Scattering

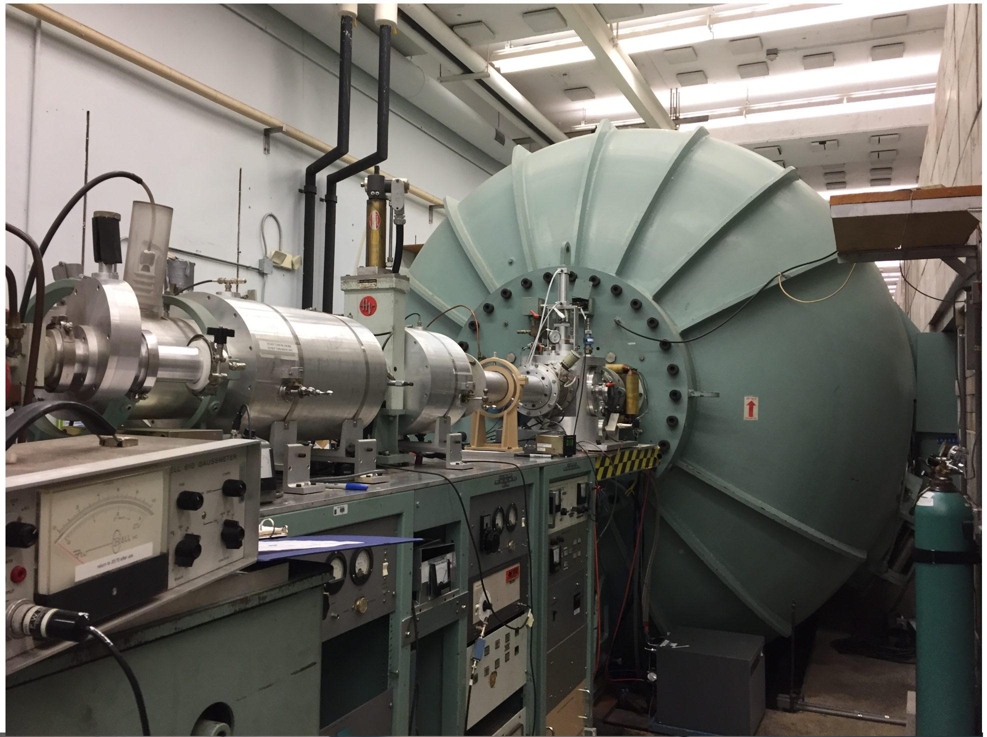
Nuclear Physics 722/822
Sebastian Kuhn

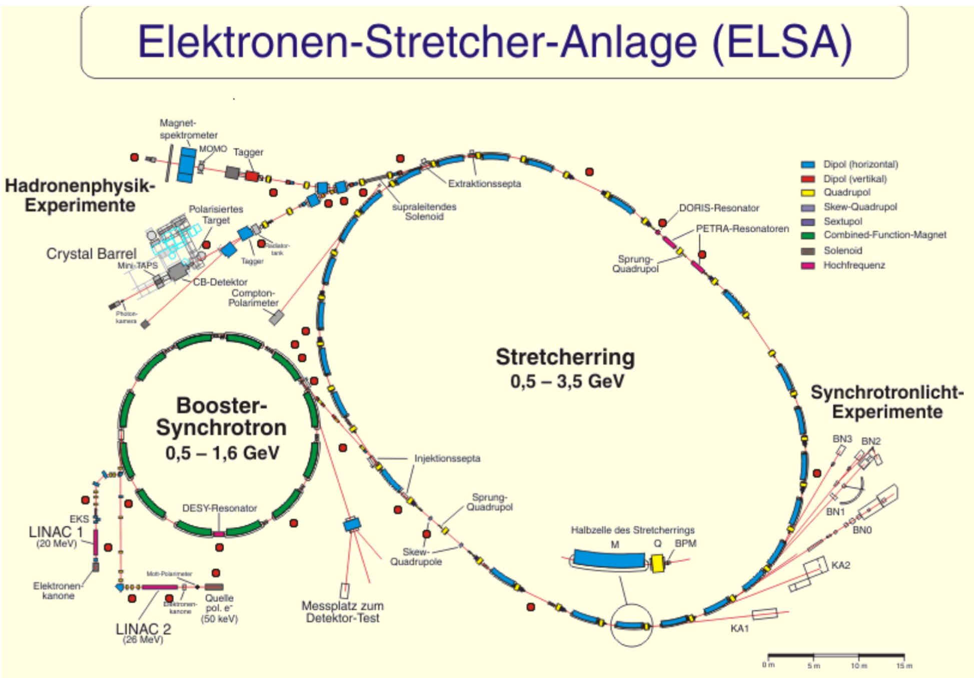
What Do we Need?

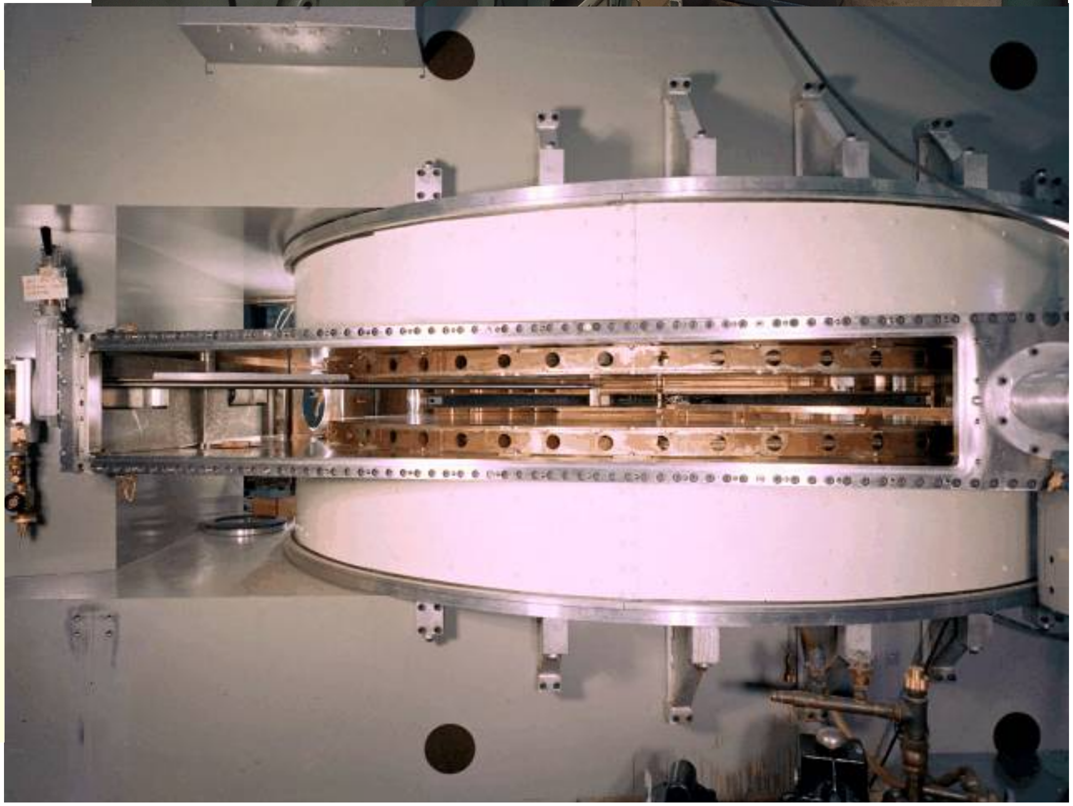
- (Polarized) Beam
- Electrons or muons → γ^*; pions, protons, antiprotons, light nuclei, heavy ions…
- (Polarized) Targets (or counterrotating beams)
- Protons, deuterons, ^{3}He, heavier nuclei, heavy ions, antiprotons
- Detectors
- For scattered/produced electrons/muons/… and hadrons/nuclei
- Facilities

Low-Medium Energy Accelerators

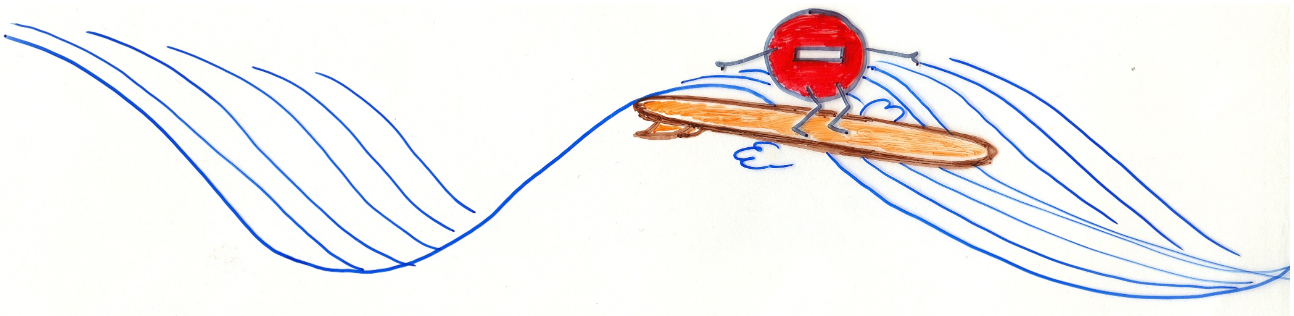
- (Tandem) Van de Graaf
- Cyclotron
- Synchrotron







High-energy Accelerators

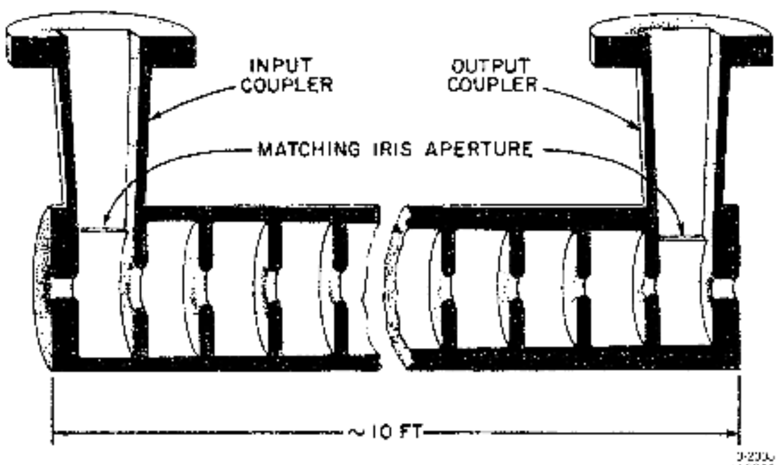


Surf the microwaves!

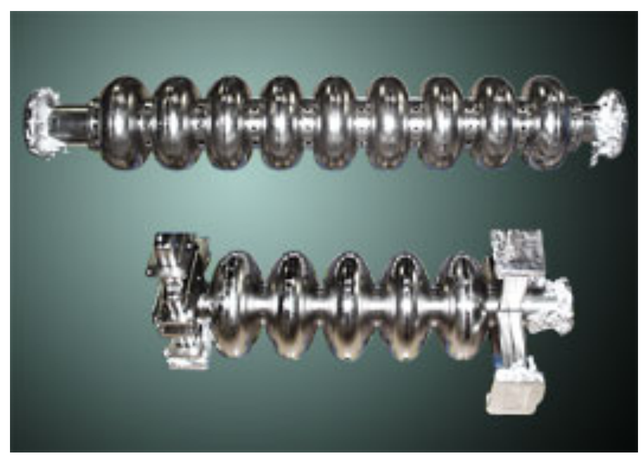
Accelerating cavity: disk loaded cylindrical wave guide

use TM_{01} mode to get a longitudinal electric field

match phase and velocity



DESY



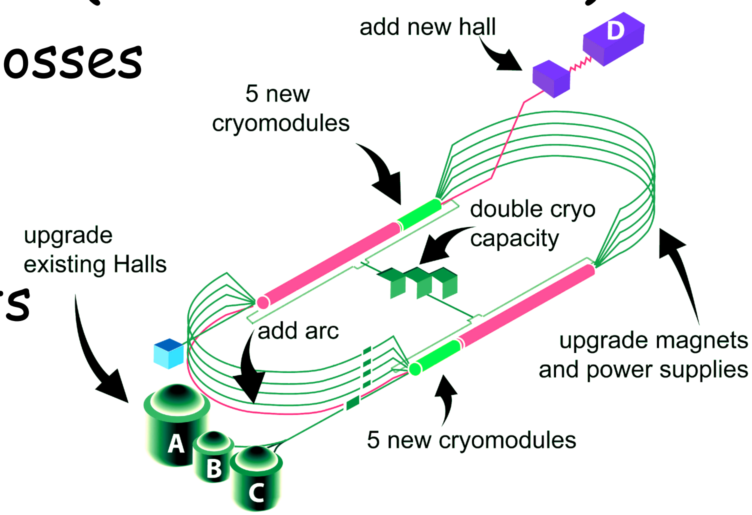
new

old

Jefferson Lab

High Energy Accelerators – 2 Examples

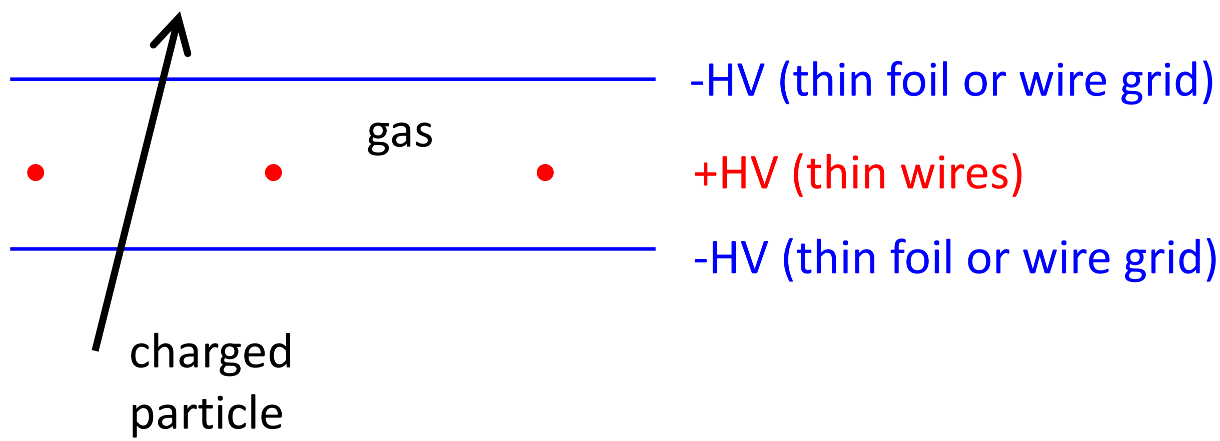
- Superconducting Linear Accelerators (CEBAF at JLab)
  - 2K niobium cavities, very low resistive losses
  - Recirculate few times, 100's of μA
  - High gradient (5-50 MeV/m $\Rightarrow$ 4-12 GeV)
  - CW extracted beam on external targets
  - Thick targets $\Rightarrow$ high luminosity



- Storage rings (HERA at DESY, RHIC at BNL, LHC at CERN)
  - Large circulating currents (mA)
  - Recirculate millions of times
  - Require only modest (re)acceleration
  - CW internal beam on thin gas targets or counterrotating beams
    (typically lower Luminosity)

Typical Detector Elements

Wire chambers measure position (and angle)



1. Charged particle passes through wire chamber and knocks out electrons from the gas.
2. Electrons drift in the E field to the cathode wire, colliding with gas molecules
3. Close to the wire, the mean free path times the electric field is large enough to ionize the gas molecules. Avalanche!
4. Read the signal on the cathode wire (time gives distance)

Similar: $G_{as}E_{lectron}M_{ultiplier}s$, µMEGAs,...

