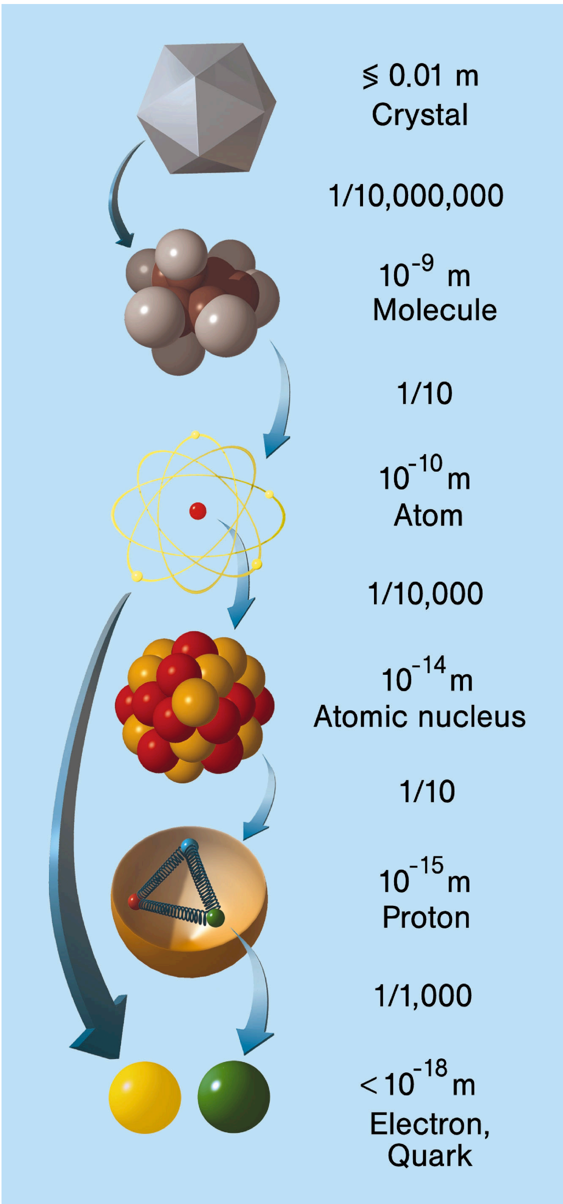
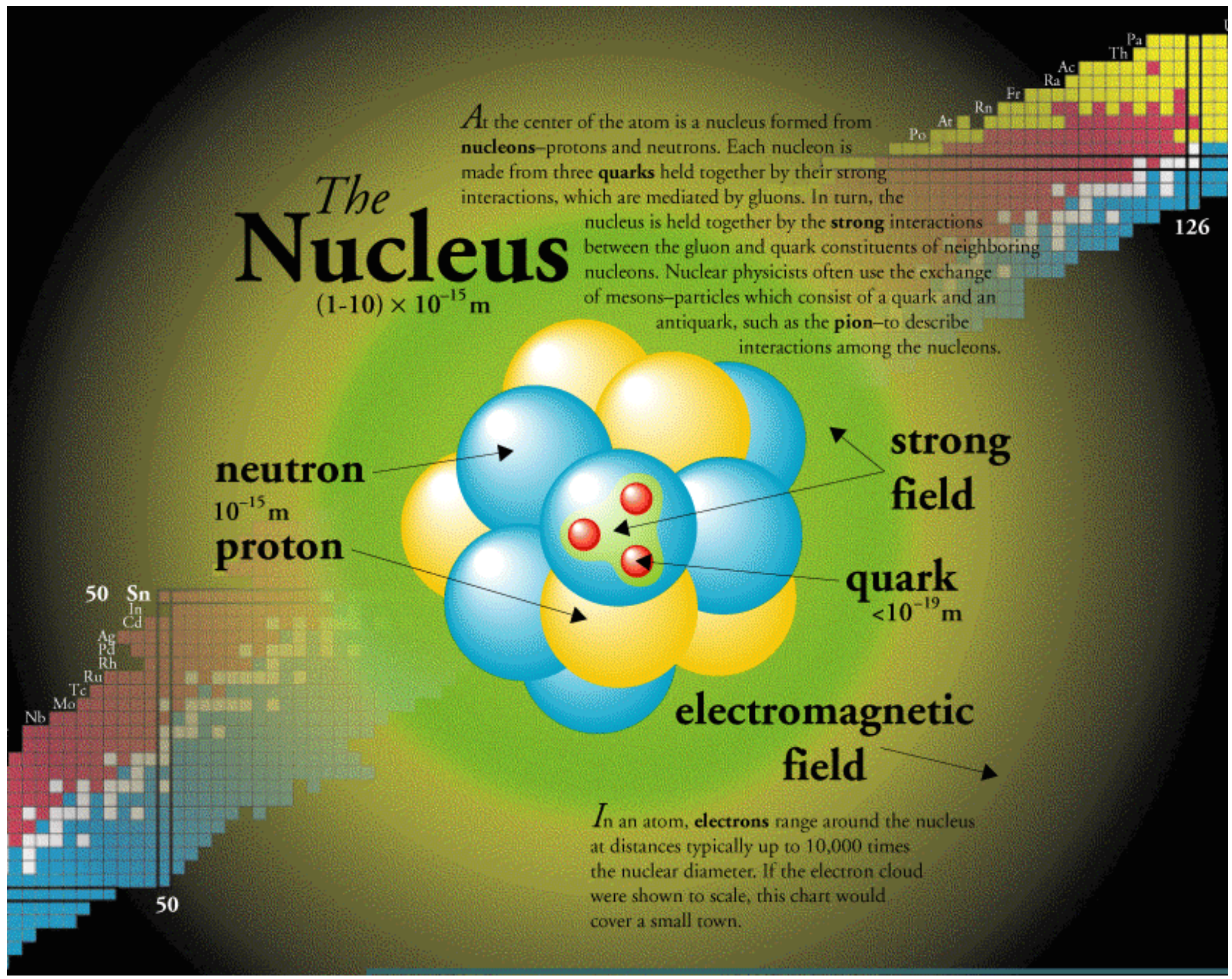
Part 2

NUCLEON STRUCTURE

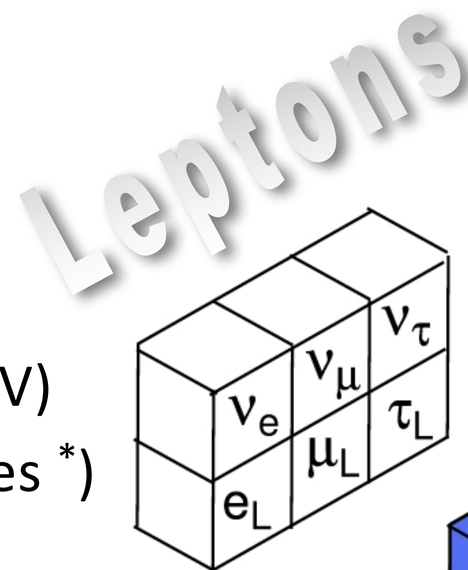
The Structure of Matter



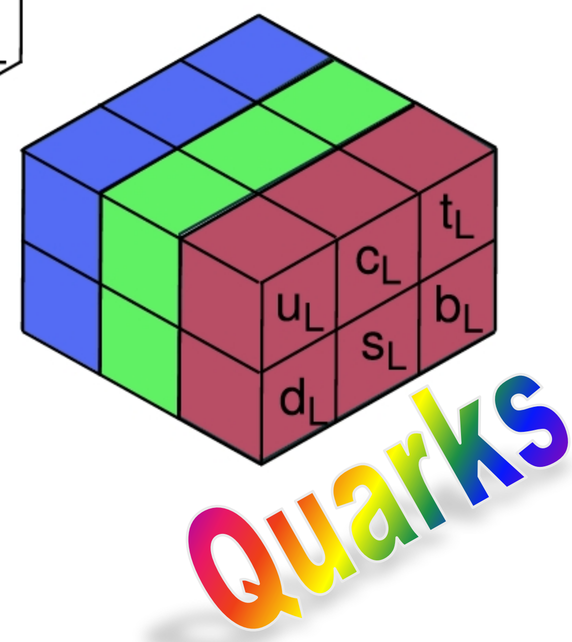


Matter Particles

- Make up visible matter
- Pointlike (<10^{-18} m), Fundamental $^{*)}$
- Have mass (from < ½ eV to 178,000,000,000 eV = 178 GeV)
- Distinct from their antiparticles *)
- Fermions (Spin ½) ⇒ they "defend" their space (Pauli Principle) and can only be created in particle-antiparticle pairs
- Can be "virtual", but make up matter being (nearly) "real"
- "stable" (against strong decays; lifetimes from ∞ to 10^{-24} s)



3 "colors" = 3 different charges: red, green, blue



x2 for R, x2 for antiparticles

$^{*)}$ Until further notice

Forces and Force Carriers

- Mediate Interactions (Forces) - form “Waves”
- Pointlike, Fundamental
- Massless *)
- Some are their own antiparticles (photon, Z^0, graviton)
- Spin 1, 2 -> Bosons (tend to cluster together, can be produced in arbitrary numbers)
- Can be real, but carry forces as virtual particles
- Some are absolutely stable (γ, gluons, gravitons)

*) See next slide

BOSONS force carriers spin = 0, 1, 2, ...

