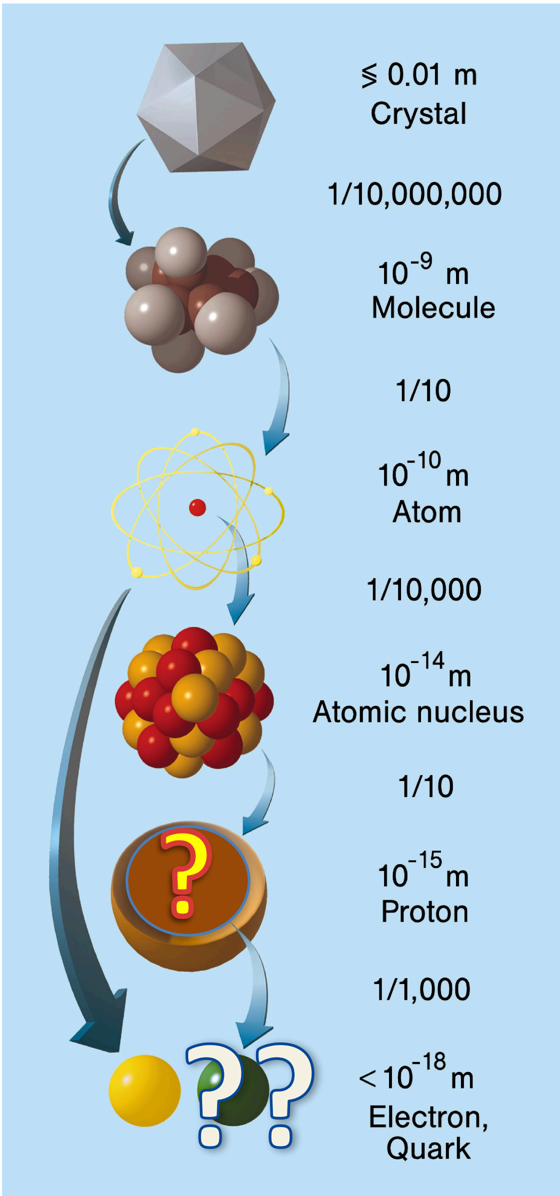
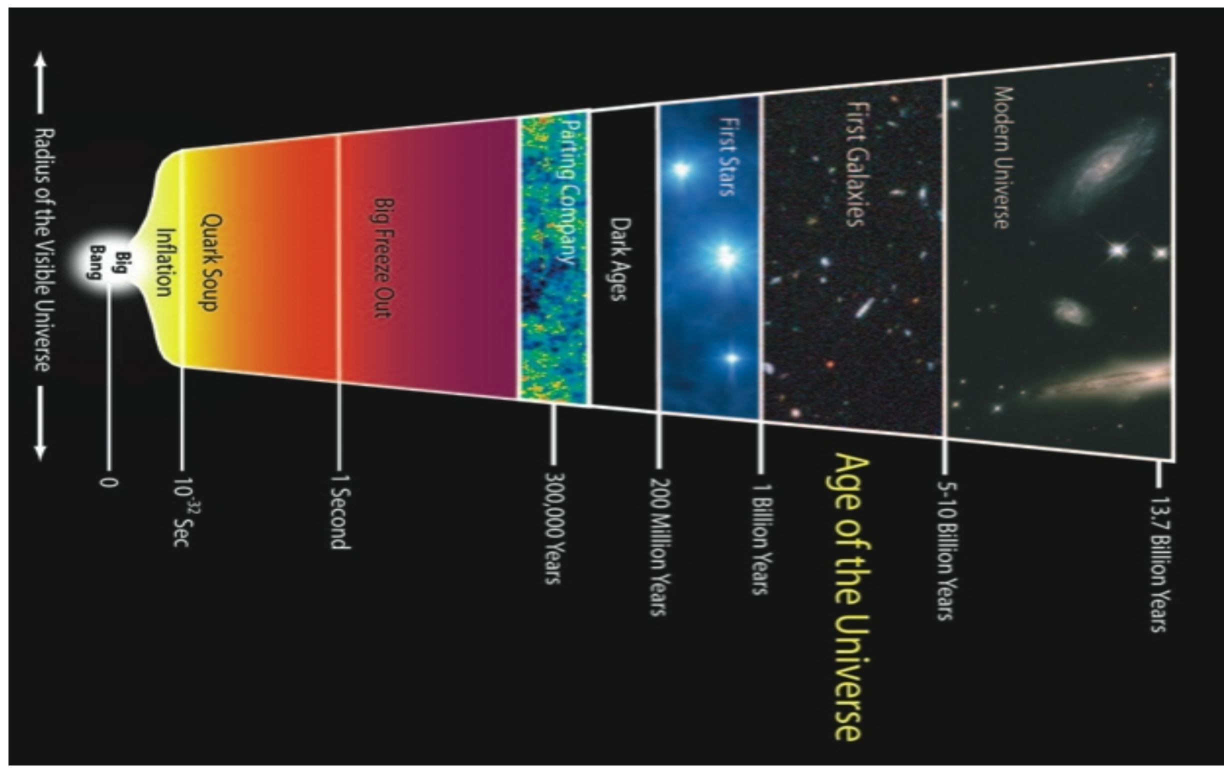
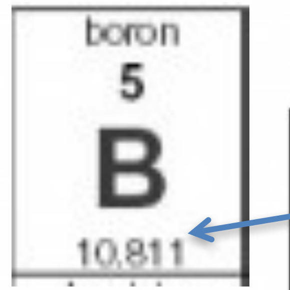
The Structure of Matter