Applications: VDC, Multi-layer drift chamber (track → $\vec{p}$), $T_{ime}P_{rojection}C_{hamber}$

Typical Detector Elements

Scintillators: time ($\Rightarrow \beta \Rightarrow$ particle type) and energy measurement
(typical resolution: down to 50-100 ps for plastic)

- Typically a doped plastic or crystal (eg: Ge, NaI, BaF_2)
- Charged particle passes through scintillator (or neutral particle interacts) and excites atomic electrons. These de-excite and emit light.
- Minimum energy loss (when $\beta\gamma \approx 1$) is $dE/dx = 2$ MeV/(g/cm^2)

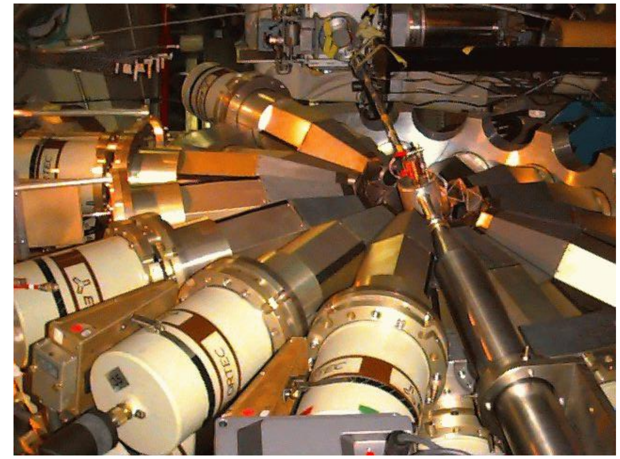
Cherenkov counter: threshold velocity measurement.

- Typically an empty box with smoke (ie: a gas) and mirrors
- Local light speed is $v = 1/n < c$
- Particles travelling faster than v will emit Cherenkov light (an electromagnetic 'sonic boom') $\Rightarrow$ threshold CC (yes/no)
- The opening angle of the Cherenkov cone is related to the particle's velocity $\Rightarrow$ R~ing~I~maging~CH~erenkov~ (measure $\beta \Rightarrow$ particle type)

Also: T~ransition~R~adiation~D~etector~, DIRC,

Typical Detector Elements

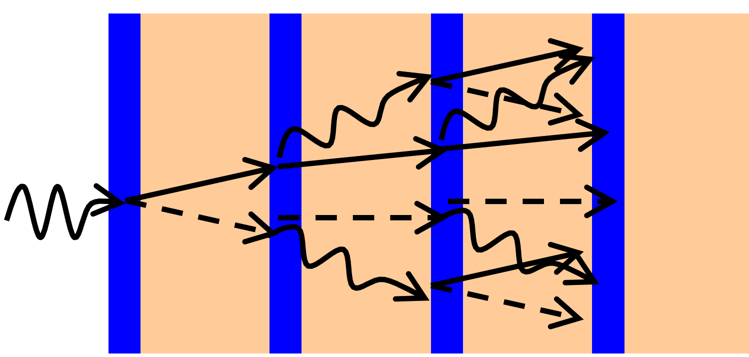
Photon Counters ->

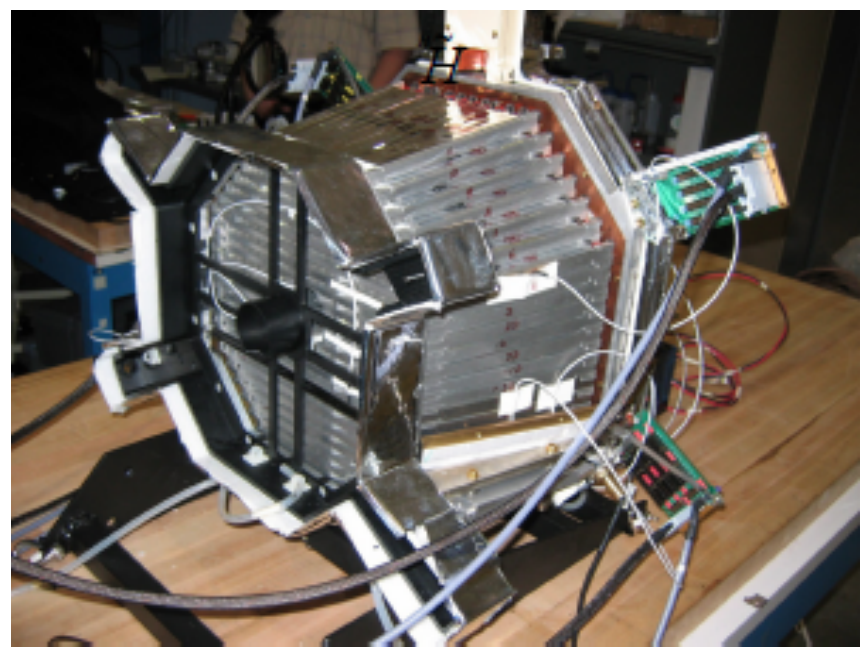


Electromagnetic shower counters:
measure energy (+ time), discriminate electrons and detect neutral particles

- Electrons and photons passing through material shower
- After one radiation length of material on average:
  - Electrons emit a bremsstrahlung photon
  - Photons convert to an electron/positron pair or Compton-scatter
- After ≈10 radiation lengths, one e^- or γ is now ~1000 particles
- Simple design: alternating layers of lead (R_L = 6 mm) and scintillant

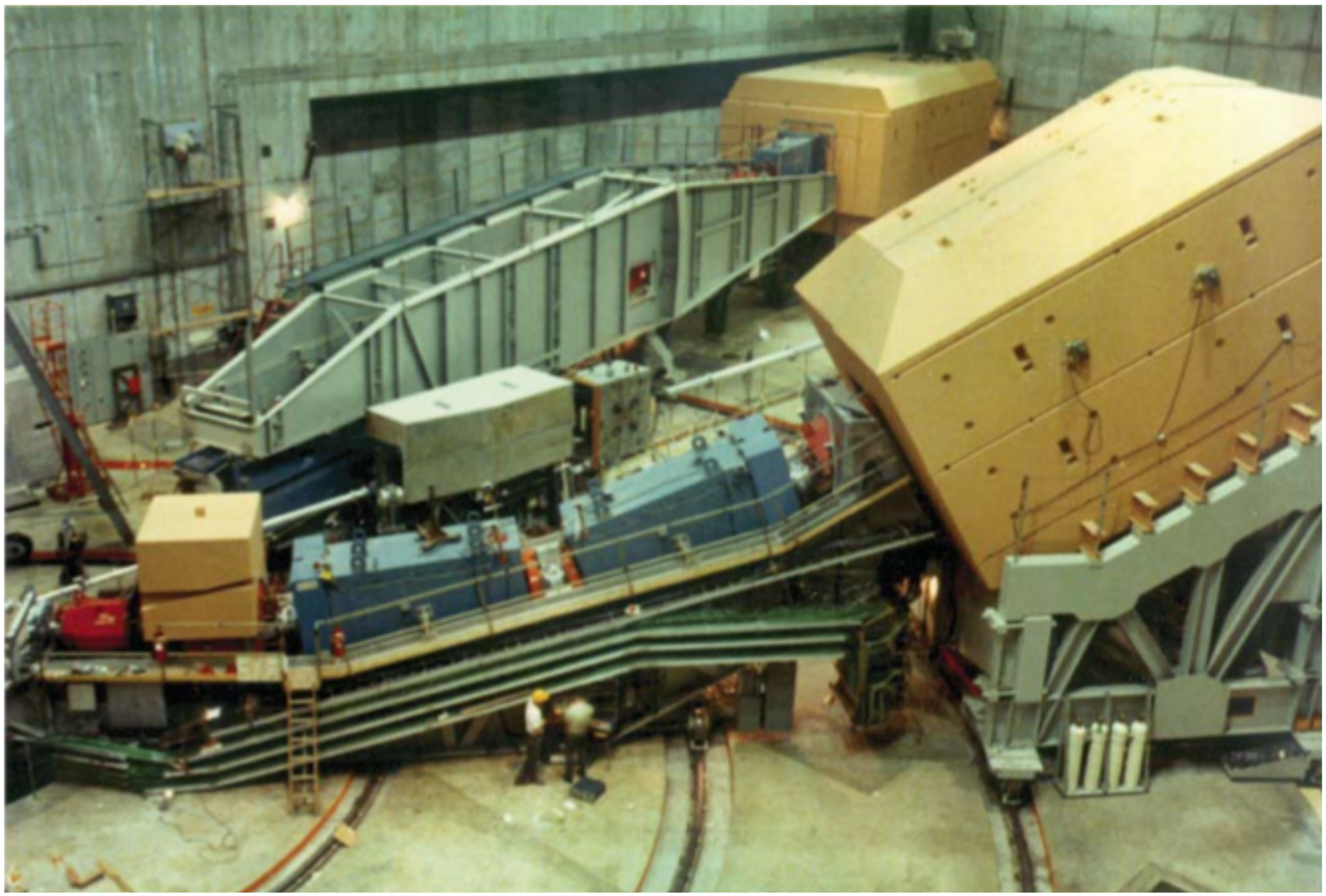
  Higher resolution: Heavy metal glass (Pb glass, $PbWO_4$) combine both
  - Particles shower in the lead
  - Charged particles deposit energy in the scintillant



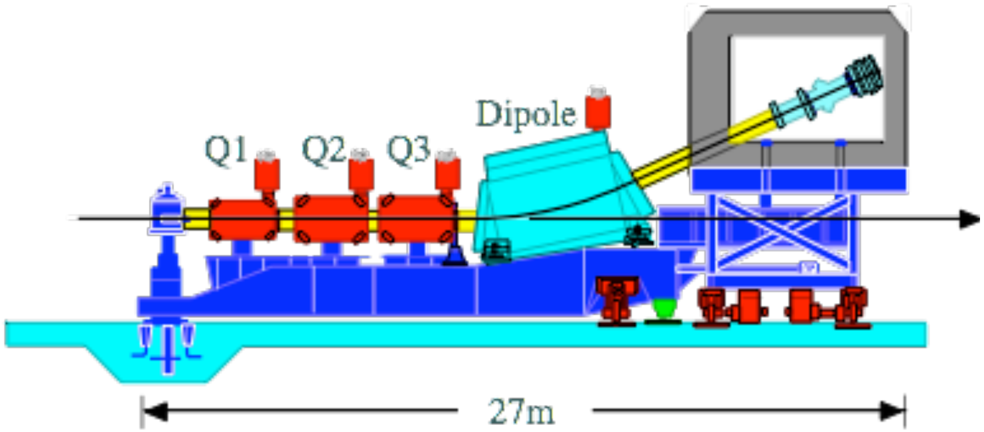


Also: Hadronic Calorimeter, μ counter…

Detectors+Magnets = Spectrometers

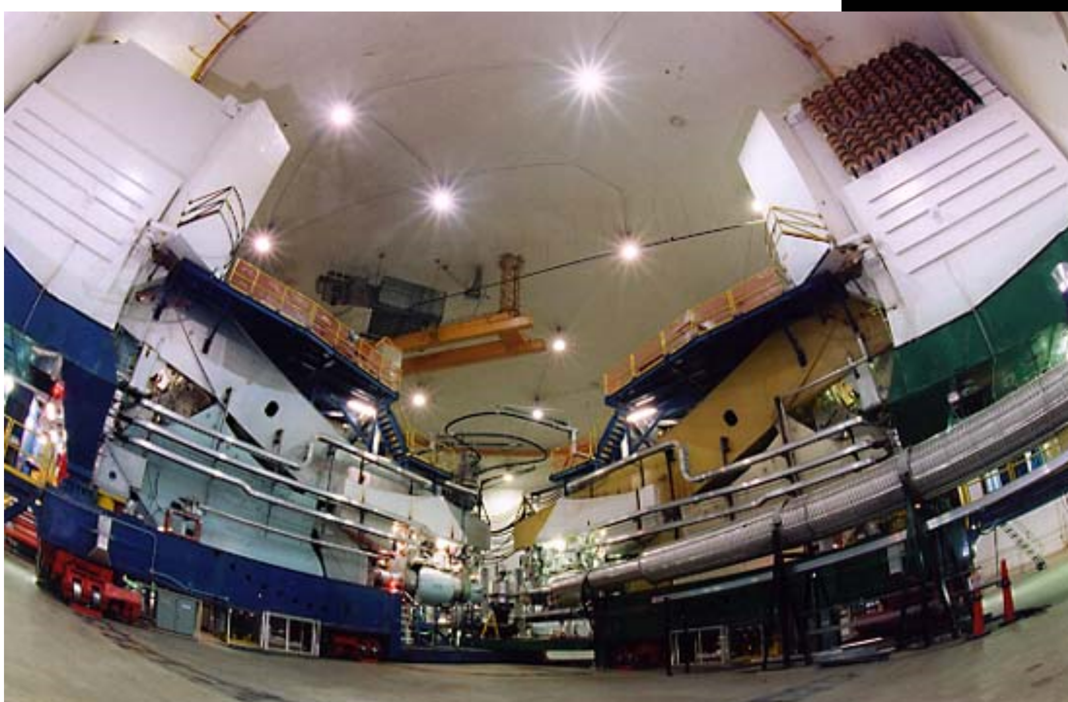


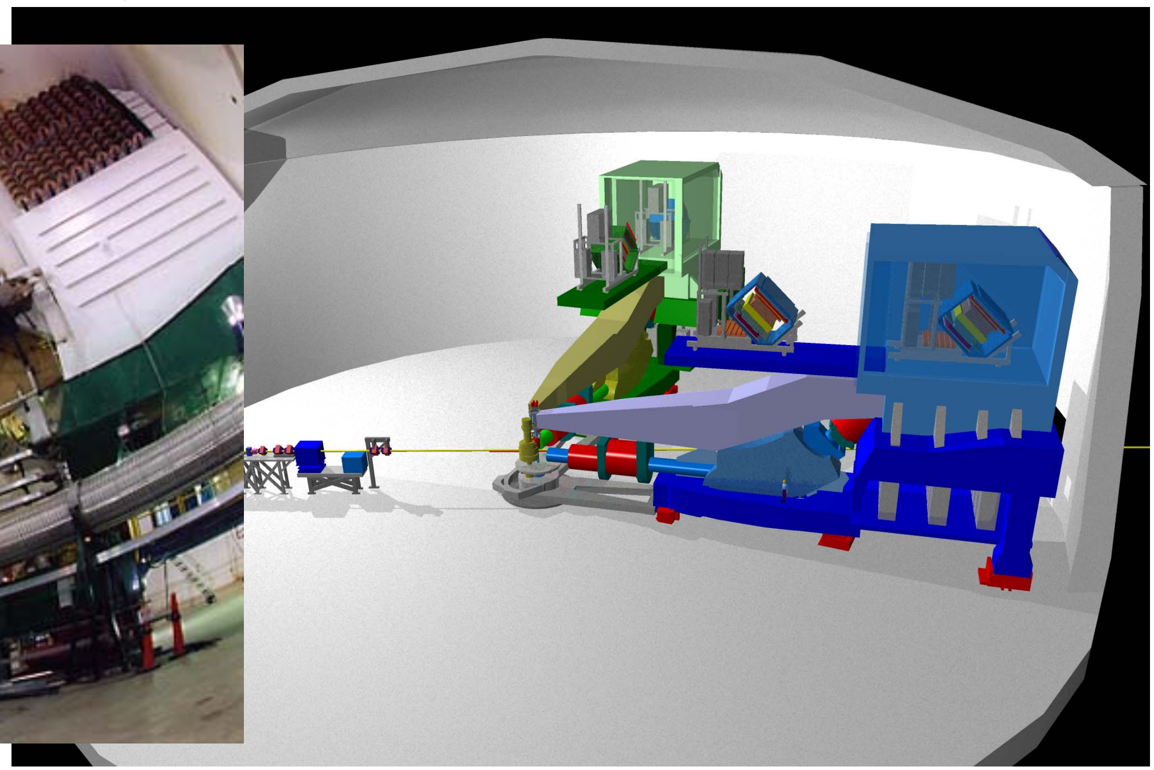
SLAC End Station A – where the quarks were discovered experimentally