Unified Electroweak spin = 1		
Name	Mass GeV/c^2	Electric charge
γ photon	0	0
W^-	80.4	−1
W^+	80.4	+1
Z^0	91.187	0

Strong (color) spin = 1		
Name	Mass GeV/c^2	Electric charge
g gluon	0	0

Gravitation spin = 2		
Name	Mass GeV/c^2	Electric charge
g graviton	0	0

Note: gluons come in 8 possible combinations of color/anticolor (9^{th} is “sterile” – doesn’t exist)

Simple (Constituent) Quark Model

Flavor	Isospin I	I_3	Strangeness S	Charge Q	Baryon Number B
U	1/2	+1/2	0	+2/3	1/3
D	1/2	−1/2	0	−1/3	1/3
S	0	0	−1	−1/3	1/3

$$|\Delta^{++}\uparrow\rangle = |U\uparrow U\uparrow U\uparrow\rangle$$

$$|\Delta^{+}\uparrow\rangle = 1/\sqrt{3}\,(|U\uparrow U\uparrow D\uparrow\rangle + |U\uparrow D\uparrow U\uparrow\rangle + |D\uparrow U\uparrow U\uparrow\rangle)$$

The case of the proton is a bit more complicated, since the wave function cannot be symmetric in spin and flavor separately. The most intuitive way to derive the proton wave function is by observing that 2 of the 3 quarks are equal (U), and therefore their relative spin wave function should be symmetric also. This leads to the conclusion that the two U–quarks couple their spins to a total spin of one. Let's denote the case where this spin has a z-projection of +1 as $(UU\Uparrow) := |U\uparrow U\uparrow\rangle$, while the projection with $S_z = 0$ will be indicated by $(UU\Rightarrow) := 1/\sqrt{2}\,(|U\uparrow U\downarrow\rangle + |U\downarrow U\uparrow\rangle)$. We can now combine the spin 1/2 of the remaining D quark with the spin 1 of the UU pair in two ways to get total spin and projection 1/2; the proper way follows simply from insertion of the correct Clebsch-Gordon coefficients:

$$|P\uparrow\rangle = 1/\sqrt{3}\left(\sqrt{2}|(UU\Uparrow)D\downarrow\rangle - |(UU\Rightarrow)D\uparrow\rangle\right). \qquad (2)$$

Constituent Quark Model:

- SU(6)-symmetric wave function of the proton in the quark model:

$$|p\uparrow\rangle = \frac{1}{\sqrt{18}}\left(3u\uparrow[ud]_{S=0} + u\uparrow[ud]_{S=1} - \sqrt{2}u\downarrow[ud]_{S=1} - \sqrt{2}d\uparrow[uu]_{S=1} - 2d\downarrow[uu]_{S=1}\right)$$

- In this model: d/u = 1/2, Δu/u = 2/3, Δd/d = -1/3 for all *x*
  => A_{1p} = 5/9, A_{1n} = 0, A_{1D} = 1/3 *)

- Hyperfine structure effect: S=1 suppressed => d/u = 0, Δu/u = 1, Δd/d = -1/3
  for *x* -> 1 => A_{1p} = 1, A_{1n} = 1, A_{1D} = 1

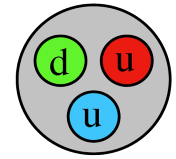
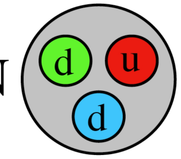
- pQCD: helicity conservation (q↑↑p) => d/u =2/(9+1) = 1/5, Δu/u = 1, Δd/d = 1
  for *x* -> 1

- Wave function of the neutron via isospin rotation:
  replace u -> d and d -> u => using experiments with protons and neutrons one can extract information on u, d, Δu and Δd in the valence quark region.

*) $$A_{1p} = \frac{4/9\cdot u\cdot \Delta u/u + 1/9\cdot d\cdot \Delta d/d}{4/9\cdot u + 1/9\cdot d} = \frac{4\cdot \Delta u/u + (d/u)\cdot \Delta d/d}{4 + (d/u)}$$

Hadronic Particle Zoo

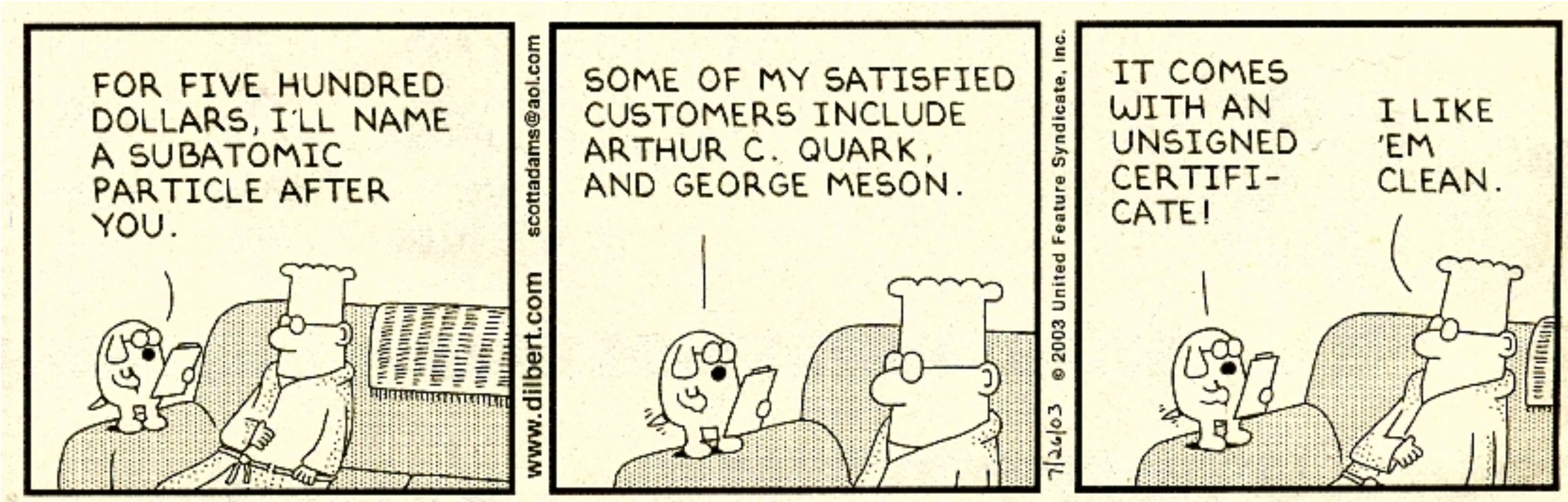
- what can one build from quarks?

P  N 

HADRONS

Family Name	Particle Name	Particle Symbol	Antiparticle Symbol	Composition	Mass	Electric Charge	Lifetime in Seconds
baryon	proton	p or p^+	$\bar{p}$	uud	1,836	+1	stable
	neutron	n or n^0	$\bar{n}$	udd	1,839	0	887
	lambda	Λ^0	$\bar{\Lambda}^0$	uds	2,183	0	2.6×10^{-10}
	lambda-c	Λ^+_c	$\bar{\Lambda}^-_c$	udc	4,471	+1	2.1×10^{-13}
	lambda-b	Λ^0_b	$\bar{\Lambda}^0_b$	udb	11,000	0	1.1×10^{-12}
	sigma	Σ^+	$\bar{\Sigma}^-$	uus	2,328	+1	0.8×10^{-10}
		Σ^0	$\bar{\Sigma}^0$	$\frac{(ud \pm du)s}{\sqrt{2}}$	2,334	0	7.4×10^{-20}
		Σ^-	$\bar{\Sigma}^+$	dds	2,343	−1	1.5×10^{-10}
	xi	Ξ^0	$\bar{\Xi}^0$	uss	2,573	0	2.9×10^{-10}
		Ξ^-	$\bar{\Xi}^+$	dss	2,585	−1	1.6×10^{-10}
	xi-c	Ξ^0_c	$\bar{\Xi}^0_c$	dsc	4,834	0	9.8×10^{-14}
		Ξ^+_c	$\bar{\Xi}^-_c$	usc	4,826	+1	3.5×10^{-13}
	omega	Ω^-	Ω^+	sss	3,272	−1	0.8×10^{-10}
	omega-c	Ω^0_c	$\bar{\Omega}^0_c$	ssc	5,292	0	6.4×10^{-14}
meson	pion	π^+	π^-	$u\bar{d}$	273	+1	2.6×10^{-8}
		π^0	π^0	$\frac{(u\bar{u} - d\bar{d})}{\sqrt{2}}$	264	0	8.4×10^{-17}
	kaon*	K^+	K^-	$u\bar{s}$	966	+1	1.2×10^{-8}
		K^0	$\bar{K}^0$	$d\bar{s}$	974	0	8.9×10^{-11} 5.2×10^{-8}
	J/psi	J or Ψ	J or Ψ	$c\bar{c}$	6,060	0	1.0×10^{-20}
	omega	ω	ω	$\frac{(u\bar{u} + d\bar{d})}{\sqrt{2}}$	1,532	0	6.6×10^{-23}
	eta	η	η	$\frac{(u\bar{u} + d\bar{d})}{\sqrt{2}}$	1,071	0	3.5×10^{-22}
	eta-c	η_c	η_c	$c\bar{c}$	5,832	0	3.1×10^{-22}
	B	B^0	$\bar{B}^0$	$d\bar{b}$	10,331	0	1.6×10^{-12}
		B^+	B^-	$u\bar{b}$	10,331	+1	1.6×10^{-12}
	B-s	B^0_s	$\bar{B}^0_s$	$s\bar{b}$	10,507	0	1.6×10^{-12}
	D	D_0	$\bar{D}_0$	$c\bar{u}$	3,649	0	4.2×10^{-13}
		D^+	D^-	$c\bar{d}$	3,658	+1	1.1×10^{-12}
	D-s	D^+_s	D^-_s	$c\bar{s}$	3,852	+1	4.7×10^{-13}
	chi	χ^0_c	χ^0_c	$c\bar{c}$	6,687	0	3.0×10^{-22}
	psi	Ψ^0_c	Ψ^0_c	$c\bar{c}$	7,213	0	1.5×10^{-20}
	upsilon	Υ	Υ	$b\bar{b}$	18,513	0	8.0×10^{-20}

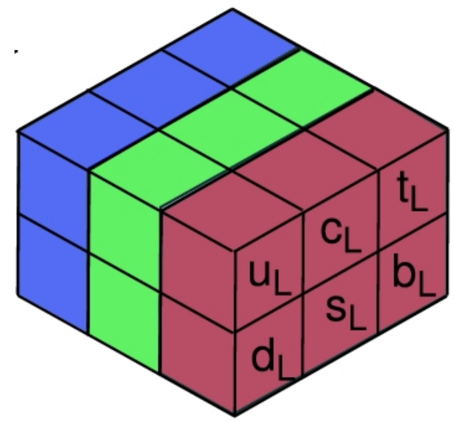
*The neutral kaon is composed of two particles; the average lifetime of each particle is given.



See also **http://particleadventure.org**

Fundamental Problem of Nuclear and Hadronic Physics

- Nearly all well-known ("visible") mass in the universe is due to hadronic matter
- Fundamental theory of hadronic matter exists since the 1960's:
  Quantum Chromo Dynamics
  - "Colored" quarks (u,d,c,s,t,b) and gluons; Lagrangian
- BUT: knowing the ingredients doesn't mean we know how to build hadrons and nuclei from them!
  - akin to the question:
    "Given bricks and mortar, how do you build a house?"



- Four related puzzles:
  - What is the "quark-gluon wave function" of known hadrons?
  - How are hadrons (nucleons) bound into nuclei?
    Does their quark-gluon wave function change inside a nucleus?
  - How do fast quarks and gluons propagate inside hadronic matter?
  - How do fast quarks and gluons turn back into observable hadrons?

Fundamental
Had

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  hadronic matter
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[PDF] The QCD Lagrangian - T39 Physics Group - TU Munich

www.t39.ph.tum.de/T39_files/Lectures_files/StrongInteraction2011/QCDkap2.pdf

CHAPTER 2. THE **QCD LAGRANGIAN**. 2.1. Preparation: Gauge invariance for QED. • Consider electrons represented by Dirac field ψ(x). Gauge transformation: .

