} I_+\sigma_z \frac{1}{\sqrt{3}} \left| \sqrt{2}d\uparrow d\uparrow u\downarrow - \frac{d\uparrow d\downarrow + d\downarrow d\uparrow}{\sqrt{2}} u\uparrow \right\rangle =$$

$$\frac{1}{3}\left\langle \sqrt{2}u\uparrow u\uparrow d\downarrow - \frac{u\uparrow u\downarrow + u\downarrow u\uparrow}{\sqrt{2}} d\uparrow \right\| \begin{matrix} \sqrt{2}u\uparrow d\uparrow u\downarrow + \sqrt{2}d\uparrow u\uparrow u\downarrow - \frac{u\uparrow d\downarrow - u\downarrow d\uparrow}{\sqrt{2}} u\uparrow \\ - \frac{-d\uparrow u\downarrow + d\downarrow u\uparrow}{\sqrt{2}} u\uparrow \end{matrix} \right\rangle =$$

$$\frac{1}{3}\left\langle \sqrt{2}u\uparrow u\uparrow d\downarrow - \frac{u\uparrow u\downarrow + u\downarrow u\uparrow}{\sqrt{2}} d\uparrow \right\| \sqrt{2}(u\uparrow u\downarrow + u\downarrow u\uparrow)d\uparrow + \frac{u\uparrow u\downarrow + u\downarrow u\uparrow}{\sqrt{2}} d\uparrow - 2\frac{u\uparrow u\uparrow d\downarrow}{\sqrt{2}} \right\rangle$$

$$= \frac{1}{3}\left\langle \sqrt{2}u\uparrow u\uparrow d\downarrow - \frac{u\uparrow u\downarrow + u\downarrow u\uparrow}{\sqrt{2}} d\uparrow \right\| 3\frac{u\uparrow u\downarrow + u\downarrow u\uparrow}{\sqrt{2}} d\uparrow - \sqrt{2}u\uparrow u\uparrow d\downarrow \right\rangle$$

$$= \frac{-2-3}{3} = -\frac{5}{3}$$

Plugging in our result from last week, we get for the proton $\Delta u - \Delta d = 4/3 + 1/3 = 5/3$, the same result (never mind the minus sign).

b) Yup, the result for a) would predict 5/18 for the difference $\Gamma_1^p - \Gamma_1^n$ between proton and neutron, which is equal to the value from last week. Of course, both of them are somewhat off: The "true" value of g_A is 1.26 (only 75% of our result), and the value for the Bjorken sum rule is even smaller than what you get for this value (0.21) at finite Q^2 (because of pQCD corrections).

Problem 2)

a) E_0 is the mass difference between neutron and proton, which is 1.2933 MeV. We get for the life time

$$\tau_n = \frac{60\pi^3\hbar}{0.47 E_0^5}\left(\frac{g_V^2}{(\hbar c)^6} + 3\frac{g_A^2}{(\hbar c)^6}\right)^{-1} = 7.20\,10^{-7}s\left[\left(\frac{g_V^2}{(\hbar c)^6} + 3\frac{g_A^2}{(\hbar c)^6}\right)/GeV^4\right]^{-1}$$

For g_V I get $g_V = -G_F \cos\theta_C$ and for g_A I get $g_A = 5/3\ G_F \cos\theta_C$

b) Using $\frac{G_F}{(\hbar c)^3} = 1.166\,10^{-5} GeV^{-2}$ and $\cos\theta_c = 0.98$, I get

$$\tau_n = 7.20\,10^{-7}s\left[\left(\frac{g_V^2}{(\hbar c)^6} + 3\frac{g_A^2}{(\hbar c)^6}\right)/GeV^4\right]^{-1} = \frac{7.20\,10^{-7}s}{\cos^2\theta_c}\left[\left(1+3\frac{25}{9}\right)\frac{G_F^2}{(\hbar c)^6}/GeV^4\right]^{-1} = 591s$$

.

If I use $g_A = 1.261 G_F \cos\theta_c$ instead, I get 956s. That’s not too far off the 887 s quoted.