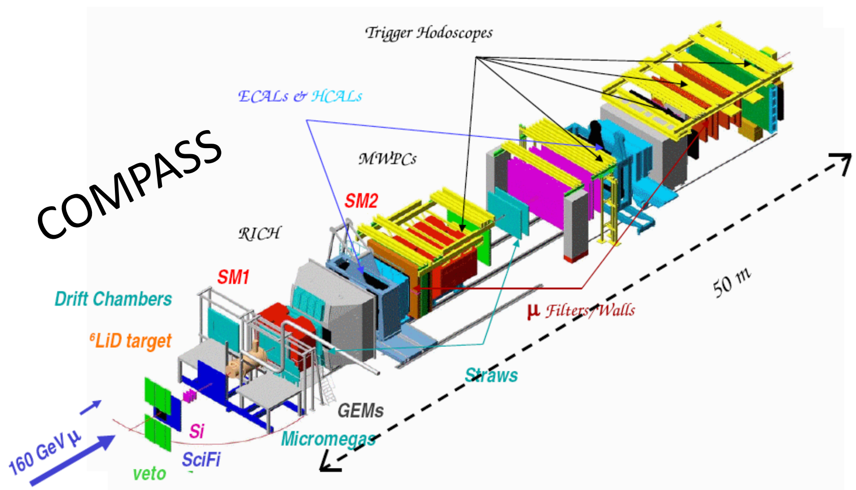
Jefferson Lab Hall A (Hall C similar)

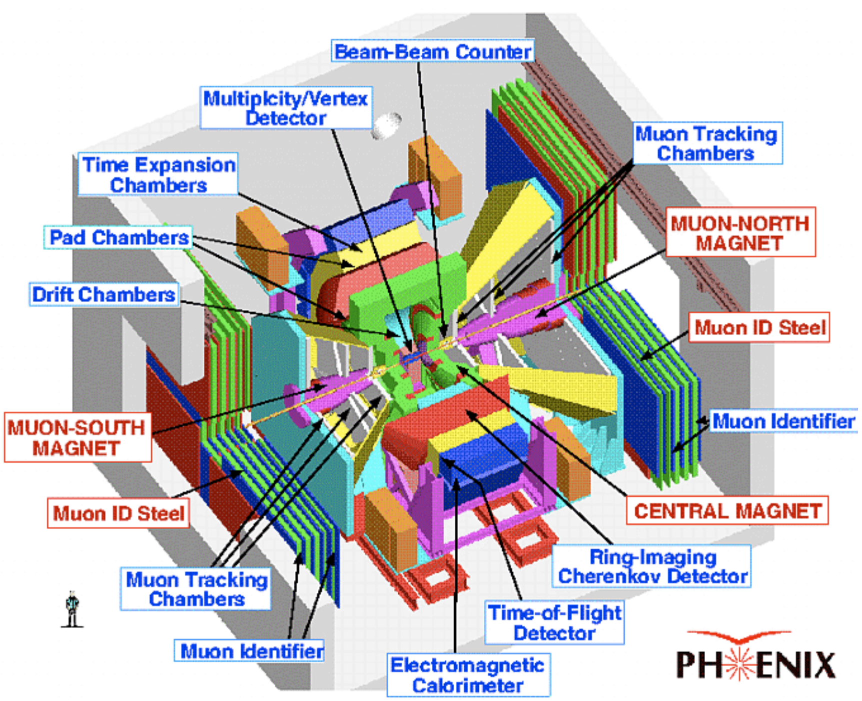
Typically small acceptance but high resolution, very good shielding
(→ high luminosity)