- Starting point: Quark fields $\psi = (\psi_{\alpha i})$

$$\begin{cases} \alpha = u,\, d,\, s,\, c,\, b,\, t \ \ \text{(flavor index)} & N_f = 6 \longleftrightarrow SU(N_f) \\ i = 1,\, 2,\, 3 \qquad \text{(color index)} & N_c = 3 \longleftrightarrow SU(3)_c \end{cases}$$

where $\psi_{\alpha i}$ is a 4-component Dirac-spinor.

Consider Quark fields with color degree of freedom and their free *Lagrangian*:

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix}, \quad \mathcal{L}_0 = \bar{\psi}\left[i\gamma_\mu\partial^\mu - m\right]\psi \tag{2.11}$$

- Local $SU(3)_c$ gauge transformations

$$\psi(x) \longrightarrow \tilde{\psi}(x) = U\,\psi(x) \tag{2.12}$$

with $U = \exp\left[-i\,\theta_a(x)\,\frac{\lambda_a}{2}\right]$ where $\theta_a(x)$ is a real function with $a = 1,\, 2,\, \cdots,\, 8$.

$$A_\mu(x) = \sum_{a=1}^{8} t_a\, A_\mu^a(x) \quad , \qquad t_a = \frac{\lambda_a}{2}$$

$$\boxed{G_{\mu\nu}^a(x) = \partial_\mu A_\nu^a(x) - \partial_\nu A_\mu^a(x) + g\, f_{abc}\, A_\mu^b(x)\, A_\nu^c(x)}$$

- QCD Lagrangian:

$$\boxed{\mathcal{L}_{\text{QCD}} = \bar{\psi}\left(i\gamma_\mu D^\mu - m\right)\psi - \frac{1}{2}\,\text{tr}\{G_{\mu\nu}\,G^{\mu\nu}\}}$$

with $D^\mu = \partial^\mu - igA^\mu(x)$.[1]

$$\mathcal{L}_{\text{glue}} = -\frac{1}{4}\,G_{\mu\nu}^a(x)\,G_a^{\mu\nu}(x) = -\frac{1}{2}\,\text{tr}\{G_{\mu\nu}\,G^{\mu\nu}\}$$

QCD Made Simple

Frank Wilczek 22 August 2000 Physics Today

$$\mathcal{L} = \frac{1}{4g^2} G^a_{\mu\nu} G^a_{\mu\nu} + \sum_j \bar{q}_j (i\gamma^\mu D_\mu + m_j) q_j$$

where $G^a_{\mu\nu} \equiv \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + i f^a_{bc} A^b_\mu A^c_\nu$

and $D_\mu \equiv \partial_\mu + i t^a A^a_\mu$

<u>That's it!</u>

Figure 1. The QCD Lagrangian $\mathcal{L}$ displayed here is, in principle, a complete description of the strong interaction. But, in practice, it leads to equations that are notoriously hard to solve. Here m_j and q_j are the mass and quantum field of the quark of jth flavor, and A is the gluon field, with spacetime indices μ and ν and color indices a, b, c. The numerical coefficients f and t guarantee SU(3) color symmetry. Aside from the quark masses, the one coupling constant g is the only free parameter of the theory.

How many quarks? From PDG

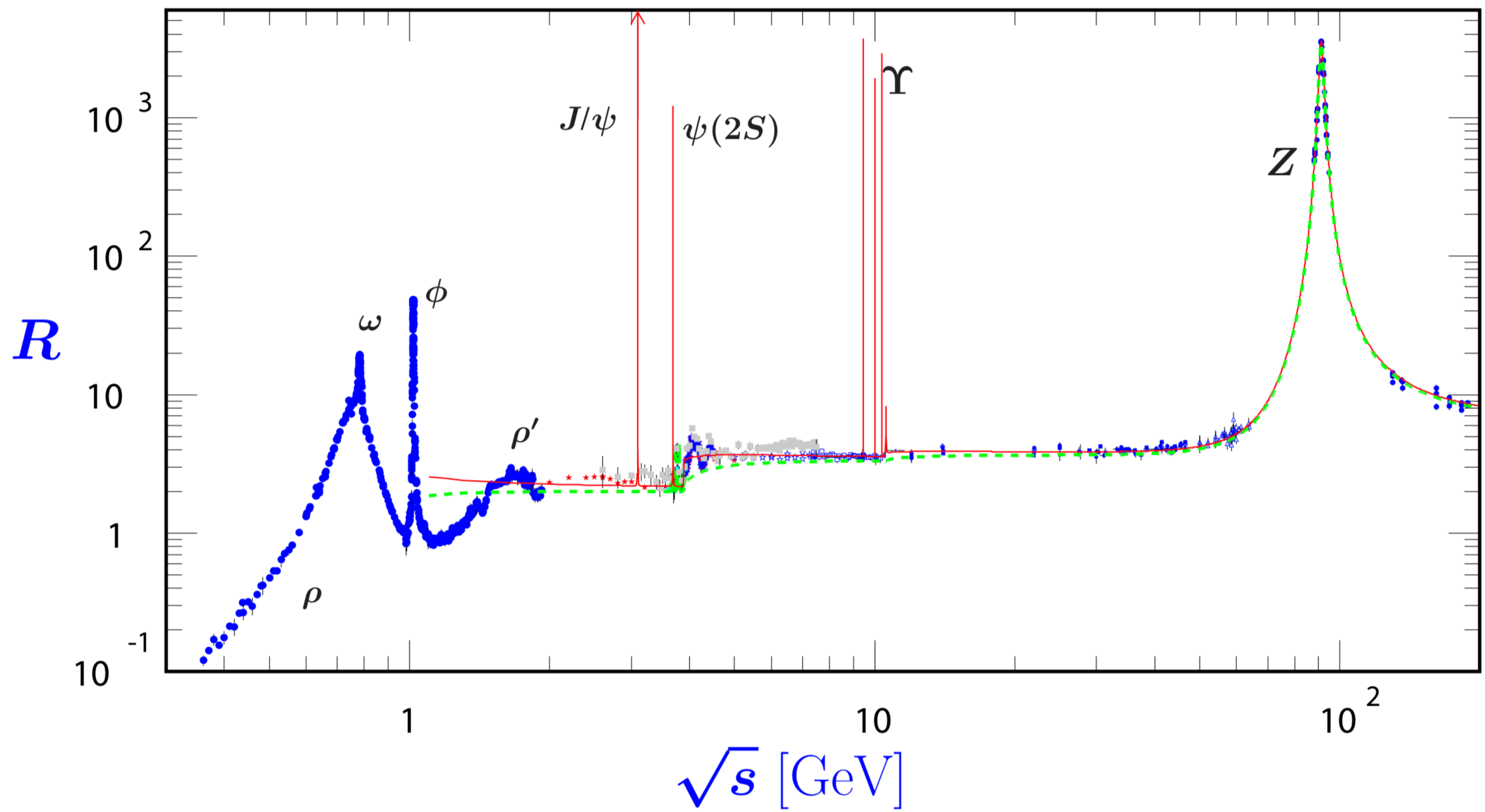
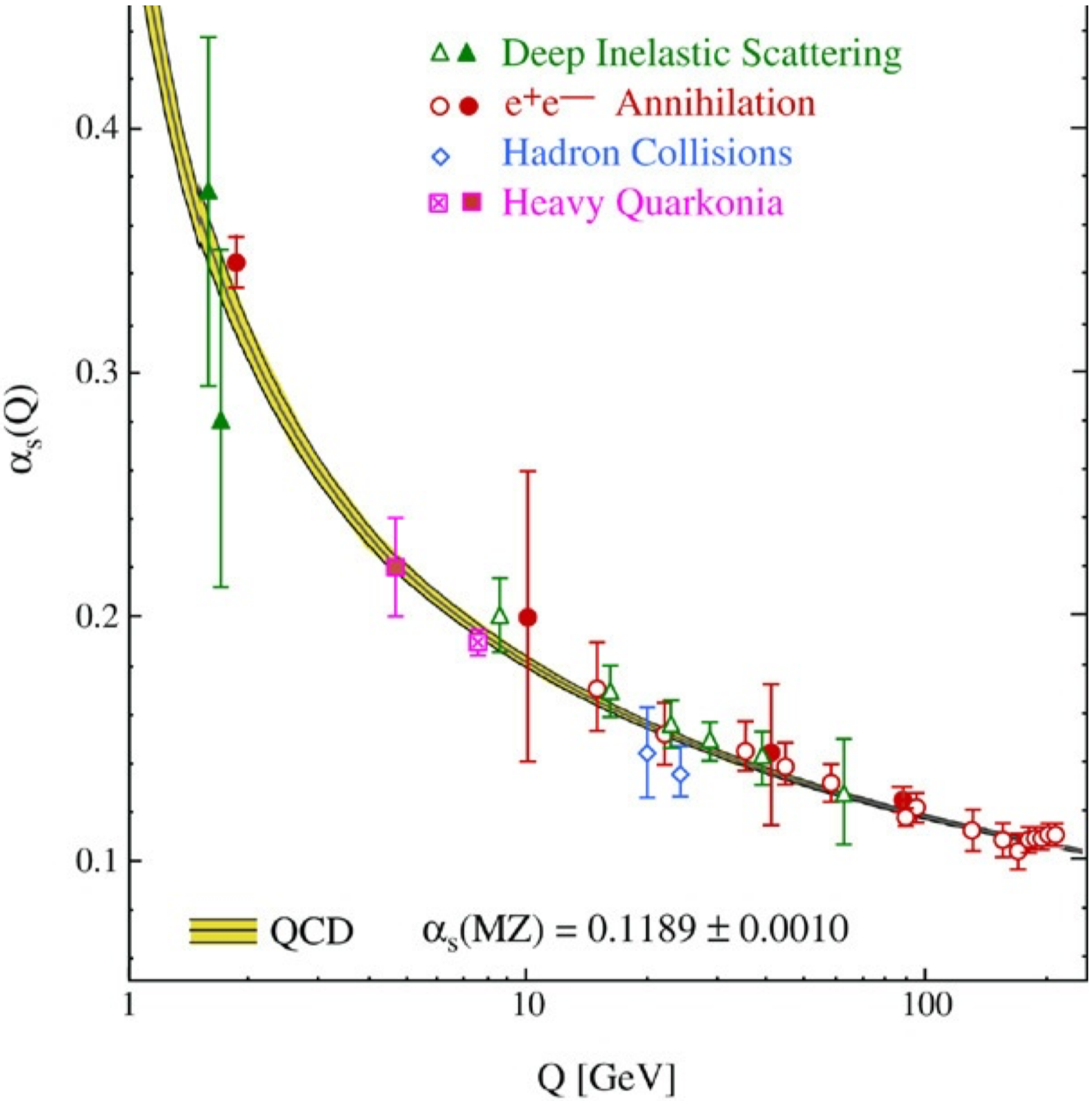