- What is the Universe made off?
- What are the most fundamental objects in Nature?
- What particles where there in the beginning (right after the big bang)?
- How do they interact?
- How do they form composite objects?

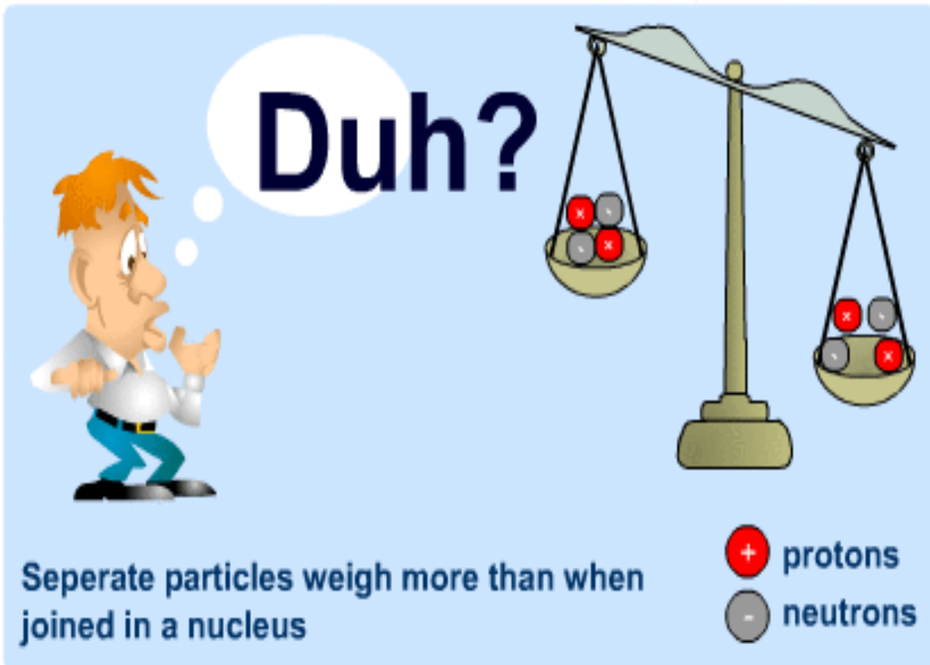


Periodic Table



hydrogen 1 H 1.0079																		helium 2 He 4.0026
lithium 3 Li 6.941	beryllium 4 Be 9.0122												boron 5 B 10.811	carbon 6 C 12.011	nitrogen 7 N 14.007	oxygen 8 O 15.999	fluorine 9 F 18.998	neon 10 Ne 20.180
sodium 11 Na 22.990	magnesium 12 Mg 24.305												aluminium 13 Al 26.982	silicon 14 Si 28.086	phosphorus 15 P 30.974	sulfur 16 S 32.065	chlorine 17 Cl 35.453	argon 18 Ar 39.948
potassium 19 K 39.098	calcium 20 Ca 40.078		scandium 21 Sc 44.956	titanium 22 Ti 47.867	vanadium 23 V 50.942	chromium 24 Cr 51.996	manganese 25 Mn 54.938	iron 26 Fe 55.845	cobalt 27 Co 58.933	nickel 28 Ni 58.693	copper 29 Cu 63.546	zinc 30 Zn 65.39	gallium 31 Ga 69.723	germanium 32 Ge 72.61	arsenic 33 As 74.922	selenium 34 Se 78.96	bromine 35 Br 79.904	krypton 36 Kr 83.80
rubidium 37 Rb 85.468	strontium 38 Sr 87.62		yttrium 39 Y 88.906	zirconium 40 Zr 91.224	niobium 41 Nb 92.906	molybdenum 42 Mo 95.94	technetium 43 Tc [98]	ruthenium 44 Ru 101.07	rhodium 45 Rh 102.91	palladium 46 Pd 106.42	silver 47 Ag 107.87	cadmium 48 Cd 112.41	indium 49 In 114.82	tin 50 Sn 118.71	antimony 51 Sb 121.76	tellurium 52 Te 127.60	iodine 53 I 126.90	xenon 54 Xe 131.29
caesium 55 Cs 132.91	barium 56 Ba 137.33	57-70 *	lutetium 71 Lu 174.97	hafnium 72 Hf 178.49	tantalum 73 Ta 180.95	tungsten 74 W 183.84	rhenium 75 Re 186.21	osmium 76 Os 190.23	iridium 77 Ir 192.22	platinum 78 Pt 195.08	gold 79 Au 196.97	mercury 80 Hg 200.59	thallium 81 Tl 204.38	lead 82 Pb 207.2	bismuth 83 Bi 208.98	polonium 84 Po [209]	astatine 85 At [210]	radon 86 Rn [222]
francium 87 Fr [223]	radium 88 Ra [226]	89-102 **	lawrencium 103 Lr [262]	rutherfordium 104 Rf [261]	dubnium 105 Db [262]	seaborgium 106 Sg [266]	bohrium 107 Bh [264]	hassium 108 Hs [269]	meitnerium 109 Mt [268]	ununnilium 110 Uun [271]	unununium 111 Uuu [272]	ununbium 112 Uub [277]		ununquadium 114 Uuq [289]				