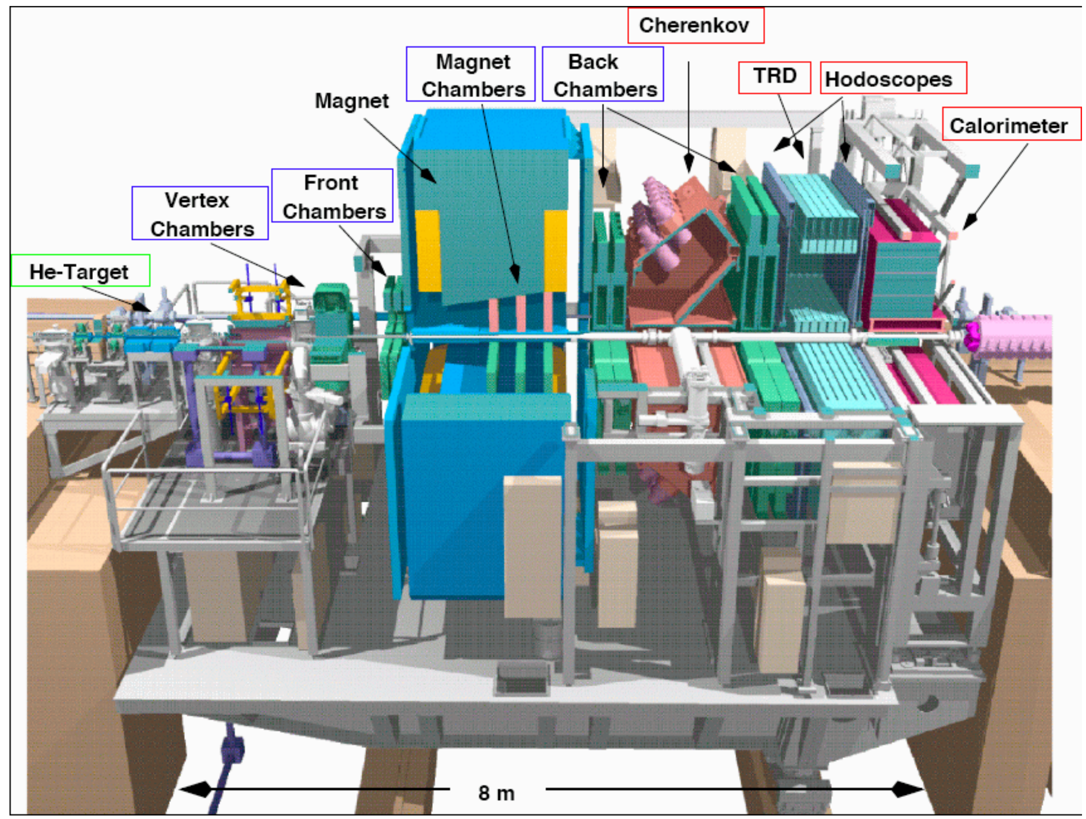




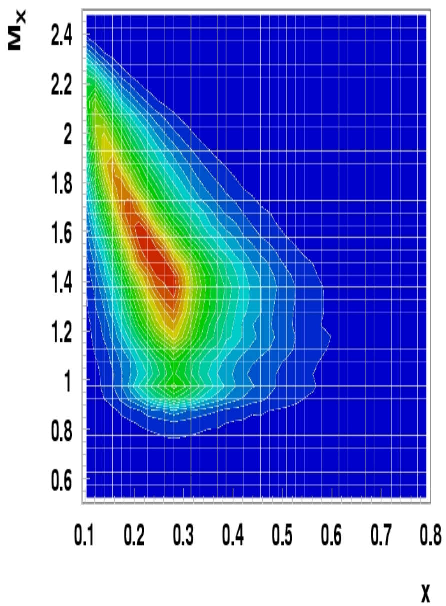
Large Acceptance Spectrometers

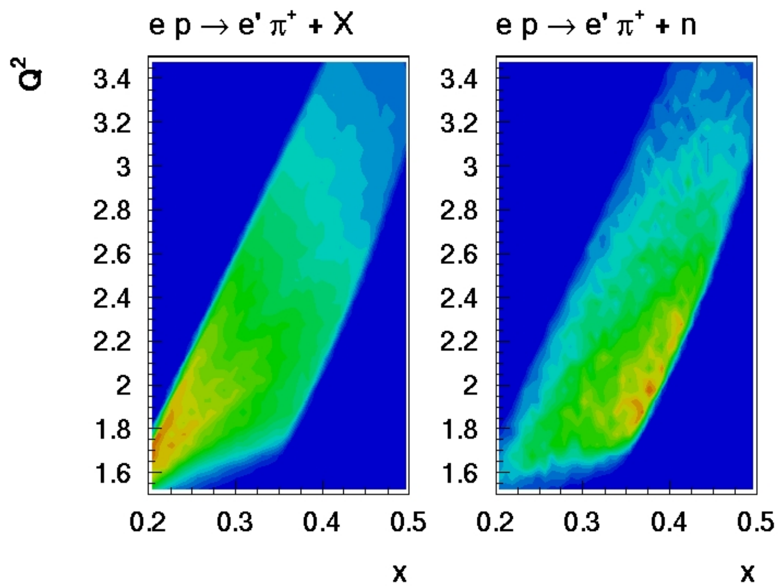


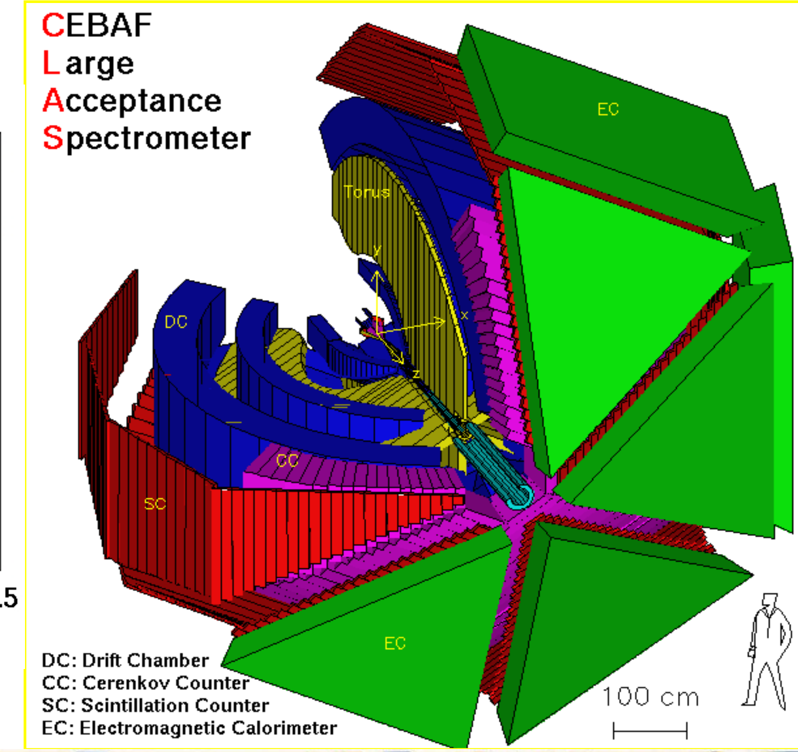


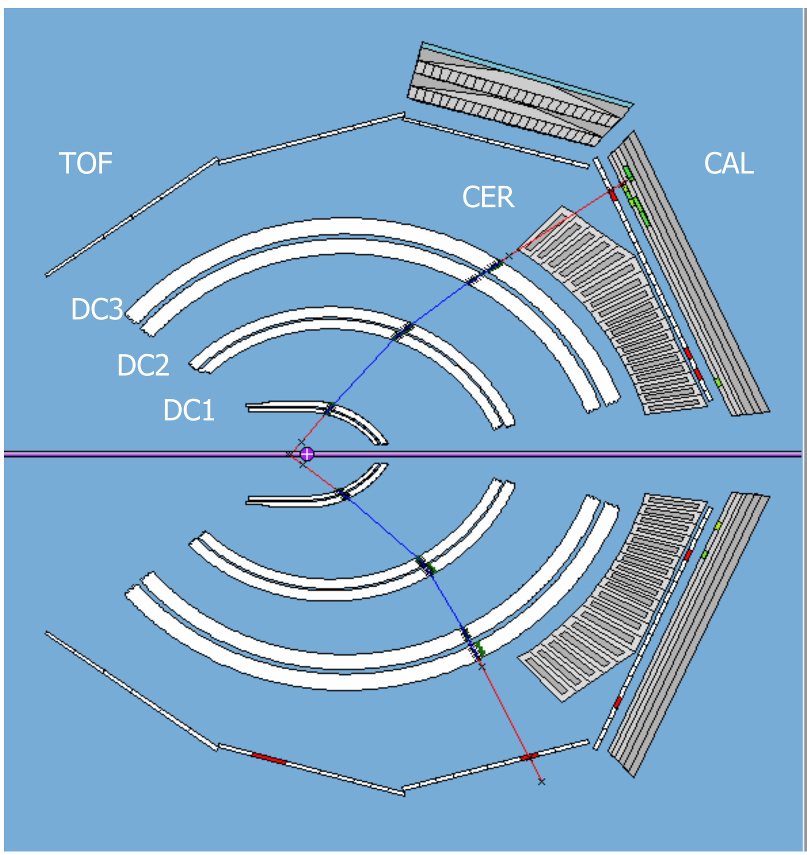


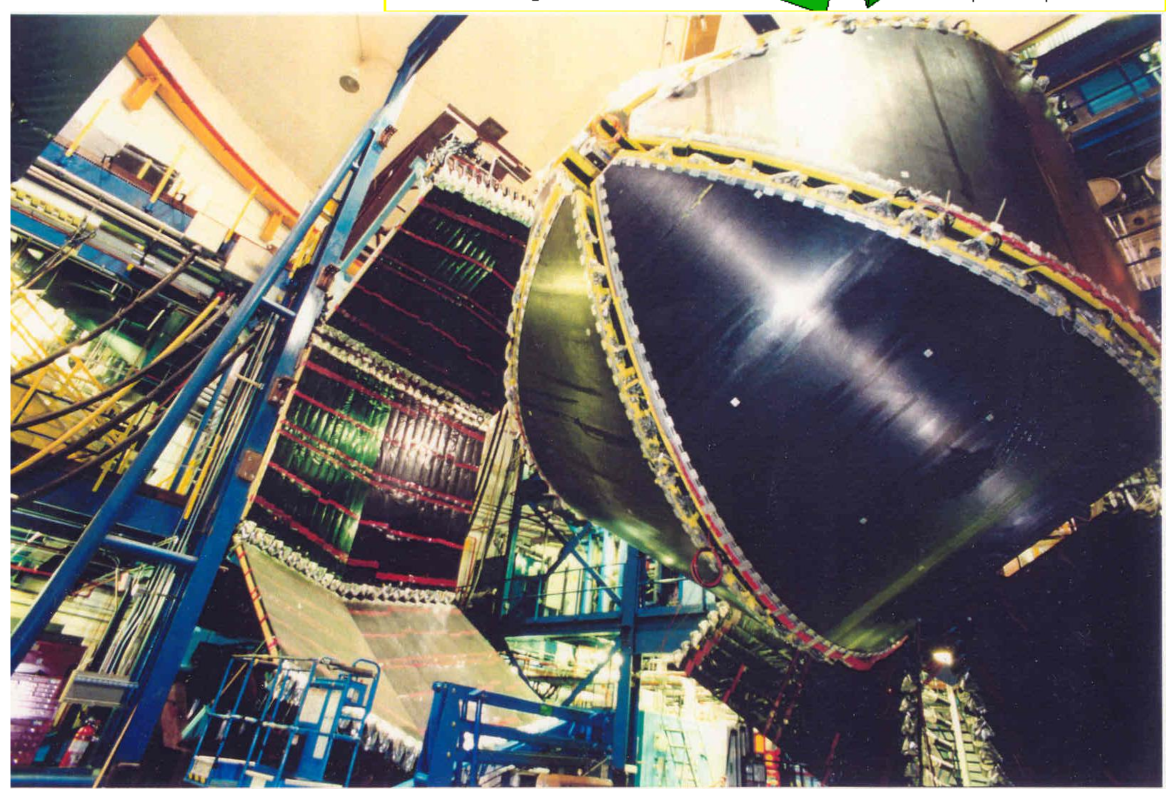
HERMES


M_X
2.4
2.2
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8
x


$e\,p \to e'\,\pi^+ + X$
$e\,p \to e'\,\pi^+ + n$
Q^2
3.4
3.2
3
2.8
2.6
2.4
2.2
2
1.8
1.6
0.2 0.3 0.4 0.5
x


CEBAF
Large
Acceptance
Spectrometer
EC
Torus
DC
CC
SC
DC: Drift Chamber
CC: Cerenkov Counter
SC: Scintillation Counter
EC: Electromagnetic Calorimeter
100 cm


TOF
CER
CAL
DC3
DC2
DC1



CLAS12

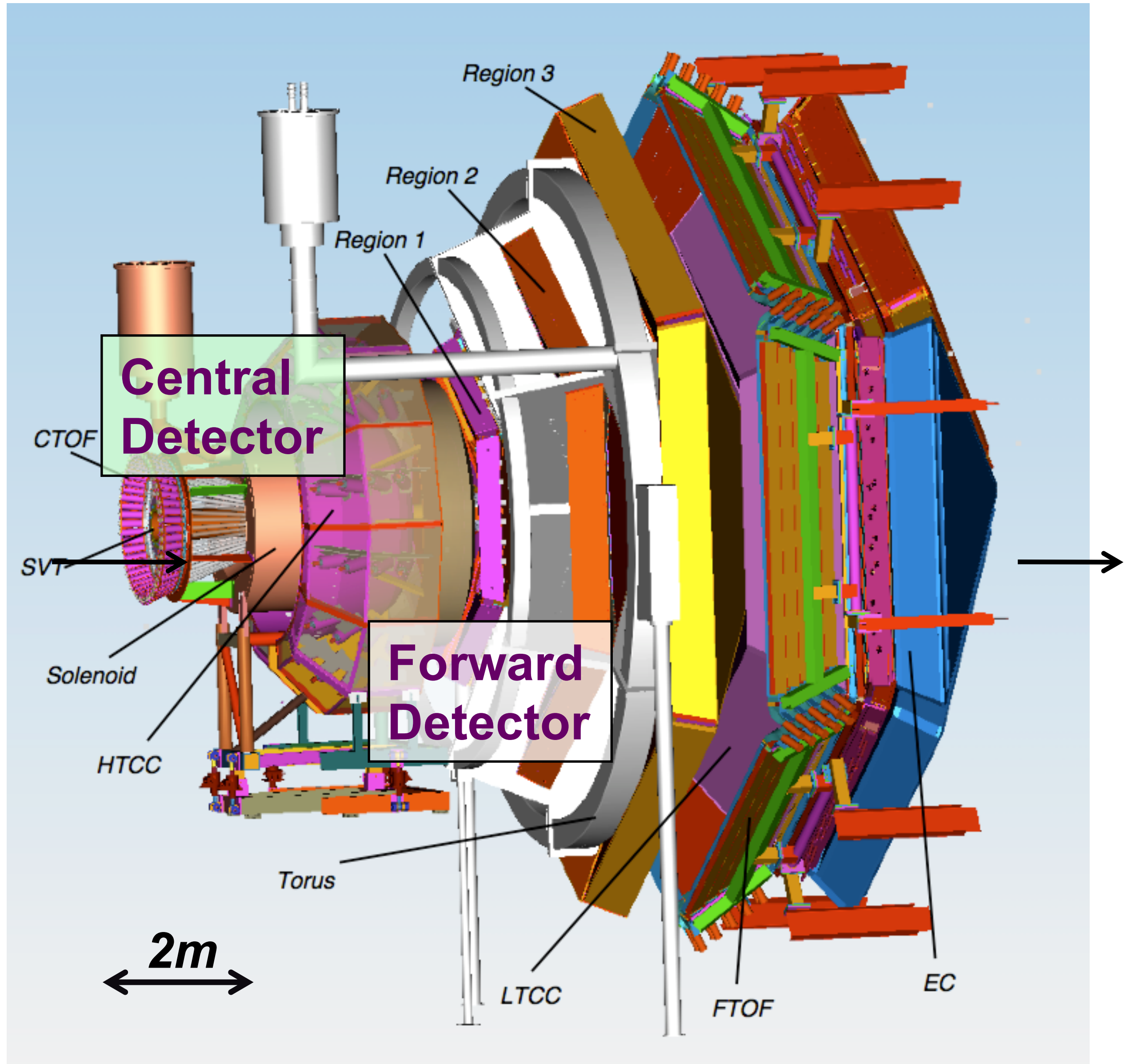
Another nearly 4π detector in Hall D (GLUEX)

Base equipment

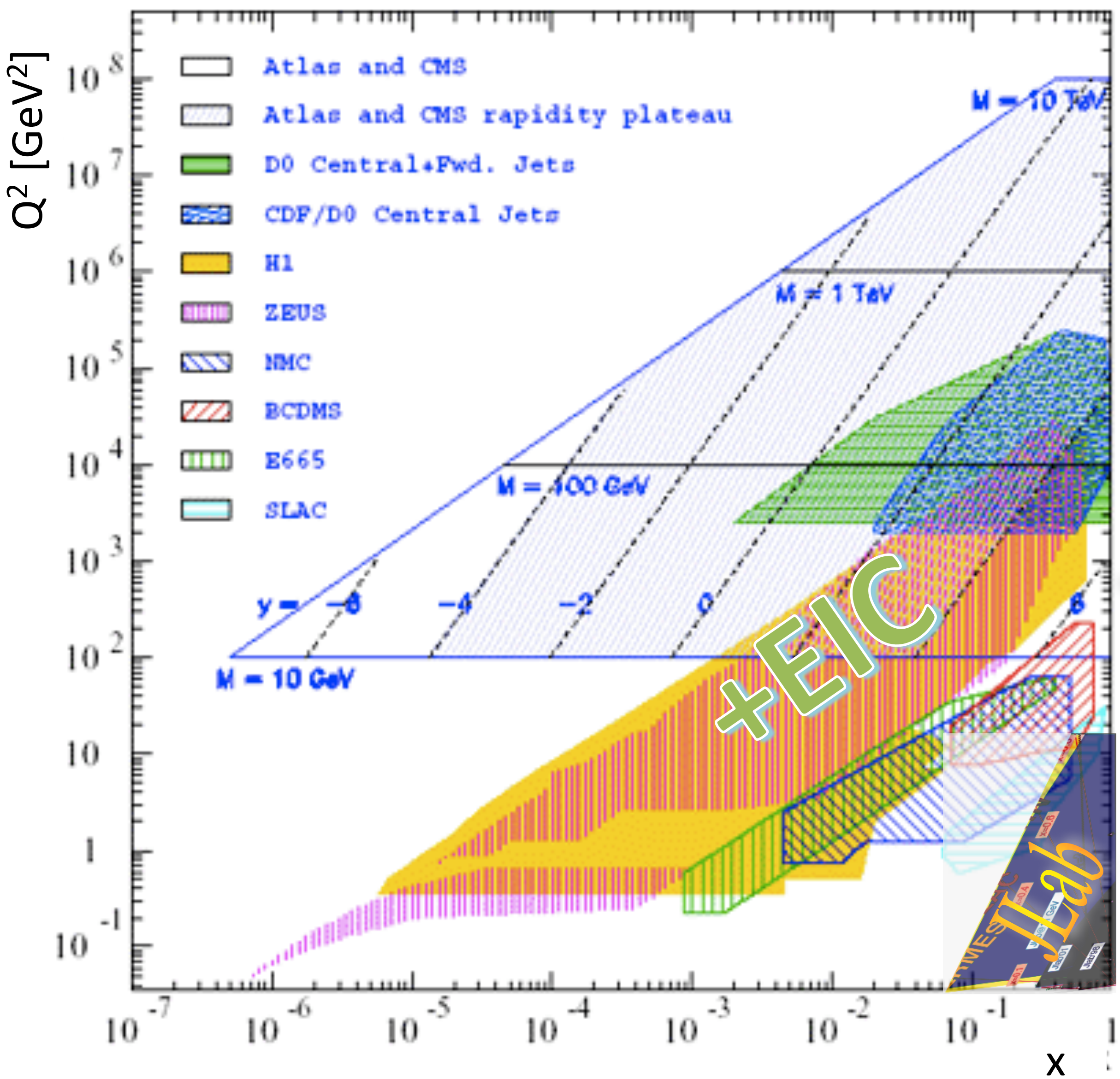
- **Forward Detector**
- TORUS magnet
- Forward vertex tracker
- HT Cherenkov Counter
- Drift chamber system
- LT Cherenkov Counter
- Forward ToF System
- Preshower calorimeter
- E.M. calorimeter
- **Central Detector**
- SOLENOID magnet
- Barrel Silicon Tracker
- Central Time-of-Flight

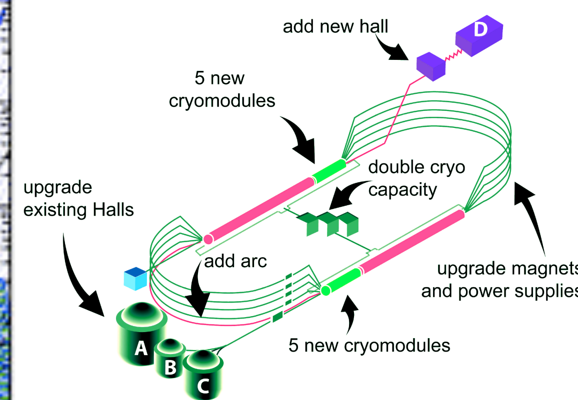
Additional equipment

- Micromegas (CD & FD)
- RICH counter (FD)
- Neutron detector (CD)
- Small angle tagger (FD)



Jefferson Lab in Perspective





JLab DIS

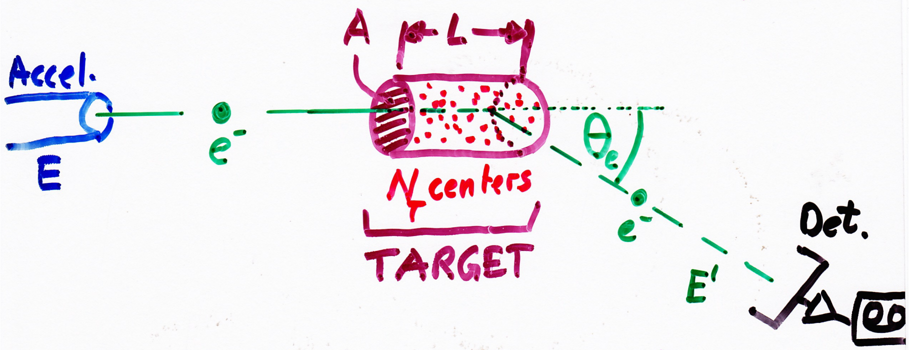
Now: 12 GeV

$Q^2 = 1 \ldots 13$ GeV^2

$x = 0.06 \ldots 0.8$

$W < 4$ GeV

Electron Scattering – what can we measure?