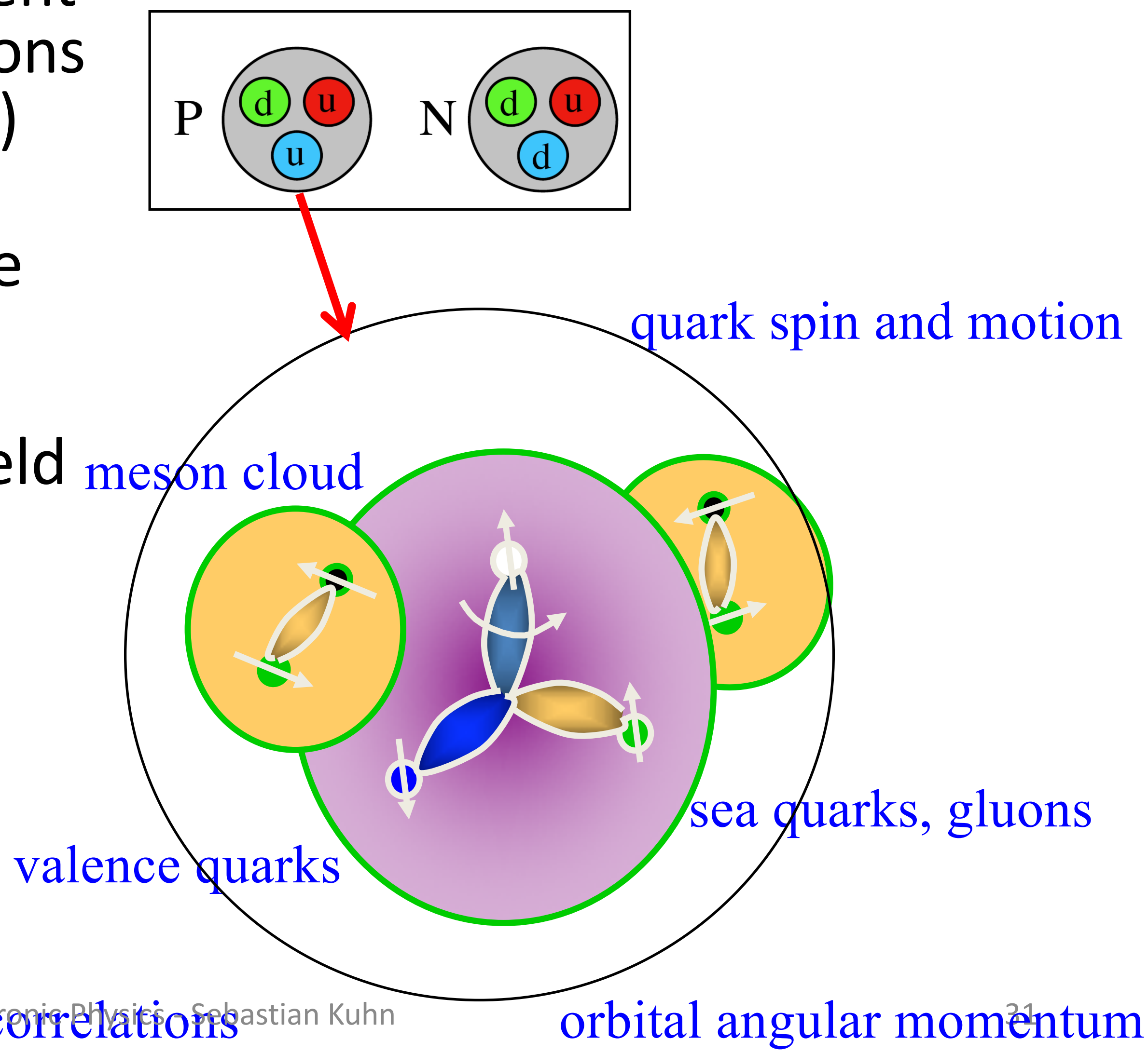
Figure 51.2: World data on the total cross section of $e^+e^- \to hadrons$ and the ratio $R(s) = \sigma(e^+e^- \to hadrons, s)/\sigma(e^+e^- \to \mu^+\mu^-, s)$. $\sigma(e^+e^- \to hadrons, s)$ is the experimental cross section corrected for initial state radiation and electron-positron vertex loops, $\sigma(e^+e^- \to \mu^+\mu^-, s) = 4\pi\alpha^2(s)/3s$. Data errors are total below 2 GeV and statistical above 2 GeV. The curves are an educative guide: the broken one (green) is a naive quark-parton model prediction, and the solid one (red) is 3-loop pQCD prediction (see "Quantum Chromodynamics" section of this *Review*, Eq. (9.7) or, for more details, K. G. Chetyrkin *et al.*, Nucl. Phys. **B586**, 56 (2000) (Erratum *ibid.* **B634**, 413 (2002)). Breit-Wigner parameterizations of J/ψ, $\psi(2S)$, and $\Upsilon(nS), n = 1, 2, 3, 4$ are also shown. The full list of references to the original data and the details of the R ratio extraction from them can be found in `[arXiv:hep-ph/0312114]`. Corresponding computer-readable data files are available at `http://pdg.lbl.gov/current/xsect/`. (Courtesy of the COMPAS (Protvino) and HEPDATA (Durham) Groups, August 2017. Corrections by P. Janot (CERN) and M. Schmitt (Northwestern U.))

Running of the Strong Coupling Constant



Hadron Structure

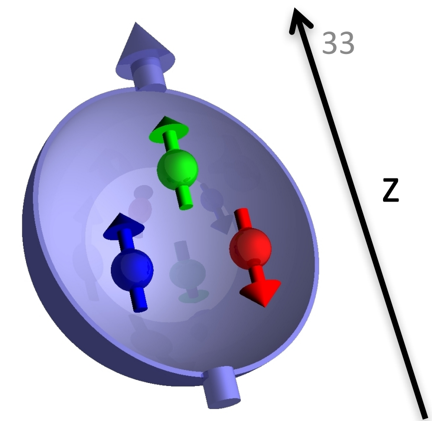
- Simple-most (constituent quark) model of nucleons (protons and neutrons)
- … becomes much more complicated once we consider the full relativistic quantum field theory called QCD
- Effective theories: Quark model, χPT, sum rules, …
- and Lattice QCD!



How Do We Study Hadron/Nuclear Structure?

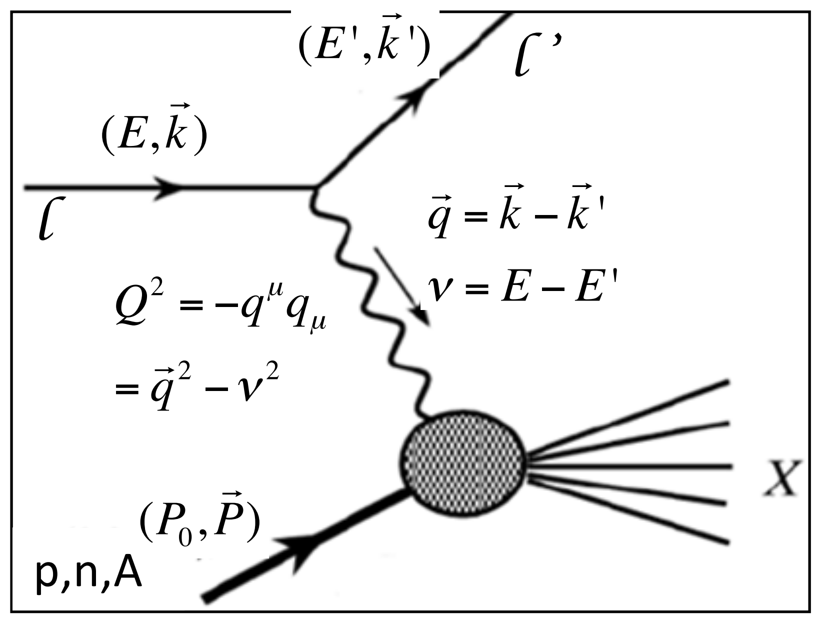
- Energy levels: Nuclear and particle (baryon, meson) masses, excitation spectra, excited state decays ->
  Spectroscopy *(What exists?)*
- Elastic and inelastic scattering, particle production
  Reactions (*Relationships?*)
- Probing the internal structure directly
  Imaging (*Shape and Content?*)
- Particular way to encode this: Structure Functions
  – *"Parton wave function"?*
    5(6)-dim. Wigner distribution ➔ …

Parton Distribution Functions



- The 1D world of nucleon/nuclear collinear structure:
  - Take a nucleon/nucleus
  - Move it real fast along z
    $\Rightarrow$ light cone momentum
    $P_+ = P_0 + P_z$ (>>M)
  - Select a "parton" (quark, gluon) inside
  - Measure **its** l.c. momentum
    $p_+ = p_0 + p_z$ (m≈0)
  - $\Rightarrow$ Momentum Fraction x = p_+/P_+ *)
  - $\boxed{\text{In DIS **): } p_+/P_+ \approx \xi = (q_z - \nu)/M \approx x_{Bj} = Q^2/2M\nu}$
  - Probability: $f_1^i(x), i = u, d, s, \ldots, G$

In the following, will often write "$q_i(x)$" for $f_1^i(x)$



*) Advantage: Boost-independent along *z*

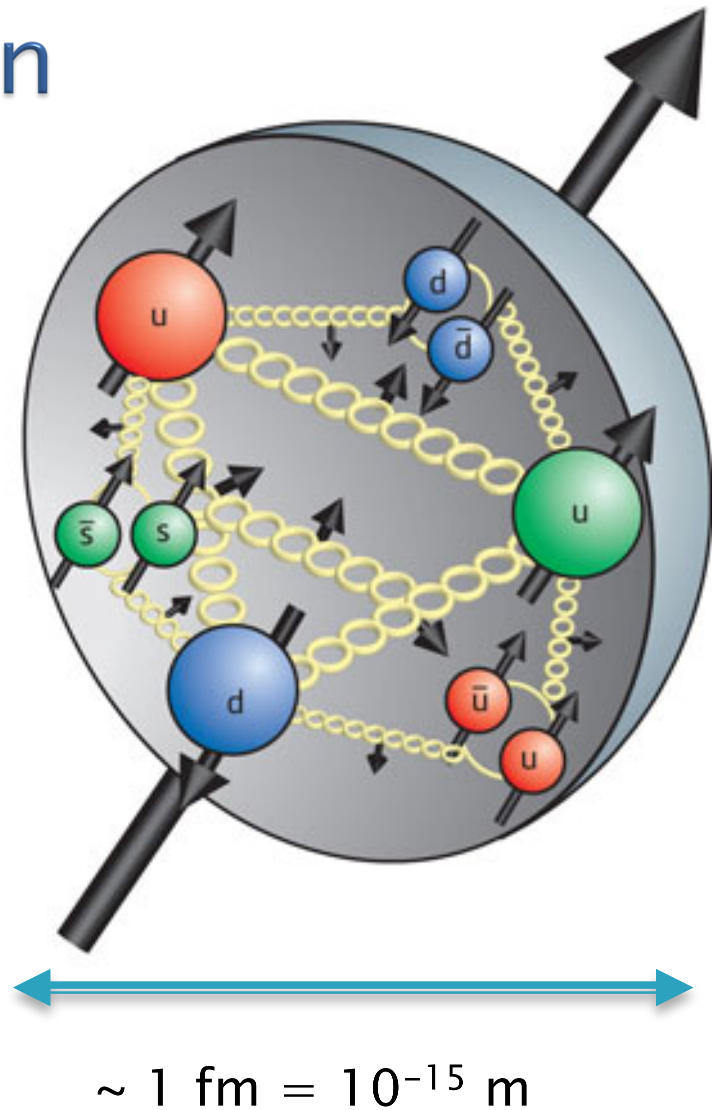
**) DIS = "Deep Inelastic (Lepton) Scattering"

Reminder:

The structure of the nucleon

Three ”valence” quarks plus gluons plus quark/anti-quark pairs.