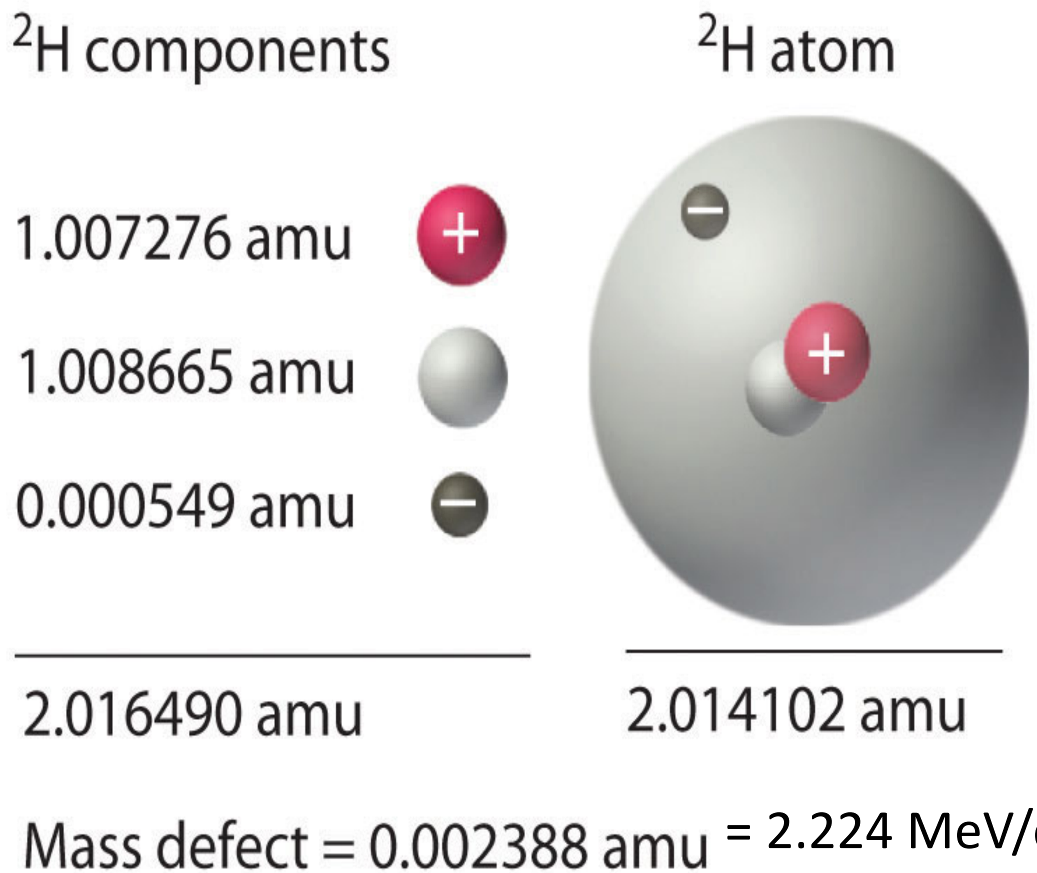
*Lanthanide series	lanthanum 57 La 138.91	cerium 58 Ce 140.12	praseodymium 59 Pr 140.91	neodymium 60 Nd 144.24	promethium 61 Pm [145]	samarium 62 Sm 150.36	europium 63 Eu 151.96	gadolinium 64 Gd 157.25	terbium 65 Tb 158.93	dysprosium 66 Dy 162.50	holmium 67 Ho 164.93	erbium 68 Er 167.26	thulium 69 Tm 168.93	ytterbium 70 Yb 173.04
**Actinide series	actinium 89 Ac [227]	thorium 90 Th 232.04	protactinium 91 Pa 231.04	uranium 92 U 238.03	neptunium 93 Np [237]	plutonium 94 Pu [244]	americium 95 Am [243]	curium 96 Cm [247]	berkelium 97 Bk [247]	californium 98 Cf [251]	einsteinium 99 Es [252]	fermium 100 Fm [257]	mendelevium 101 Md [258]	nobelium 102 No [259]


Duh?
Seperate particles weigh more than when joined in a nucleus
protons
neutrons

In atomic and nuclear physics, masses are typically given in atomic mass units (a.m.u. or *u*);
$1\ u = 1.66054 \cdot 10^{-27}$ kg =
931.494 MeV/c^2
=$M(^{12}C)/12$

Proton : 938.27 MeV =
Neutron : 939.57 MeV =

M_A = "A_{nom}"*u
"A_{nom}" ≈ A := Z + N
"A_{nom}": Atomic mass #
A = Baryon Number