What is the likelihood to find the electron scattered into the detector?

$$P \sim n_T \cdot L = \frac{N_T}{A\,L}\cdot L = \frac{N_T}{A}\;{}^{*}$$

$\Rightarrow$ call $\Delta\sigma = P/\left(\frac{N_T}{A}\right)$ (cross section)

$\Delta\sigma$ DEPENDS on the kinematics (E, E', θ_e) and is $\approx$ proportional to SIZE of kinematic bin spanned by the detector

*Note: $\frac{N_T}{A} = \rho\left[\frac{g}{cm^3}\right]\cdot L\,[cm]\cdot \frac{\text{Avogadro}}{\text{Atomic Weight [u]}}$

Count rate (L = luminosity):

$$\dot{N} = P\cdot \dot{n}_{el} = \Delta\sigma\cdot\frac{N_T}{A}\dot{n}_{el} = \Delta\sigma\cdot\frac{N_T}{A}\frac{I}{e} = \Delta\sigma\cdot L$$

In general:

$$\Delta\sigma = \Delta\sigma\left(E'\ldots E'+\Delta E', \theta_e\ldots\theta_e+\Delta\theta_e, \varphi\ldots\varphi+\Delta\varphi\right)$$

Limit of infinitesimal acceptance:

$$\Delta\sigma = \frac{d^3\sigma}{dE'\,d\theta_e\,d\varphi}\left(E',\theta_e,\varphi\right)\Delta E'\Delta\theta_e\Delta\varphi = \frac{d\sigma}{dE'\,d\Omega}\Delta E'\Delta\Omega$$

(use Jacobian to transform variables)

In case of more particles/observables and finite phase space:

$$\Delta\sigma = \iiint_{\substack{Phase\\Space}} \frac{d^n\sigma}{dk_1 dk_2\ldots dk_n}\left(k_1\ldots k_n\right) Acc\left(k_1\ldots k_n\right) dk_1 dk_2\ldots dk_n$$

where Acceptance is defined by detector boundaries and bin sizes as well as number of observed particles

(inclusive = only 1, semi-inclusive, exclusive)

Kinematic Variables

$$\nu = E - E' = \left|\vec{k}\right| - \left|\vec{k}'\right|$$

$$\vec{q} = \vec{k} - \vec{k}'; \quad q^{\mu} = (\nu, \vec{q}) = k^{\mu} - k'^{\mu}$$

$$Q^2 = -\left(k - k'\right)^2 = \vec{q}^{\,2} - \nu^2 \approx 4EE'\sin^2\frac{\theta_e}{2}$$

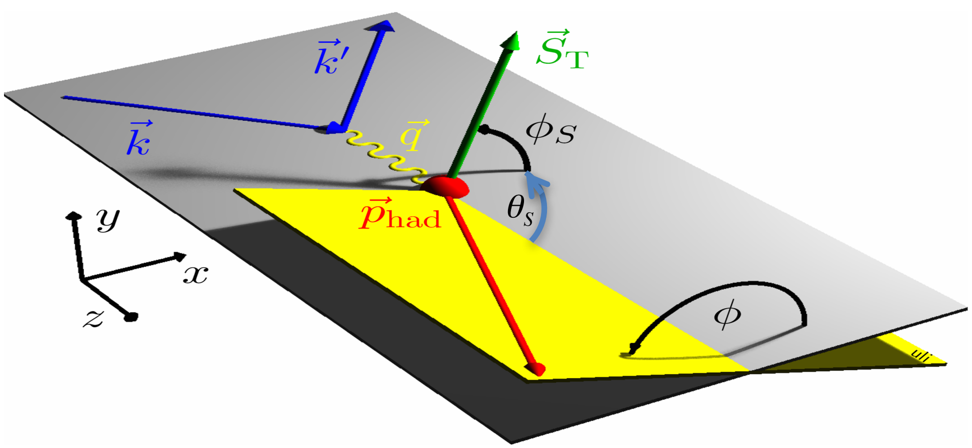
$$y = \frac{q^{\mu}P_{\mu}}{k^{\mu}P_{\mu}} = \frac{\nu}{E}; \quad x = \frac{Q^2}{2q^{\mu}P_{\mu}} = \frac{Q^2}{2M\nu}$$

$$W = \sqrt{(P_{\mu} + q^{\mu})^2} = \sqrt{M^2 + (1/x - 1)Q^2}$$

Lepton variables

Inclusive

Semi-Inclusive



Hadron variables

$$P^{\mu}_{had} = \left(z\nu, \sqrt{z^2\nu^2 - m_h^2 - P_{hT}^2}, \vec{P}_{hT}\right)$$

$$z = E_h / \nu$$

$$\vec{P}_{hT} = \left|\vec{P}_{hT}\right|(\sin\theta_h \cos\phi_h, \sin\theta_h \sin\phi_h)$$

$$x_F = \frac{\vec{P}_h^{c.m.} \cdot \hat{q}}{\left|\vec{P}_h^{c.m.}\right|(\max)}$$

Rapidity $\quad \operatorname{artanh}\frac{|\mathbf{p}|c}{E} = \frac{1}{2}\ln\frac{E + |\mathbf{p}|c}{E - |\mathbf{p}|c}$

Pseudo-Rapidity $\quad \eta = -\ln\left[\tan\left(\frac{\theta}{2}\right)\right]$

Electron Scattering – what can we measure?

What is the likelihood to find the electron scattered into the detector?

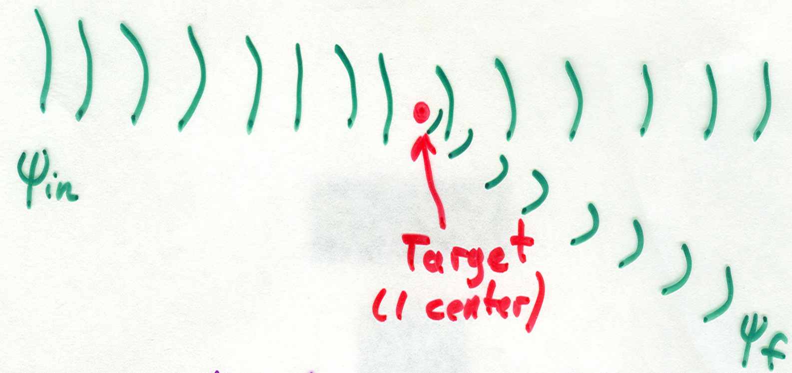
$$P \sim n_T \cdot L = \frac{N_T}{A\,L} \cdot L = \frac{N_T}{A} \;{}^{*}$$

$\Rightarrow$ call $\Delta\sigma = P/\left(\frac{N_T}{A}\right)$ (cross section)

$\Delta\sigma$ DEPENDS on the kinematics (E, E', θ_e) and is $\approx$ proportional to SIZE of kinematic bin spanned by the detector

*Note: $\frac{N_T}{A} = \rho\left[\frac{g}{cm^3}\right] \cdot L\,[cm] \cdot \frac{\text{Avogadro}}{\text{Atomic Weight [u]}}$

Electron Scattering – Theorist's View



What is the transition rate $W_{i\to f}$?

$$\dot{N}_{e,f} = \dot{N}_{e,in} \cdot P(i\to f) = I_{e,in} \cdot \frac{N_T}{A} \cdot \Delta\sigma$$

$$= \frac{I_{e,in}}{A} \cdot N_T \cdot \Delta\sigma = \left|\vec{j}_{e,in}\right|_z \cdot N_T \cdot \Delta\sigma$$

$$\Rightarrow W_{i\to f} = j_{in} \cdot \Delta\sigma$$

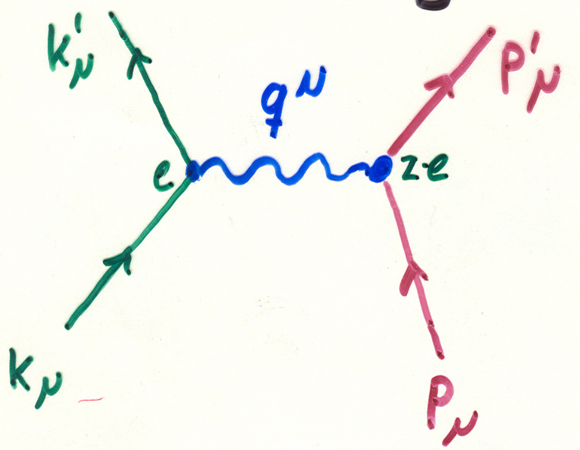
Fermi's GOLDEN Rule:

$$W_{i\to f} = \frac{2\pi}{\hbar} \left|\mathcal{M}_{fi}\right|^2 \Delta\phi$$

Phase space spanned by detector/kinematic bin

$$\mathcal{M}_{fi} = \langle \psi_f \,|\, H_{int} \,|\, \psi_{in} \rangle$$

Elastic Scattering – Feynman diagram



$$\mathfrak{M}_{fi} = e\, j_\nu \left(-\frac{1}{q^\nu q_\nu}\right) z j^\nu \qquad Q^2 := -q^\nu q_\nu$$

Based on $\Box^2 A_\nu = j_\nu \Rightarrow A_\nu = \left(\frac{1}{iq}\right)^2 j_\nu$

and $H_{int} = A_\nu j^\nu = V\rho - \vec{A}\cdot\vec{j}$

$$\Rightarrow \Delta\sigma = \frac{4z^2\alpha^2(\hbar c)^2}{Q^4}\left(1-\beta^2\sin^2\frac{\theta}{2}\right)^{*)}\left(1+2\frac{\nu^2}{Q^2}\tan^2\frac{\theta}{2}\right)^{*)}\cdot$$

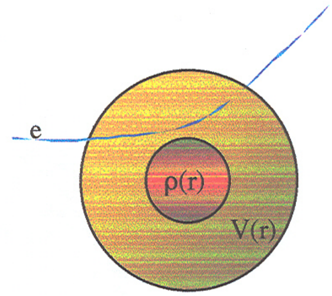
$$\cdot \int d^3\vec{k}'\,\delta(E'-E'_{el}) \left(= E'^2\cdot\Delta\Omega\right)$$

*) magnetic interaction due to electron spin

*) magnetic interaction due to target spin

Form Factors

Low-medium energy: Distribution of charge and magnetism inside the hadron



$$\vec{\nabla}^2 V(\mathbf{r}) = -4\pi\rho(\mathbf{r}) \quad \Rightarrow$$

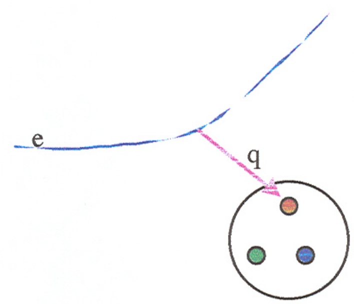
$$q^2 V \propto \int e^{i\mathbf{qr}}\rho(\mathbf{r})d^3\mathbf{r} = F(q)$$

$$\mathcal{H} \approx -eV \propto \frac{F(q)}{q^2}$$

$$\frac{d\sigma}{d\Omega} \propto \left|\langle f|\mathcal{H}|i\rangle\right|^2 \propto \frac{F^2(q)}{q^4}$$

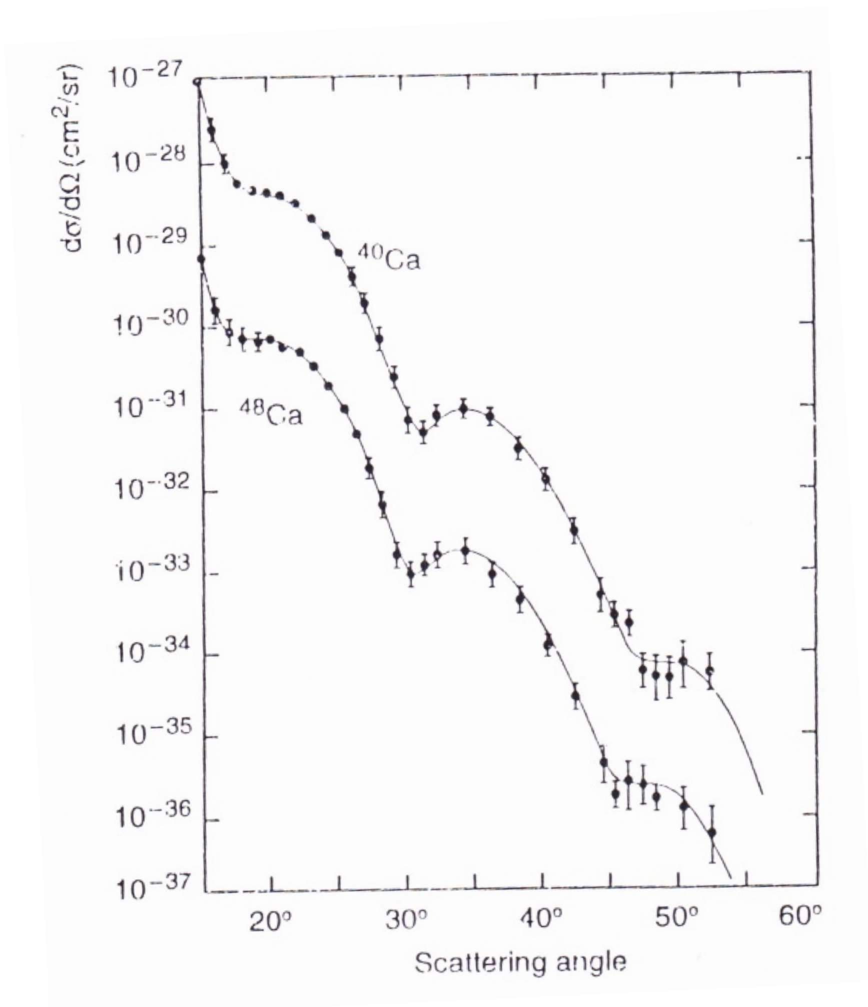
Ex: $\rho(r) = a\cdot e^{-\alpha r} \Rightarrow F(q^2) = (1+ q^2/\alpha^2)^{-2}$ (Dipole Form)

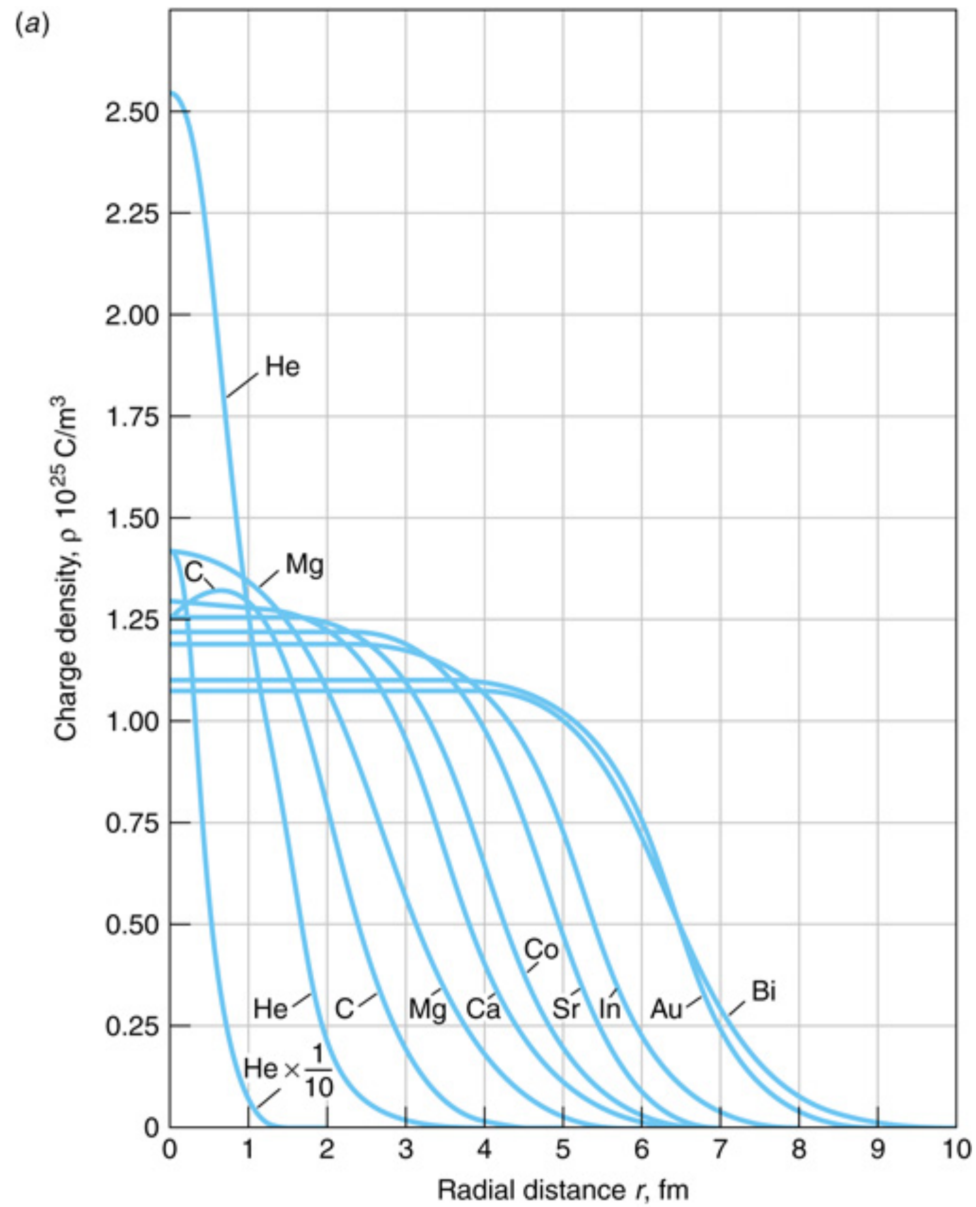
High energy: "Stability" of internal structure against hard "blows"



Can the hadron absorb a high momentum virtual photon without breaking apart?