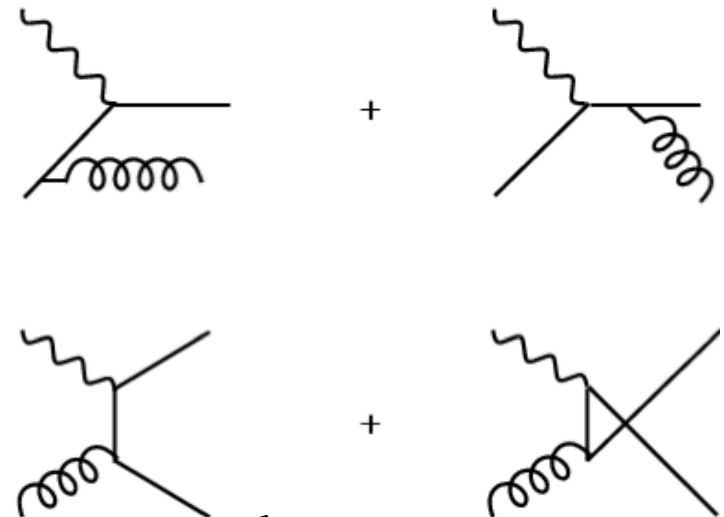
- How do the quarks and gluons interact to form a proton or neutron?
- Where does the spin of the nucleon come from?
- Why is the radius of the proton different when measured by electrons and muons?
- Why are protons and neutrons modified in a nucleus?



Parton Distribution Functions and NLO pQCD

Two effects modify simple parton picture:

1) (Gluon) radiative corrections change elementary cross section



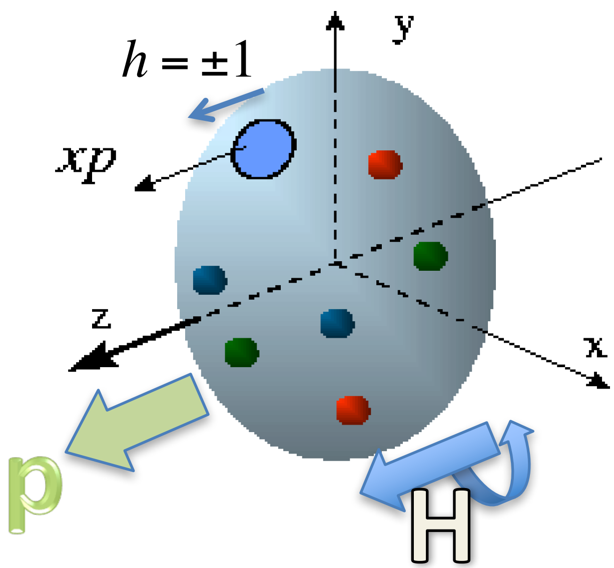
$$g_1(x,Q^2)_{pQCD} = \frac{1}{2}\sum_q^{N_f} e_q^2 [(\Delta q + \Delta\bar{q}) \otimes (1 + \frac{\alpha_s(Q^2)}{2\pi}\delta C_q) + \frac{\alpha_s(Q^2)}{2\pi}\Delta G \otimes \frac{\delta C_G}{N_f}]$$

$\delta C_q, \delta C_G - Wilson$ coefficient functions

2) pQCD evolution makes PDFs Q^2-dependent

Inclusive lepton scattering

Parton model: DIS can access



$$q(x;Q^2), \langle h \cdot H \rangle q(x;Q^2)$$

Traditional "1-D" Parton Distributions (PDFs)
(integrated over many variables)

Callan-Gross

Wandzura-Wilczek

$$F_1(x) = \tfrac{1}{2}\sum_i e_i^2 q_i(x) \ \left(\text{and } \boxed{F_2(x) \approx 2xF_1(x)}\right)$$

$$g_1(x) = \tfrac{1}{2}\sum_i e_i^2 \Delta q_i(x) \ \left(\text{and } g_2(x) \approx \boxed{-g_1(x) + \int_x^1 \frac{g_1(y)}{y} dy}\right)$$

At finite Q^2: pQCD evolution ($q(x,Q^2)$, $\Delta q(x,Q^2)$ $\Rightarrow$ DGLAP equations), and gluon radiation

$$g_1(x,Q^2)_{pQCD} = \frac{1}{2}\sum_q^{N_f} e_q^2 [(\Delta q + \Delta \bar{q}) \otimes (1 + \frac{\alpha_s(Q^2)}{2\pi}\delta C_q) + \frac{\alpha_s(Q^2)}{2\pi}\Delta G \otimes \frac{\delta C_G}{N_f}]$$

$\Rightarrow$ access to gluons. $\delta C_q, \delta C_G - Wilson$ coefficient functions

SIDIS: Tag the flavor of the struck quark with the leading FS hadron $\Rightarrow$ separate $q_i(x,Q^2)$, $\Delta q_i(x,Q^2)$

Jefferson Lab kinematics: $Q^2 \approx M^2 \Rightarrow$ target mass effects, higher twist contributions and resonance excitations

- Non-zero $R = \frac{F_2}{2xF_1}\left(\frac{4M^2x^2}{Q^2}+1\right)-1$, $g_2^{HT}(x) = g_2(x) - g_2^{WW}(x)$
- Further Q^2-dependence (power series in $\frac{1}{Q^n}$)

⇒ Our 1D View of the Nucleon

(depends on *x* and the resolution of the virtual photon ~ $1/Q^2$)

W = final state invariant mass = $\sqrt{M^2 + \left(\frac{1}{x} - 1\right) Q^2}$

Low Q^2:

ChPT

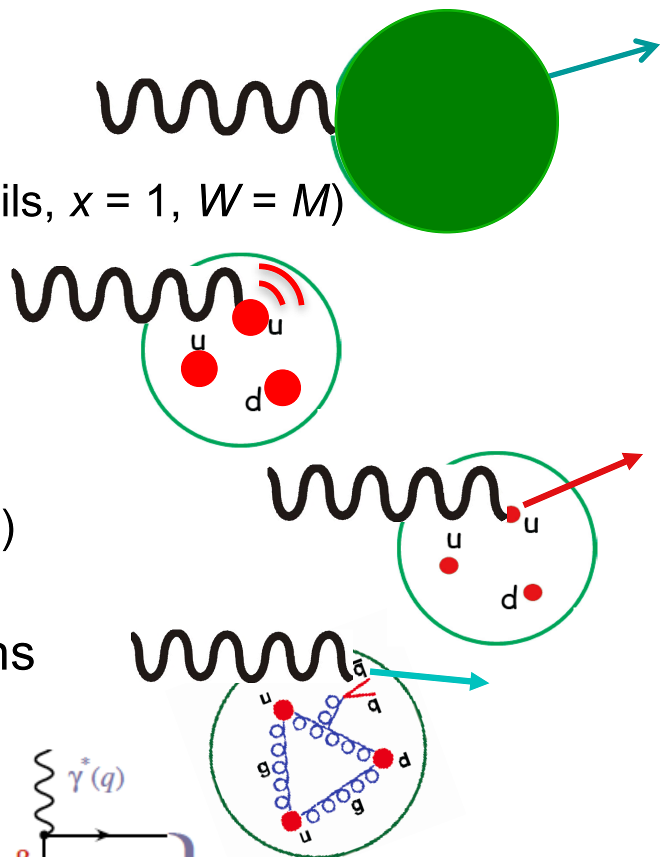
CQM

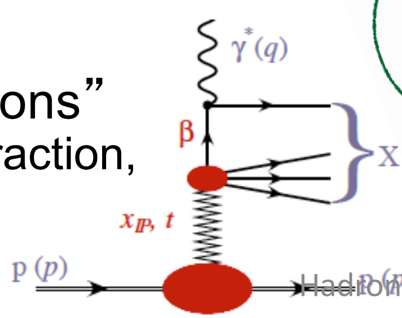
OPE

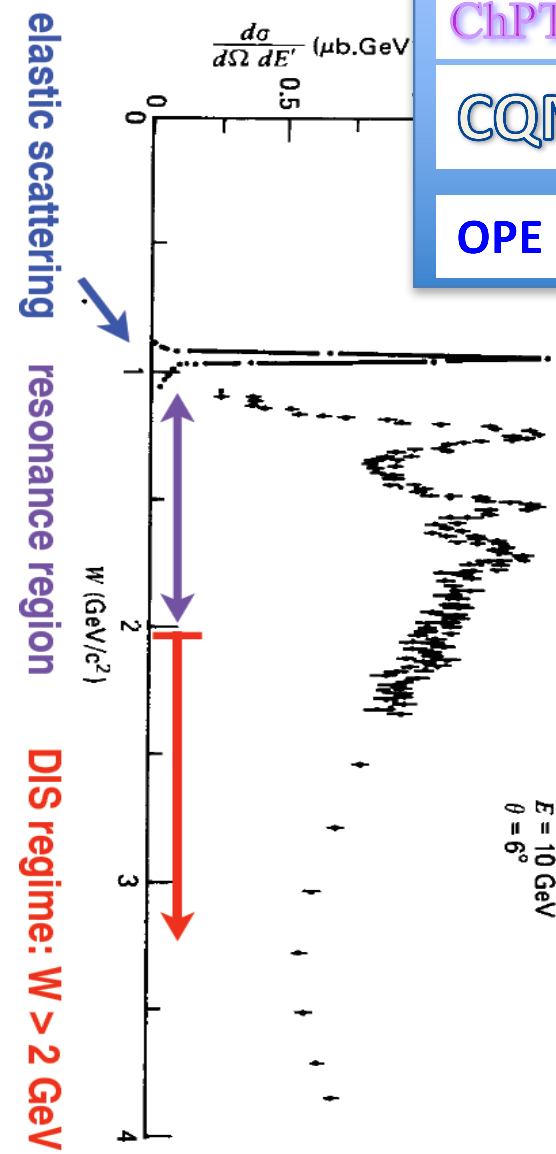
JLab

DIS

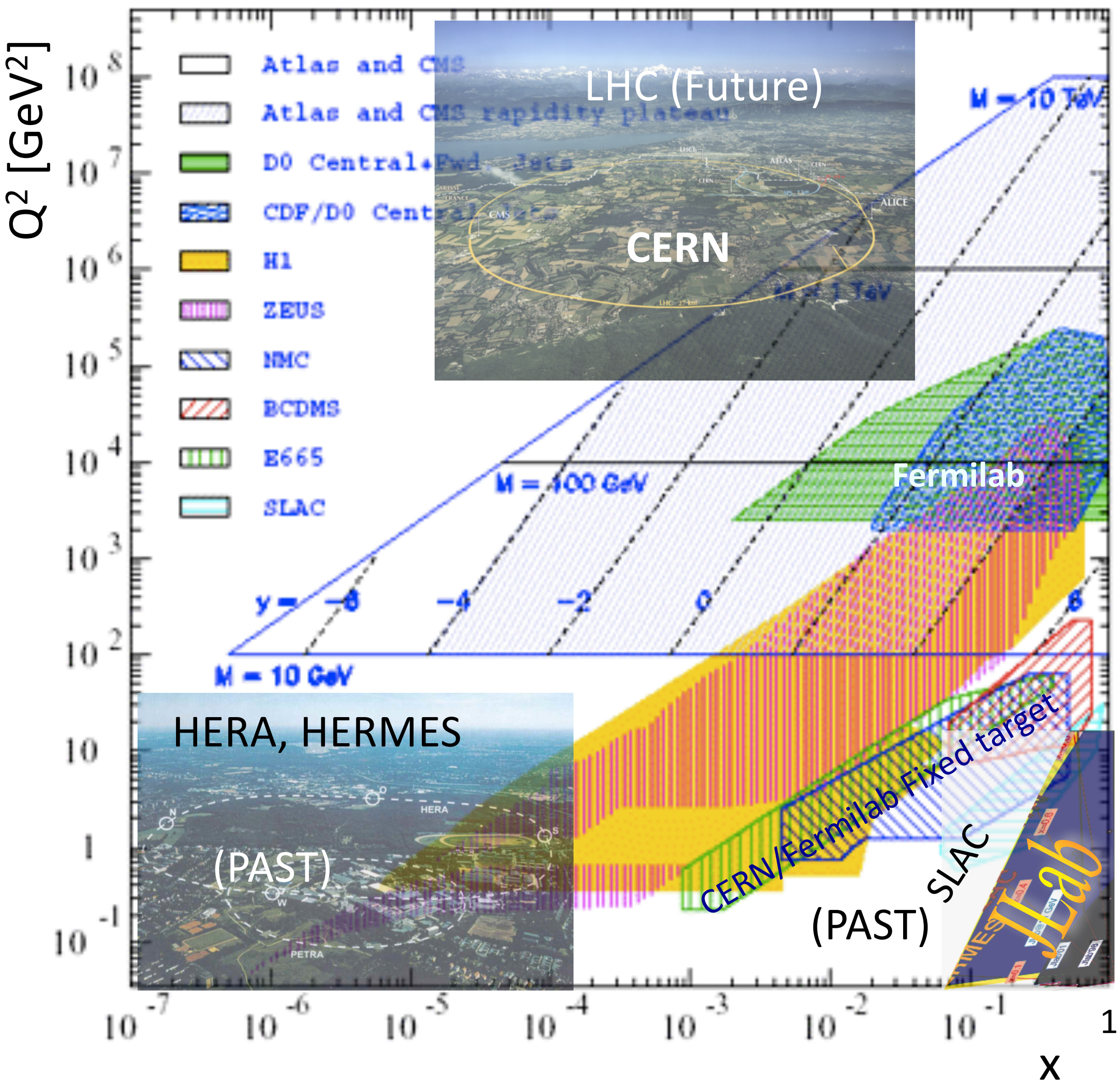
- Elastic scattering
  (Whole system recoils, $x = 1$, $W = M$)
- Resonances
  ($x < 1$, $W < 2$ GeV)
- Valence quarks
  ($x \geq 0.3$, $W > 2$ GeV)
- Sea quarks, gluons
  ($x < 0.3$)
- "Wee Partons"
  ($x \to 0$, Diffraction, Pomerons)