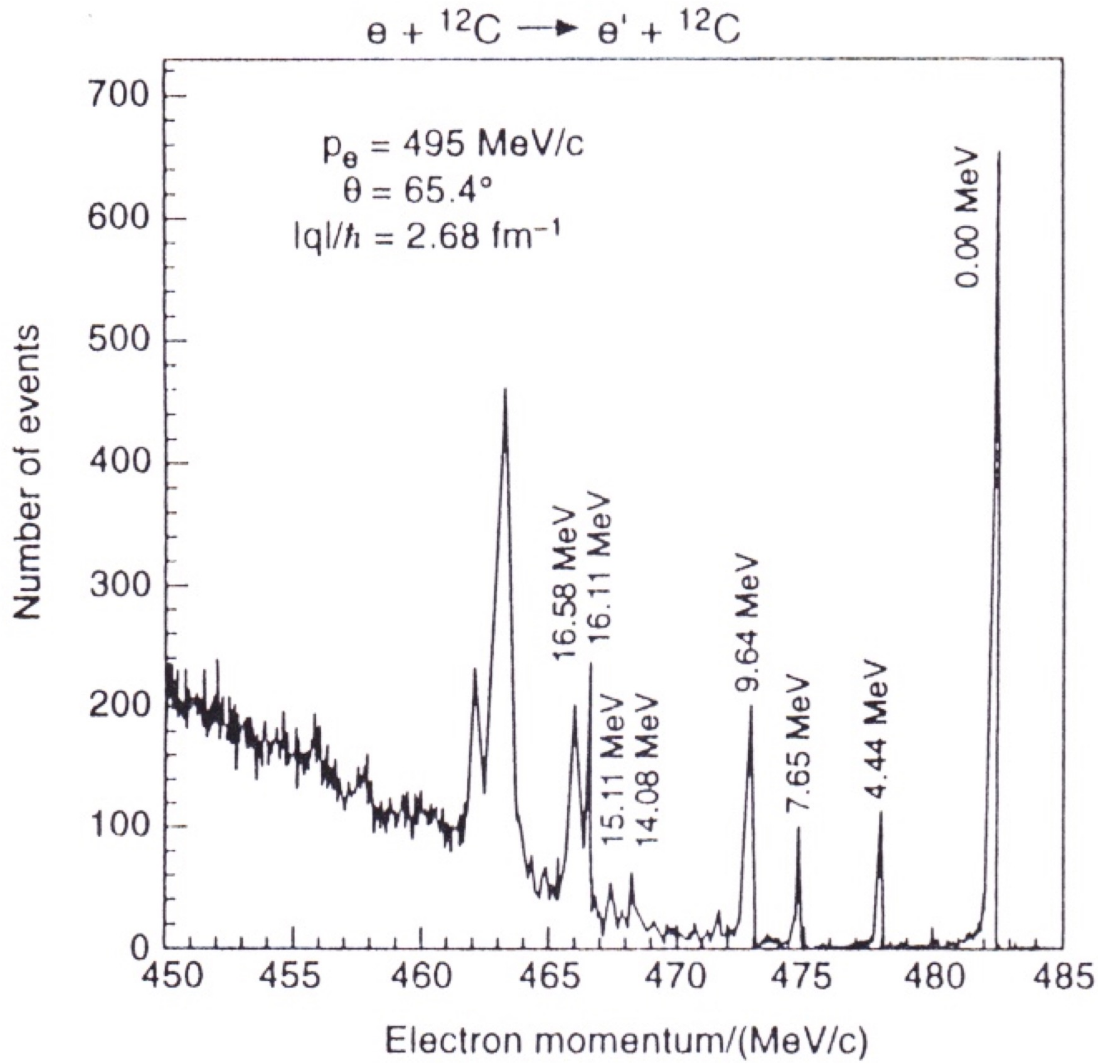
Elastic Scattering off Nuclei





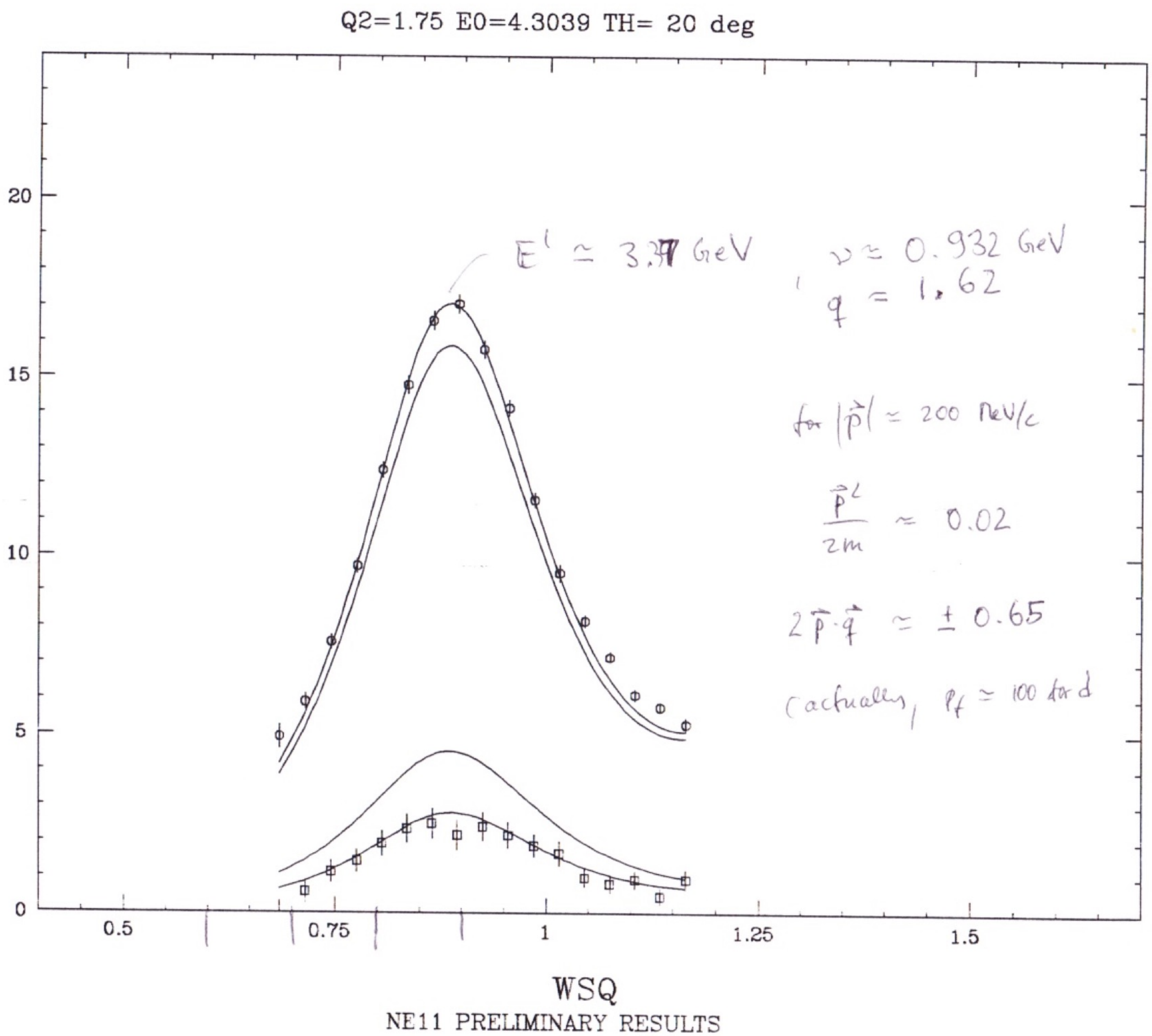
-> Nuclear radii, density distributions

Inelastic Scattering off Nuclei



-> Excited states of nuclei, transition matrix elements

Quasi-elastic Scattering off Nuclei



-> scattering of single nucleons bound in the nucleus; momentum distribution, energy of residual system

Quasi-elastic Scattering off Nuclei with coincindent detection of proton

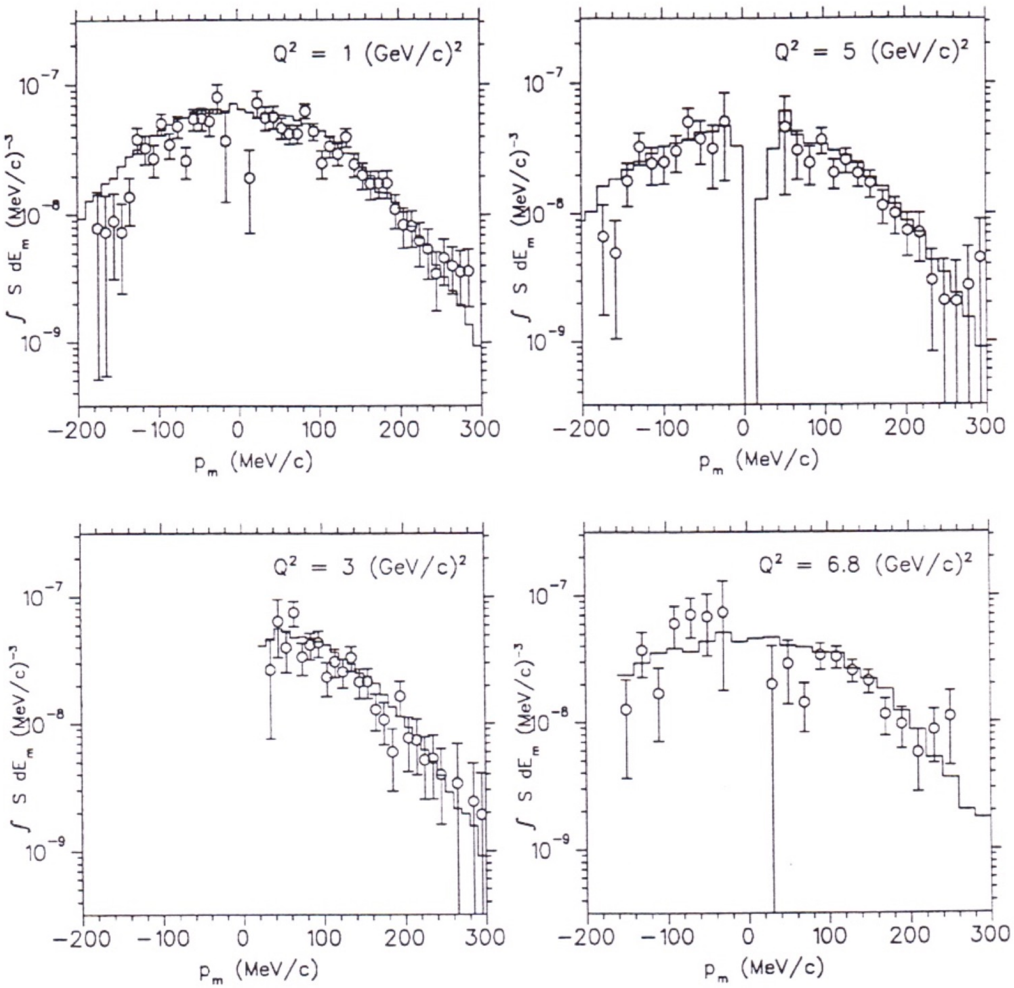
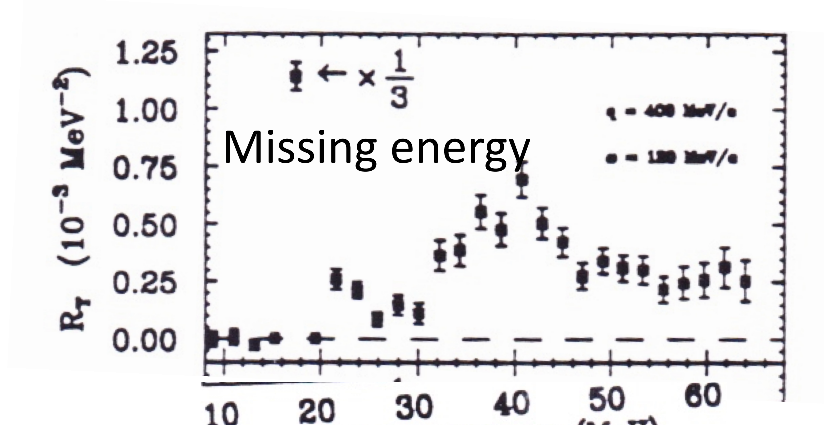


Figure 5-12: Extracted $\rho(p_m)$ for the 1s shell of ^{12}C. The spectral function has been integrated over $30 < E_m < 80$ MeV. The solid line represents the result of the PWIA calculation, normalized to the measured transparency; the data points are shown with statistical errors only.