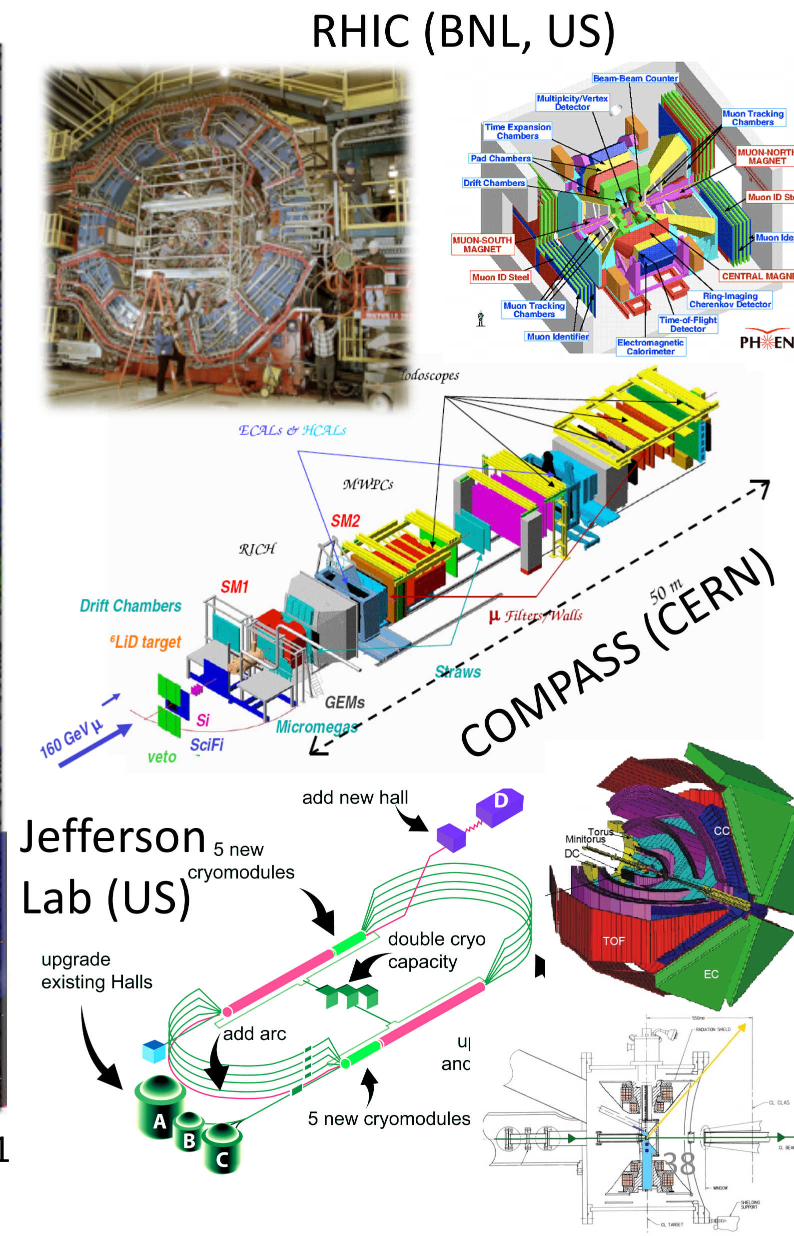






Experimental Facilities





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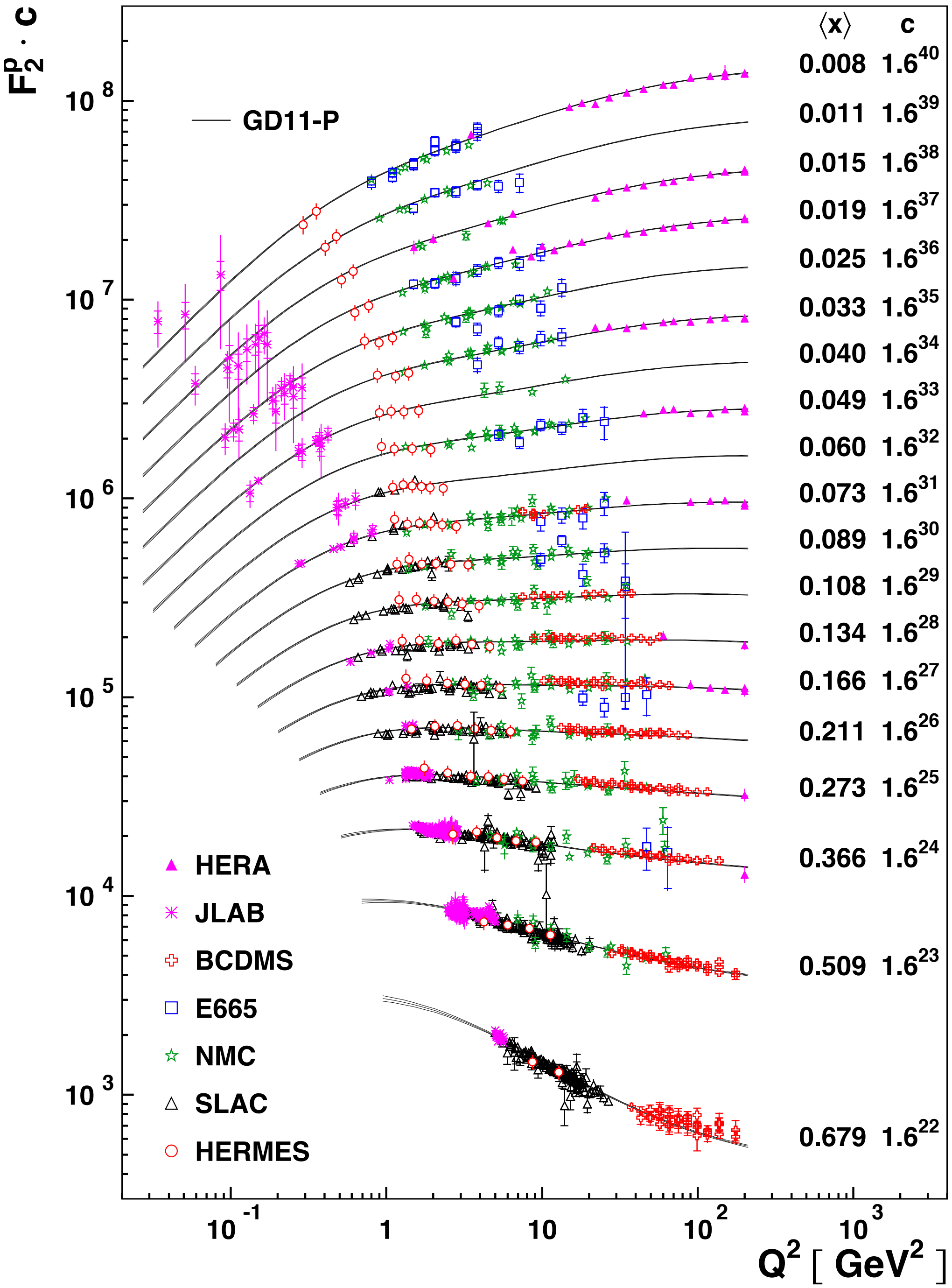
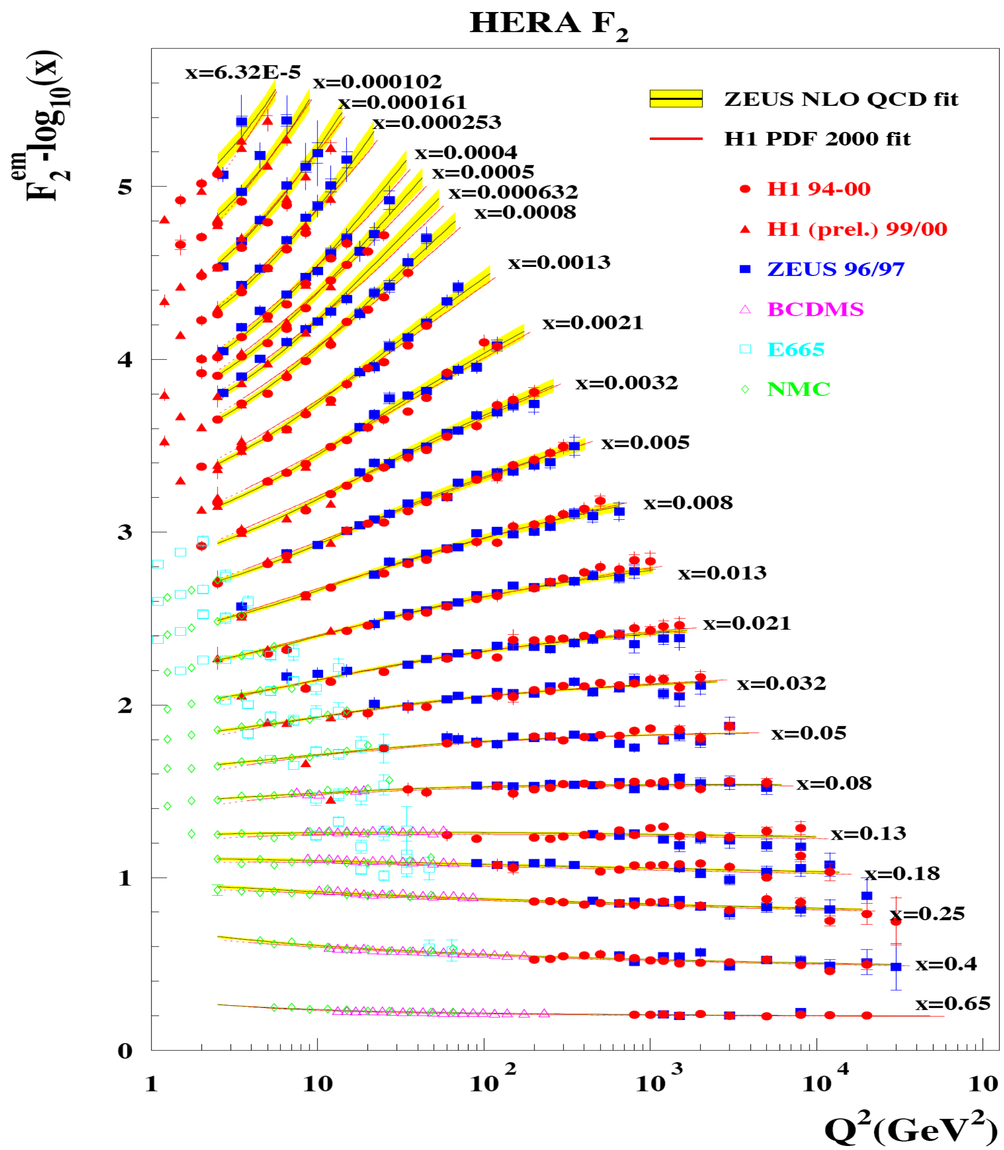
Results

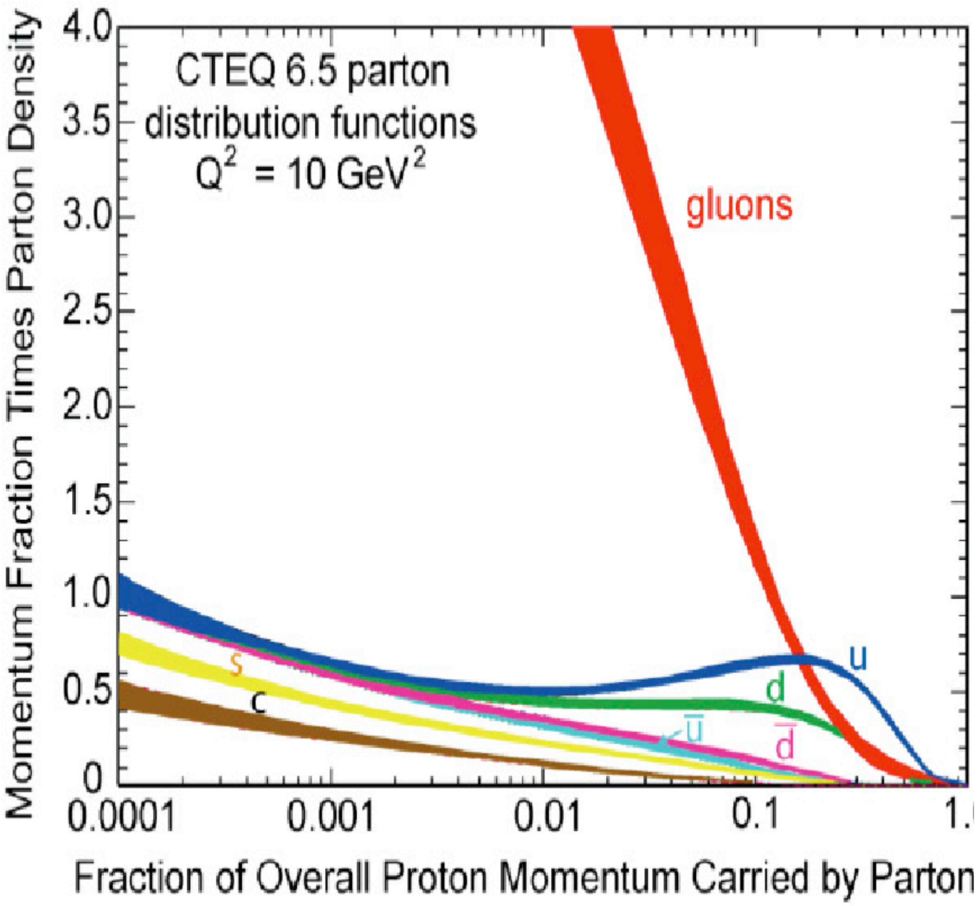


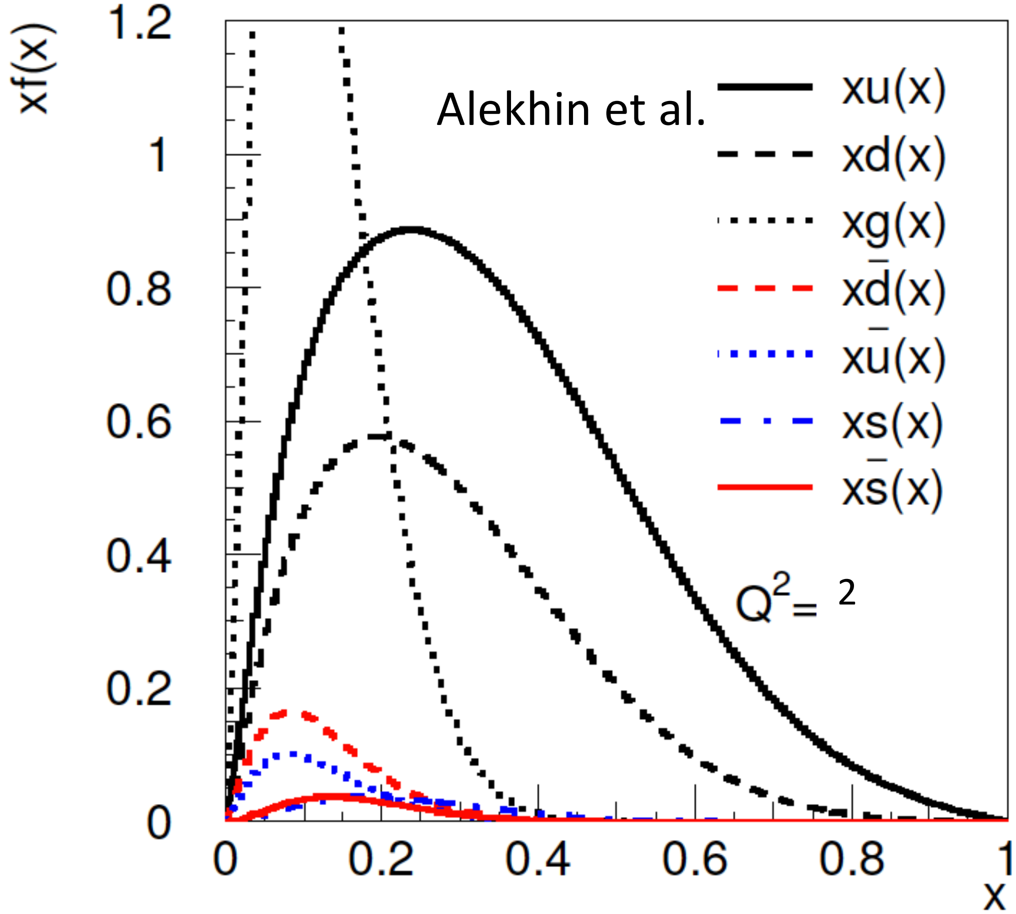
$F_2^p \cdot c$
GD11-P
⟨x⟩ c
0.008 1.6^40
0.011 1.6^39
0.015 1.6^38
0.019 1.6^37
0.025 1.6^36
0.033 1.6^35
0.040 1.6^34
0.049 1.6^33
0.060 1.6^32
0.073 1.6^31
0.089 1.6^30
0.108 1.6^29
0.134 1.6^28
0.166 1.6^27
0.211 1.6^26
0.273 1.6^25
0.366 1.6^24
0.509 1.6^23
0.679 1.6^22
HERA
JLAB
BCDMS
E665
NMC
SLAC
HERMES
Q^2 [GeV^2]

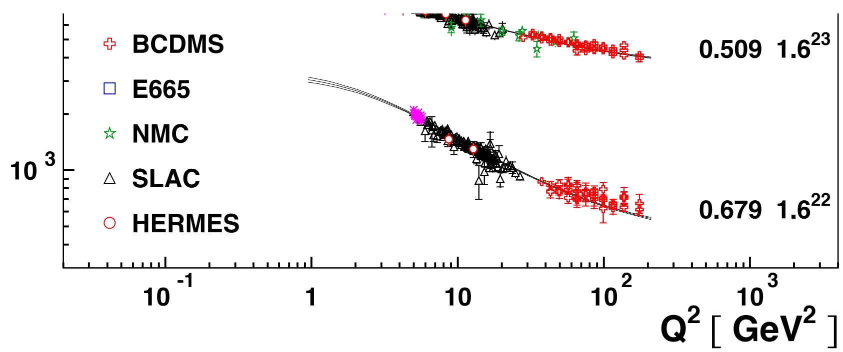


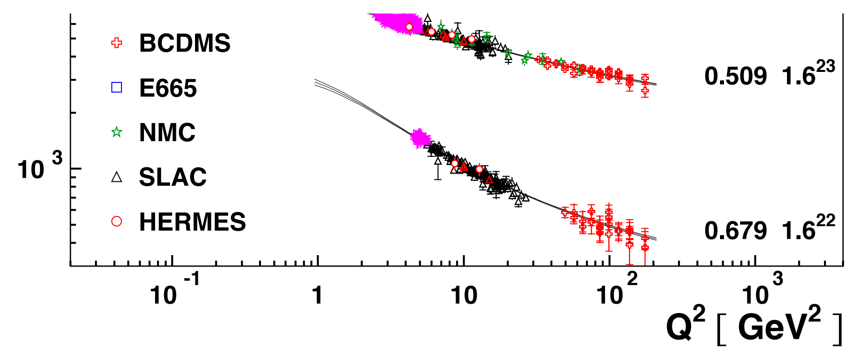
HERA F_2
F_2^{em}-$\log_{10}(x)$
x=6.32E-5
x=0.000102
x=0.000161
x=0.000253
x=0.0004
x=0.0005
x=0.000632
x=0.0008
x=0.0013
x=0.0021
x=0.0032
x=0.005
x=0.008
x=0.013
x=0.021
x=0.032
x=0.05
x=0.08
x=0.13
x=0.18
x=0.25
x=0.4
x=0.65
ZEUS NLO QCD fit
H1 PDF 2000 fit
H1 94-00
H1 (prel.) 99/00
ZEUS 96/97
BCDMS
E665
NMC
$Q^2(GeV^2)$

Unpolarized Structure Functions

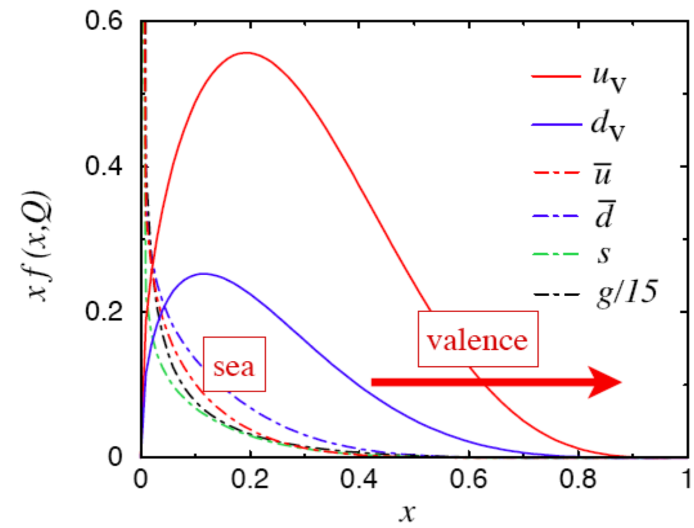




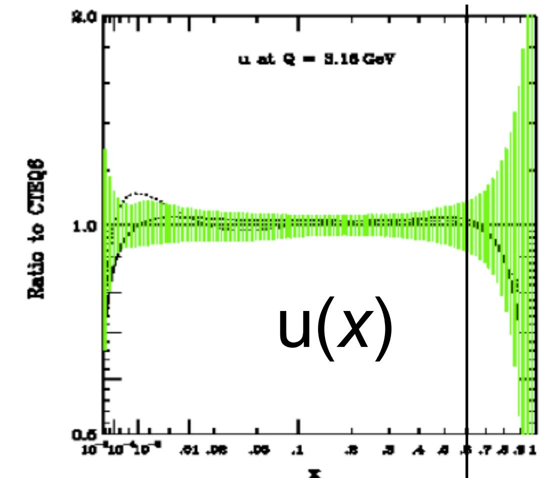


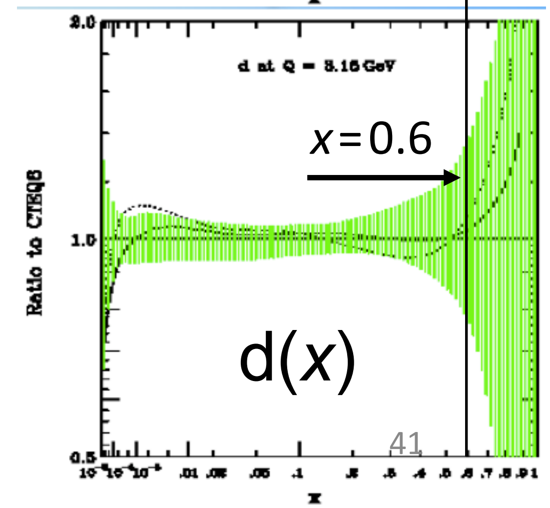


Valence PDFs

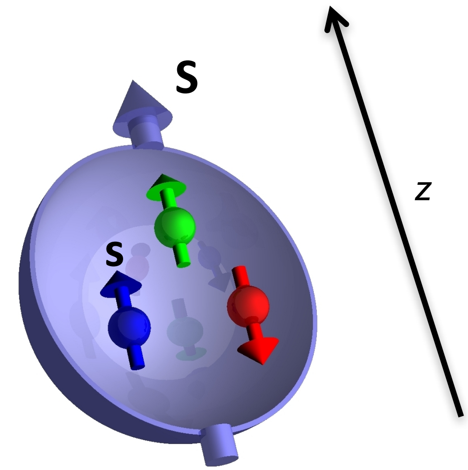


- Behavior of PDFs still unknown for $x \to 1$
  - SU(6): d/u = 1/2, Δu/u = 2/3, Δd/d = -1/3 for all x
  - Relativistic Quark model: Δu, Δd reduced
  - Hyperfine effect (1-gluon-exchange): Spectator spin 1 suppressed, d/u → 0, Δu/u → 1, Δd/d → -1/3
  - Helicity conservation: d/u → 1/5, Δu/u → 1, Δd/d → 1
  - Orbital angular momentum: can explain slower convergence to Δd/d → 1
- Plenty of data on proton → mostly constraints on u and Δu
- Knowledge on d limited by lack of free neutron target (nuclear binding effects in d, ^{3}He)
- Large x requires very high luminosity and resolution; binding effects become dominant uncertainty for the neutron





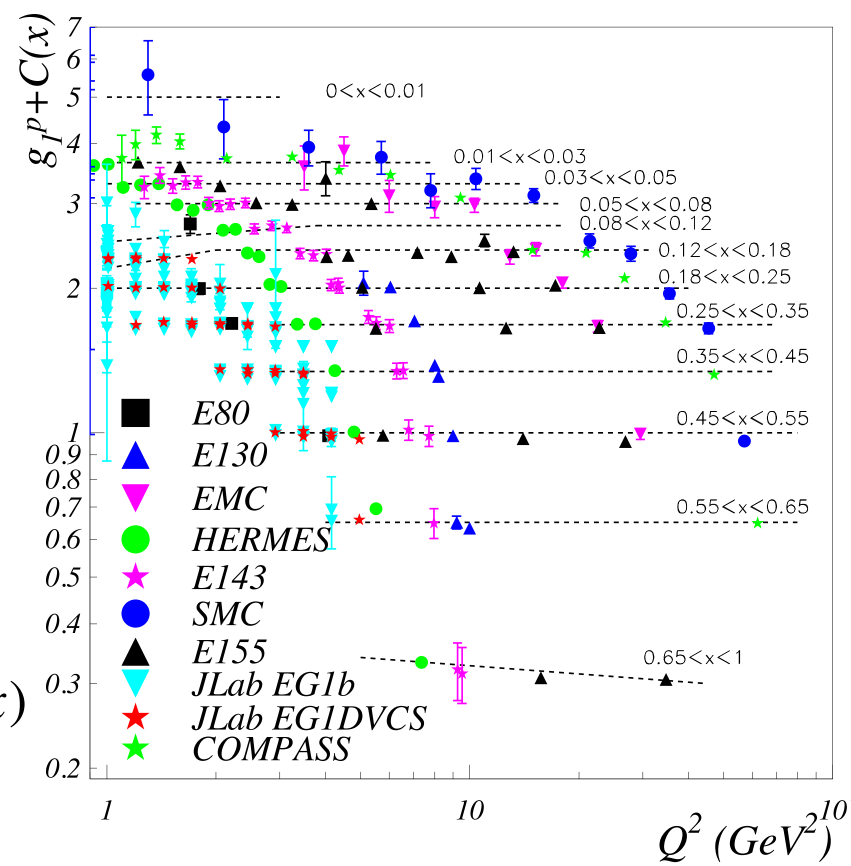
Polarized Parton Distribution Functions



- Introduce two more quantities of interest:
  - Proton spin **S**
  - Parton spin **s**
  - Now we have 3 vectors: $\hat{z}, \vec{S}, \vec{s}$
  - **But**: Every observable must be a scalar
  - **And**: Spins are axial vectors!
  - **Finally**: Must treat longitudinal and transverse directions differently (boost)
  - 2 Pseudoscalars: $H = \vec{S}\cdot\hat{z}, h = \vec{s}\cdot\hat{z}$
  - 2 transverse (2D) axial vectors: $\vec{S}_\perp, \vec{s}_\perp$
  - 2^{nd} Structure function

$$g_1^i(x) = \langle hH\rangle q_i(x) \text{ or } \langle hH\rangle G(x) = \Delta q_i(x) \text{ or } \Delta G(x)$$

$$\Delta q_i = \quad q\Uparrow\uparrow(x) - q\Uparrow\downarrow(x)$$



Can also form one more scalar: $T = \vec{S}_\perp \cdot \vec{s}_\perp$ (not measurable in DIS) → Transversity $h_1(x)$