-> momentum and energy distribution of bound proton; “Spectroscopic Factor”



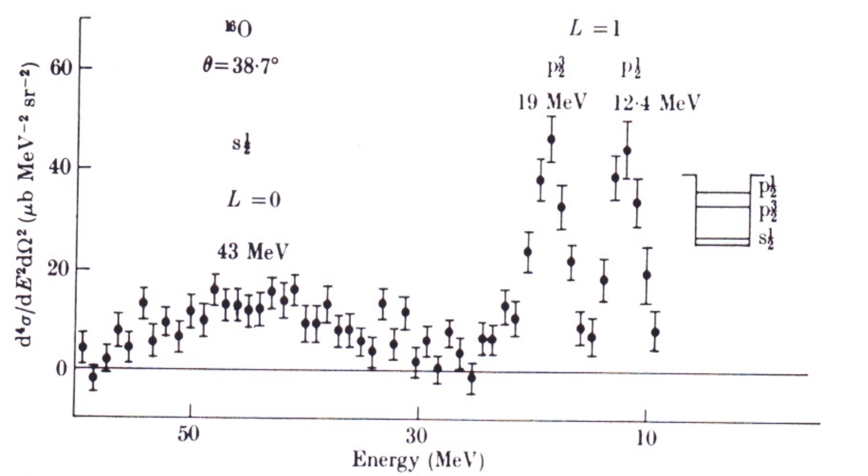
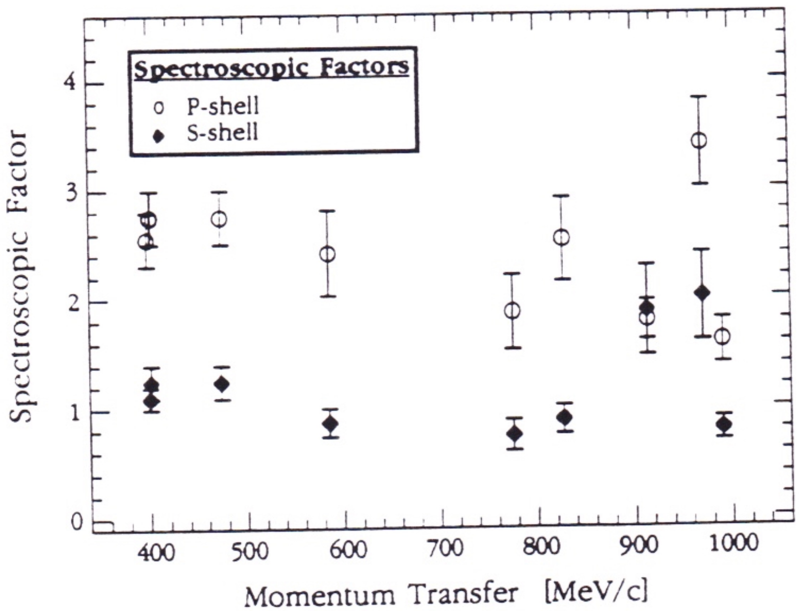
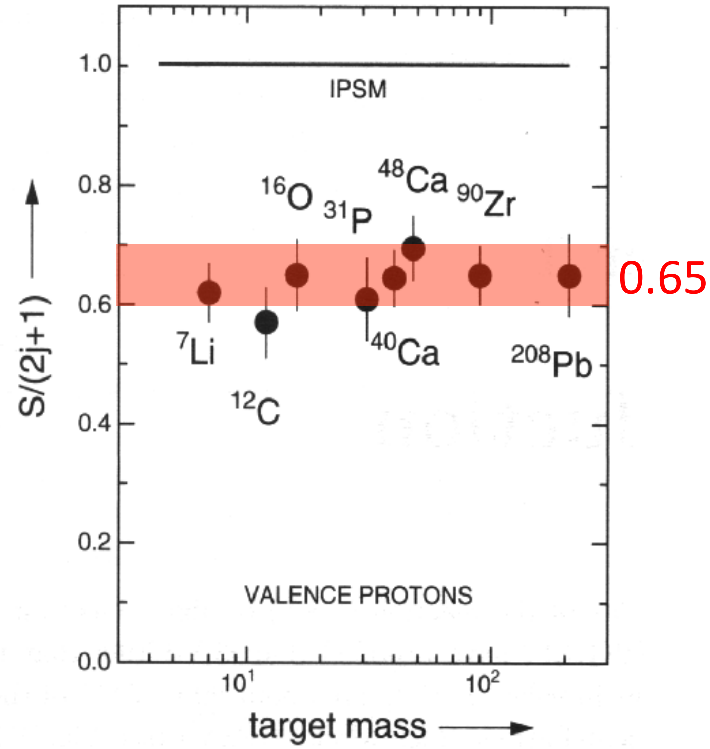
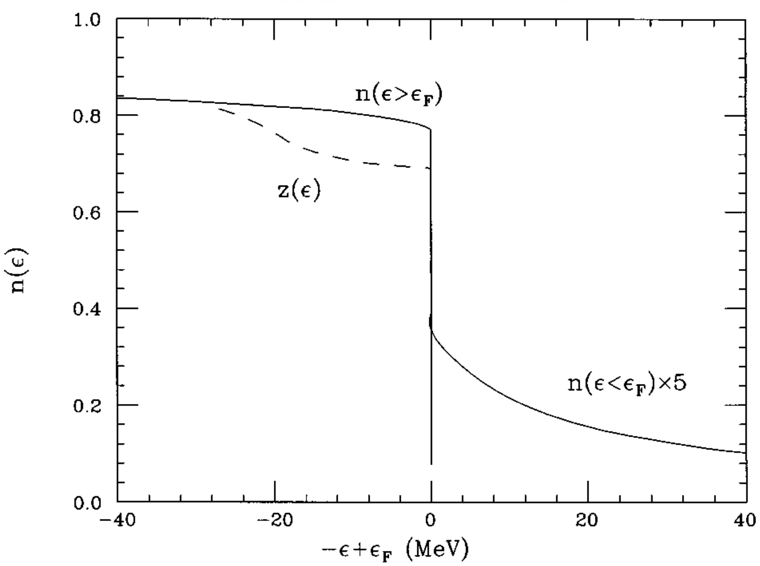


Fig. 17.8 Distribution of energies required to remove a proton from ^{16}O by the (p,2p) reaction, showing the single-particle states (Tyrén *et al.* 1958).

Quasi-elastic Scattering off Nuclei with coincident detection of proton







Spectroscopic factors show that roughly 30-35% of nucleons are not in (close in energy to) a well-defined shell.

Some kinematics – in the lab system

$$k^\mu = (E, 0, 0, E) = (E, \vec{k})$$

$$k'^\mu = (E', E'\sin\theta, 0, E'\cos\theta) = (E', \vec{k}')$$

$$q^\mu = (E-E', -E'\sin\theta, 0, E-E'\cos\theta) = (\nu, \vec{q})$$

$$Q^2 = -q^\mu q_\mu = -\nu^2 + \vec{q}^{\,2} = (E-E'\cos\theta)^2 + E'^2\sin^2\theta - (E-E')^2$$

$$= E^2 - 2EE'\cos\theta + E'^2 - E^2 - E'^2 + 2EE'$$

$$= 2EE'(1-\cos\theta) = 4EE'\sin^2\frac{\theta}{2}$$

$$p^\mu = (M, 0, 0, 0) = (M, \vec{0})$$

$$p'^\mu = p^\mu + q^\mu = (M+\nu, \vec{q})$$

$$p'^\mu p'_\mu =: W^2 = M^2 + 2M\nu + \nu^2 - \vec{q}^{\,2}$$

$$= M^2 + 2M\nu - Q^2$$

Elastic Scattering: $W^2 \overset{!}{=} M^2$

$$\Rightarrow \nu_{el} \overset{!}{=} \frac{Q^2}{2M} \quad \text{or} \quad x_{el} = \frac{Q^2}{2M\nu_{el}} \overset{!}{=} 1$$

Elastic Cross section – final form

$$\Delta\sigma = \frac{4z^2\alpha^2(\hbar c)^2}{Q^4}\cos^2\frac{\theta}{2}\left[\underbrace{\frac{G_E^2 + \tau G_M^2}{1+\tau}}_{\substack{\text{LONGITUDINAL} \\ \text{(charge)}}} + \underbrace{2\tau\tan^2\frac{\theta}{2}\,G_M^2}_{\substack{\text{TRANSVERSE} \\ \text{(magnetic)}}}\right]$$

$$\cdot E'^2 \Delta\Omega \underbrace{\frac{E'}{E}}_{\text{Recoil}}$$

$\tau = \frac{\nu^2}{Q^2}$, $G_E(Q^2)$, $G_M(Q^2)$: Form Factors

Dirac Particle: $G_E = G_M = 1$ (const.)

Anomalous magnetic moment: $G_M \approx (1+\kappa)G_E$

Extended Charge distribution:

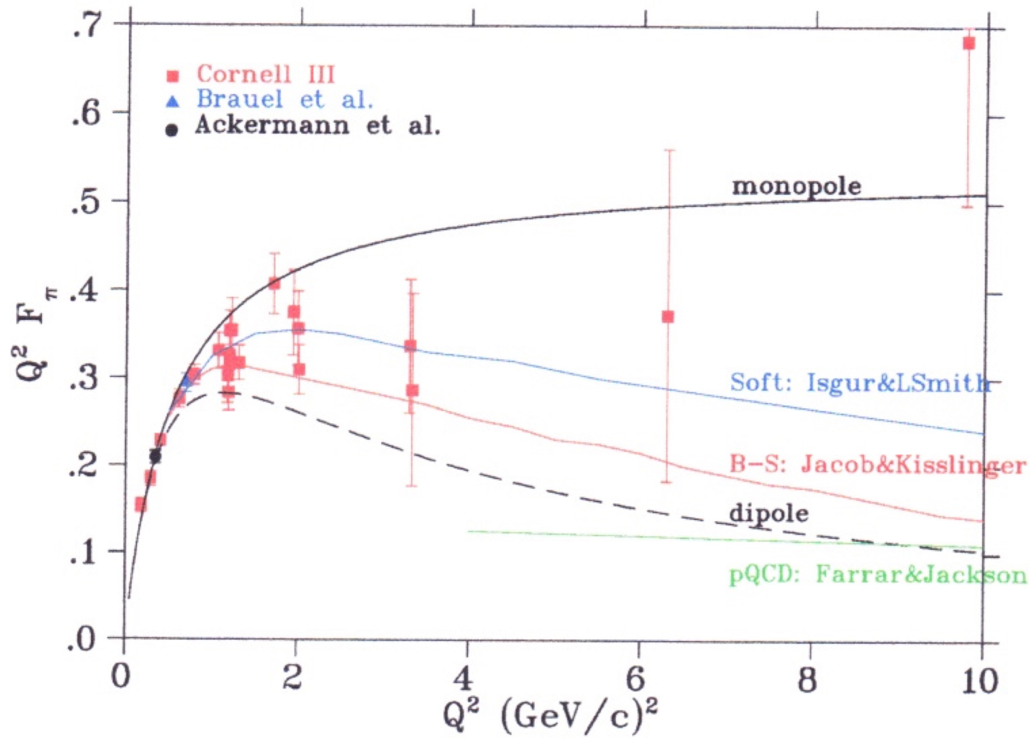
$G_E(Q^2) \simeq$ Fourier transform of $\rho(r)$

Ex: $\rho(r) \simeq \frac{a^3}{8\pi}e^{-ar} \Rightarrow G_E(Q^2) \simeq \left(\frac{1}{1+\frac{Q^2}{a^2}}\right)^2$

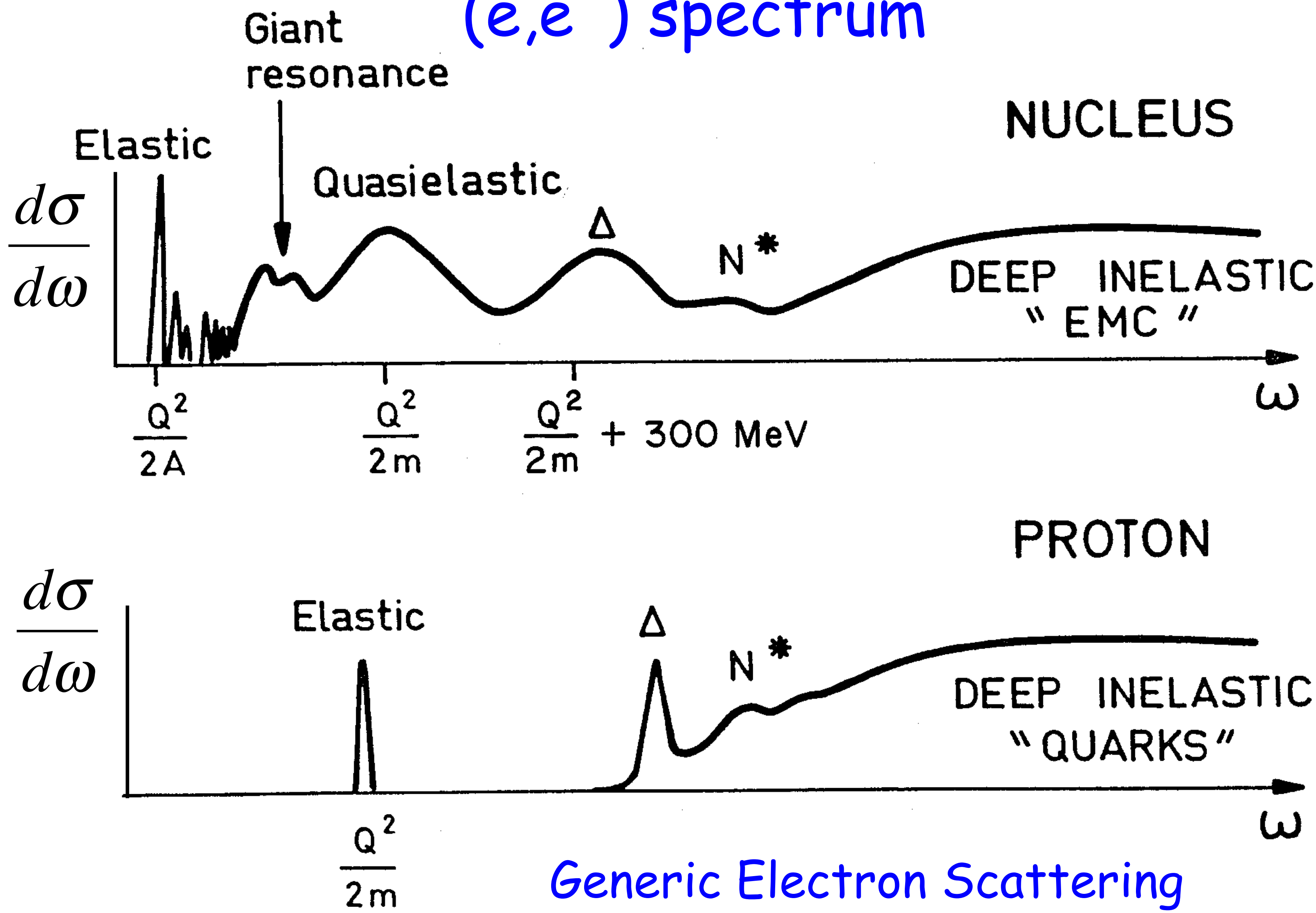
(Dipole Form). p: $a^2 = 0.71\,\text{GeV}^2$

Elastic Form Factors of Hadrons

- Example: Pion
- Nucleons discussed in separate presentation



(e,e') spectrum



Generic Electron Scattering at fixed momentum transfer

Inelastic Cross Section – What's different

Phase space factor $d^3k' = k'^2 d\Omega_{k'}\, dk'$

(δ-function drops since E' can have any value)

— conversion —

$$k'^2 d\Omega dk' = E'^2 2\pi \sin\theta d\theta dE'$$
$$= -2E'^2 d\cos\theta \, \pi dE'$$
$$= \frac{\pi E'}{E} \underbrace{(-2EE' d\cos\theta)}_{+dQ^2} \underbrace{dE'}_{|d\nu|} \quad \text{since } \nu = E - E'$$

Replace $\frac{G_E^2+\tau G_M^2}{1+\tau}$ with $W_2(Q^2,\nu)$ *)

and τG_m^2 with $W_1(Q^2,\nu) \Rightarrow$

$$\Delta\sigma = \frac{4\pi\alpha^2(\hbar c)^2}{Q^4}\frac{E'}{E}\cos^2\theta/2 \left[W_2 + 2\tan^2\frac{\theta}{2} W_1\right]\Delta Q^2 \Delta\nu$$

*) $\Rightarrow$ $W_2^{el}(Q^2,\nu) = \frac{G_E^2+\tau G_M^2}{1+\tau}\cdot\delta(\nu-\nu_{el})$

$W_1^{el}(Q^2,\nu) = \tau G_m^2 \cdot \delta(\nu-\nu_{el})$

Interpretation of $W_1(Q^2,\nu)$

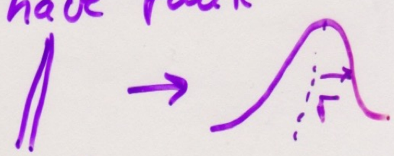
(transverse) electromagnetic transition probability to final state characterized by ν, with resolution $\sim \frac{1}{\sqrt{Q^2}}$

Transition to discrete final states (resonances):

$$W_1(Q^2,\nu) \simeq \tau G_m^2(\text{transition})\cdot\delta(\nu-\nu_R)$$

ν_R is given by $M^2_{Res} \overset{!}{=} W^2_{Res} = M^2 + 2M\nu_R - Q^2$

In reality: Resonances have finite width $\Gamma \sim \frac{1}{\tau_{life}} \Rightarrow$



(except elastic transition) δ-function

Threshold: $W^2_{min} = (M+m_\pi)^2 = (1.08\,\text{GeV})^2$

Elastic scattering

$$\frac{\Delta\sigma}{\Delta\Omega} = \frac{4\alpha^2(\hbar c)^2 E'^2 cos^2\frac{\theta}{2}}{Q^4}\frac{E'}{E}\left(\frac{G_E^2(Q^2)+\tau G_M^2(Q^2)}{1+\tau} + 2\tau\, tan^2\frac{\theta}{2}\, G_M^2(Q^2)\right)$$

where $\tau = \nu^2/Q^2$.

Inelastic Scattering

$x = \frac{p}{P}$

W: γ^*-p invariant mass

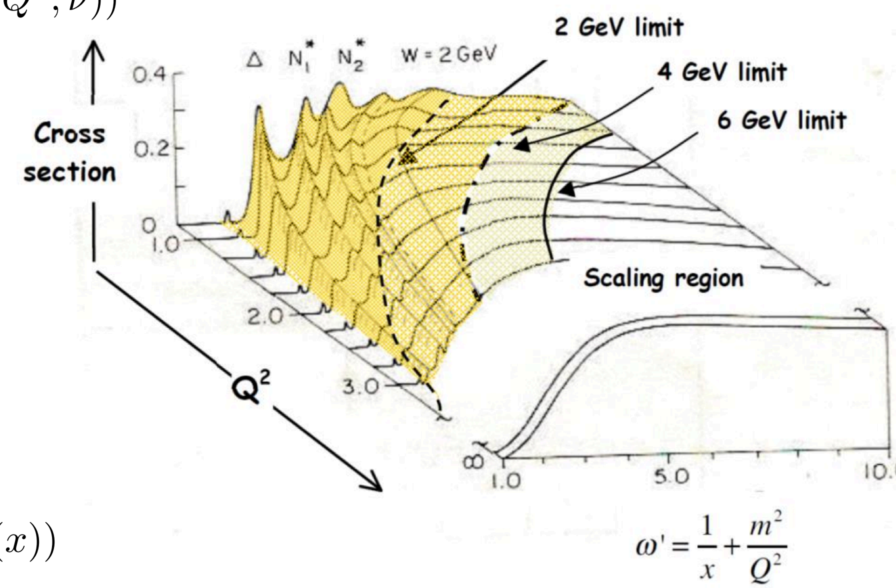
$$\frac{\Delta\sigma}{\Delta Q^2\Delta\nu} = \frac{4\pi\alpha^2(\hbar c)^2 E' cos^2(\theta/2)}{Q^4 E}(W_2(Q^2,\nu) + 2\, tan^2(\theta/2)\, W_1(Q^2,\nu))$$

$$\frac{\Delta\sigma}{\Delta Q^2\Delta\nu} = \frac{4\pi\alpha^2(\hbar c)^2 E' cos^2(\theta/2)}{Q^4 E}\frac{W_1(Q^2,\nu)}{\epsilon(1+\tau)}(1+\epsilon R(Q^2,\nu))$$

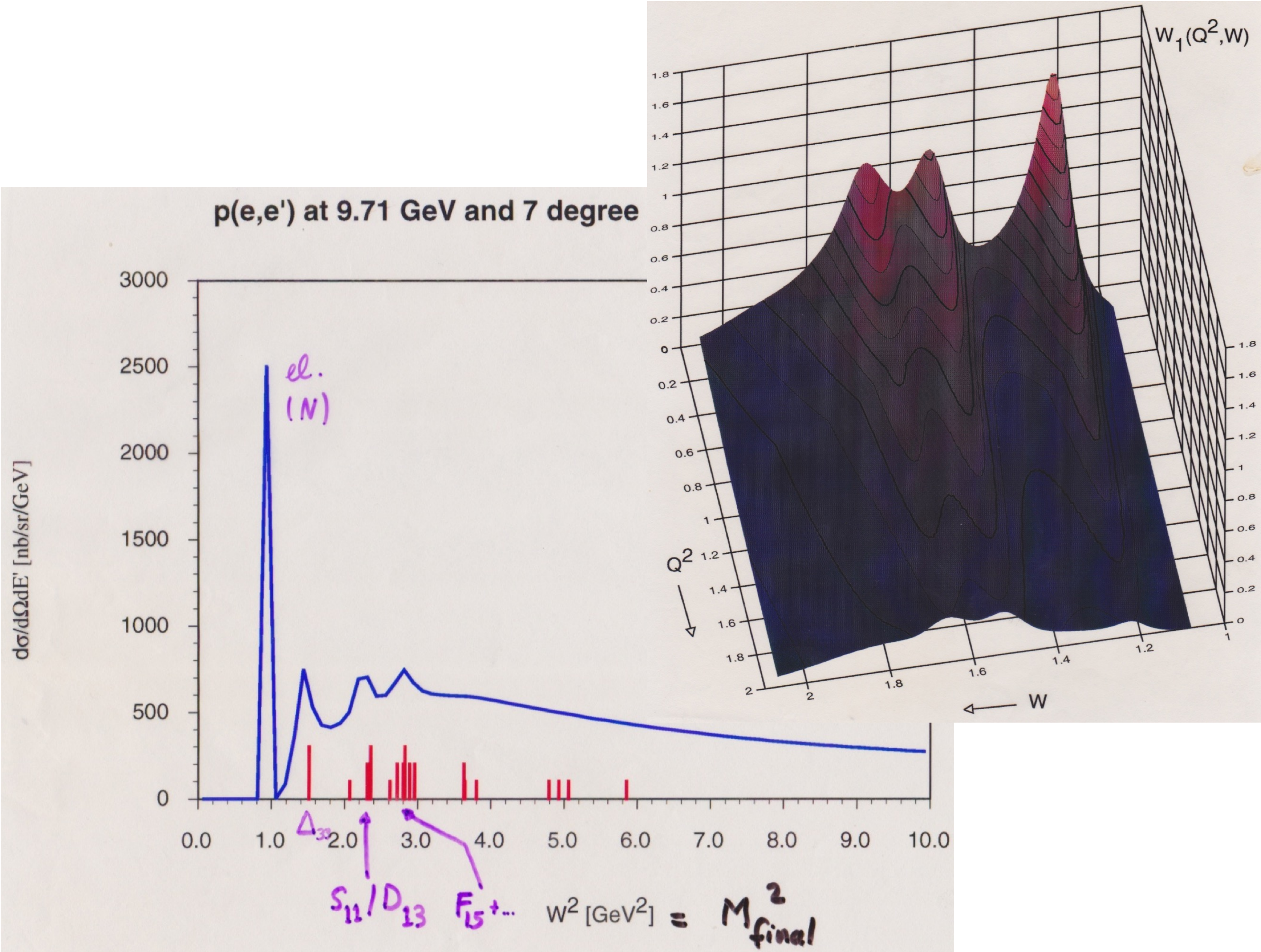
with $\epsilon = (1 + 2(1+\tau)tan^2(\theta/2))^{-1}$

$$\frac{\Delta\sigma}{\Delta Q^2\Delta\nu} = \frac{4\pi\alpha^2(\hbar c)^2 E' cos^2(\theta/2)}{Q^4 E}(\frac{1}{\nu}F_2(x) + 2\, tan^2(\theta/2)\frac{1}{M}F_1(x))$$

$$F_1(x) = MW_1(Q^2,\nu) \qquad F_2(x) = \nu W_2(Q^2,\nu)$$



Picture from F. Gross,
«Making the case for Jefferson Lab»
The first decade of Science at Jefferson Lab
JoP, Conf. Series 299 (2011) 012001



p(e,e') at 9.71 GeV and 7 degree
dσ/dΩdE' [nb/sr/GeV]
3000
2500
2000
1500
1000
500
0
0.0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
9.0
10.0
el.
(N)
Δ33
S11 / D13
F15 +...
W² [GeV²] = M²final
W1(Q²,W)
Q²
W

Deep Inelastic Scattering (DIS)

Reminder: Elastic scattering

$$\frac{\Delta\sigma}{\Delta\Omega} = \frac{4\alpha^2(\hbar c)^2 E'^2 cos^2\frac{\theta}{2}}{Q^4}\frac{E'}{E}\left(\frac{G_E^2(Q^2)+\tau G_M^2(Q^2)}{1+\tau} + 2\tau\, tan^2\frac{\theta}{2}\, G_M^2(Q^2)\right)$$

Elastic scattering from quarks:

$$\Delta\sigma = \frac{4\pi z_q^2\alpha^2(\hbar c)^2 E' cos^2(\theta/2)}{Q^4 E}(q(x)\Delta x + 2\nu^2/Q^2 tan^2(\theta/2)q(x)\Delta x)\Delta Q^2. \quad (12)$$

We can use the relation $\Delta x = -Q^2/(2M\nu^2)\Delta\nu = -x\Delta\nu/\nu$ to rewrite this as

$$\frac{\Delta\sigma}{\Delta Q^2\Delta\nu} = \frac{4\pi\alpha^2(\hbar c)^2 E' cos^2(\theta/2)}{Q^4 E}(\frac{x}{\nu}z_q^2 q(x) + \frac{1}{M}tan^2(\theta/2)z_q^2 q(x)). \quad (13)$$

Reminder: IN-Elastic scattering

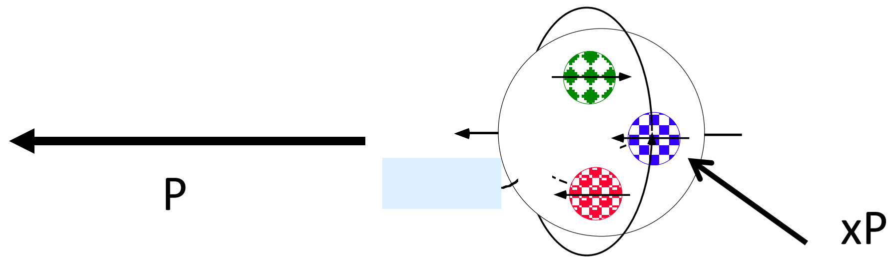
$$\frac{\Delta\sigma}{\Delta Q^2\Delta\nu} = \frac{4\pi\alpha^2(\hbar c)^2 E' cos^2(\theta/2)}{Q^4 E}(\frac{1}{\nu}F_2(x) + 2\, tan^2(\theta/2)\frac{1}{M}F_1(x))$$

NOTE: $\Rightarrow$
$F_2 = 2xF_1$
Callan-Gross

$$\Rightarrow \quad F_1(x) = \frac{1}{2}\left(\frac{4}{9}\left[u(x)+\bar{u}(x)\right] + \frac{1}{9}\left[d(x)+\bar{d}(x)+s(x)+\bar{s}(x)\right] + \ldots\right)$$

No Q^2!

Quark-Parton Structure of the Proton



q(x) $\sim \left\langle P,s \middle| \bar{q}\gamma^{\mu} q \middle| P,s \right\rangle$

Δq(x) $= q \Uparrow\uparrow(x) - q \Uparrow\downarrow(x) + \bar{q} \Uparrow\uparrow(x) - \bar{q} \Uparrow\downarrow(x) \sim \left\langle P,s \middle| \bar{q}\gamma^{\mu}\gamma^{5} q \middle| P,s \right\rangle$

"axial charge", similarly G(x) and ΔG(x) for gluons

Spin Sum Rule:

$$S_p = \frac{1}{2} = \frac{1}{2}\sum_q \Delta q + \Delta G + L_q + L_G$$

$\Delta\Sigma$

Structure Functions

$$\frac{\Delta\sigma}{\Delta Q^2 \Delta\nu} = \frac{4\pi\alpha^2(\hbar c)^2 E' cos^2(\theta/2)}{Q^4 E}(\frac{1}{\nu}F_2(x) + 2\,tan^2(\theta/2)\frac{1}{M}F_1(x))$$

$$\frac{\Delta\sigma}{\Delta Q^2 \Delta\nu}\downarrow\Uparrow - \frac{\Delta\sigma}{\Delta Q^2 \Delta\nu}\uparrow\Uparrow = \frac{4\pi\alpha^2}{M\nu Q^2 E^2}\left[\left(E + E'\cos\theta\right)\mathbf{g_1} - 2xM\mathbf{g_2}\right]$$

Unpolarized: $F_1(x,Q^2)$ and $F_2(x, Q^2)$

Polarized: $g_1(x,Q^2)$ and $g_2(x, Q^2)$

Parton model:

$$F_1(x) = \frac{1}{2}\sum_i e_i^2 q_i(x) \text{ and } F_2(x) = 2xF_1(x)$$

$$g_1(x) = \frac{1}{2}\sum_i e_i^2 \Delta q_i(x) \text{ and } g_2(x) = 0$$

$i =$ quark flavor

$e_i =$ quark charge

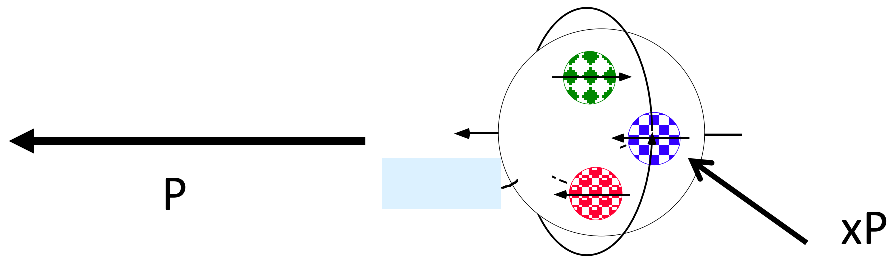
the structure functions $\mathbf{g_1}$ and $\mathbf{g_2}$ are linear combinations of $\mathbf{A_1}$ and $\mathbf{A_2}$

$$g_1(x,Q^2) = \frac{\tau}{1+\tau}(A_1 + \frac{1}{\sqrt{\tau}}A_2)F_1$$

$$g_2(x,Q^2) = \frac{\tau}{1+\tau}(\sqrt{\tau}A_2 - A_1)F_1$$

$$\tau = \frac{\nu^2}{Q^2}$$

Quark-Parton Structure of the Proton



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$$\Delta\Sigma$$

Structure Functions

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