= 2.224 MeV/c^2

Nuclear masses given in

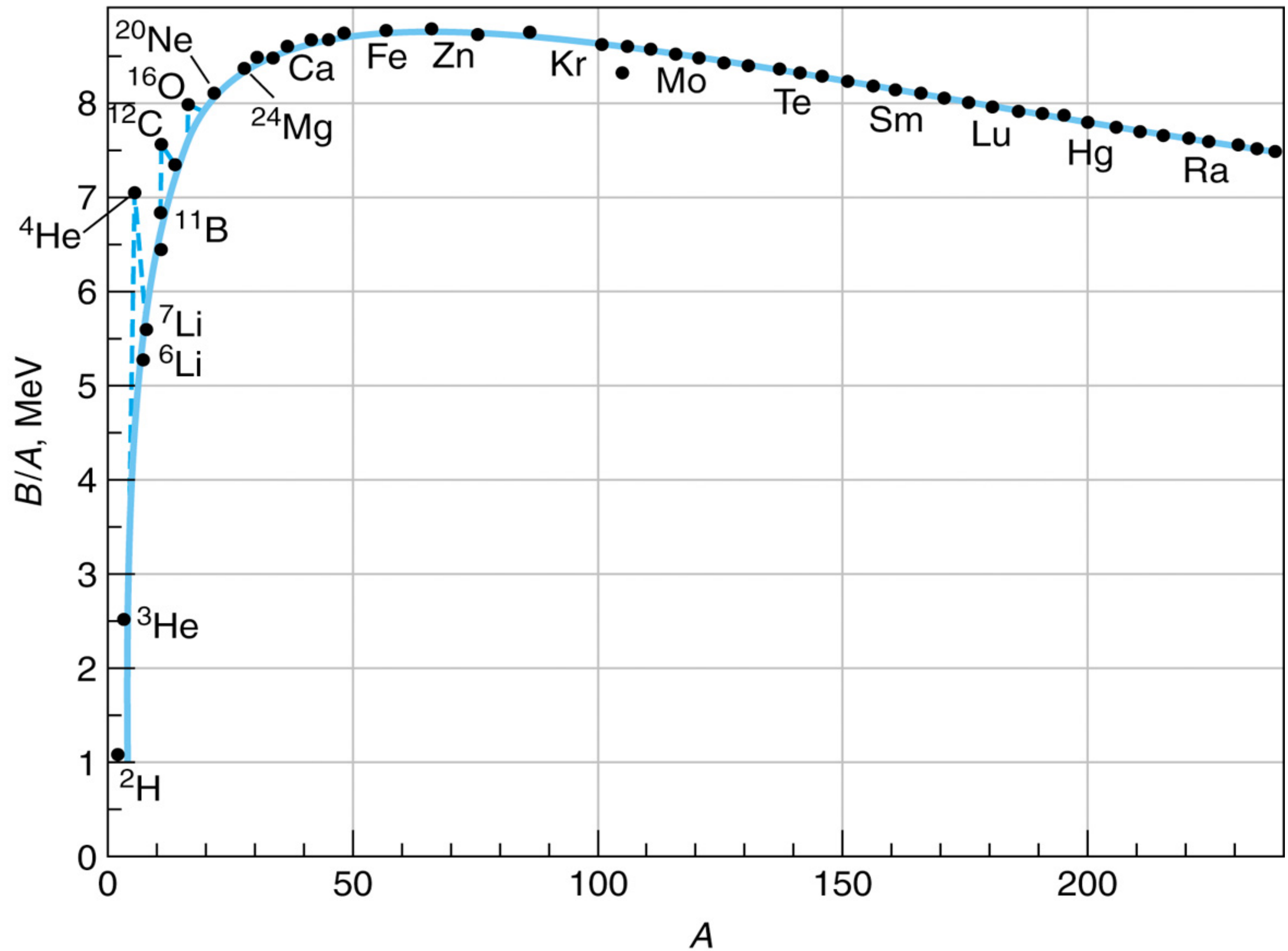
- kg
- MeV/c^2
- AMU ("A")
- "Mass Excess" $\Delta(Z,N) = M(Z,N) - (Z+N)*u$
  with u (= 1 a.m.u. = atomic mass unit) = $m(^{12}C)/12$ = 931.494 MeV/c^2

Nuclear Binding energy

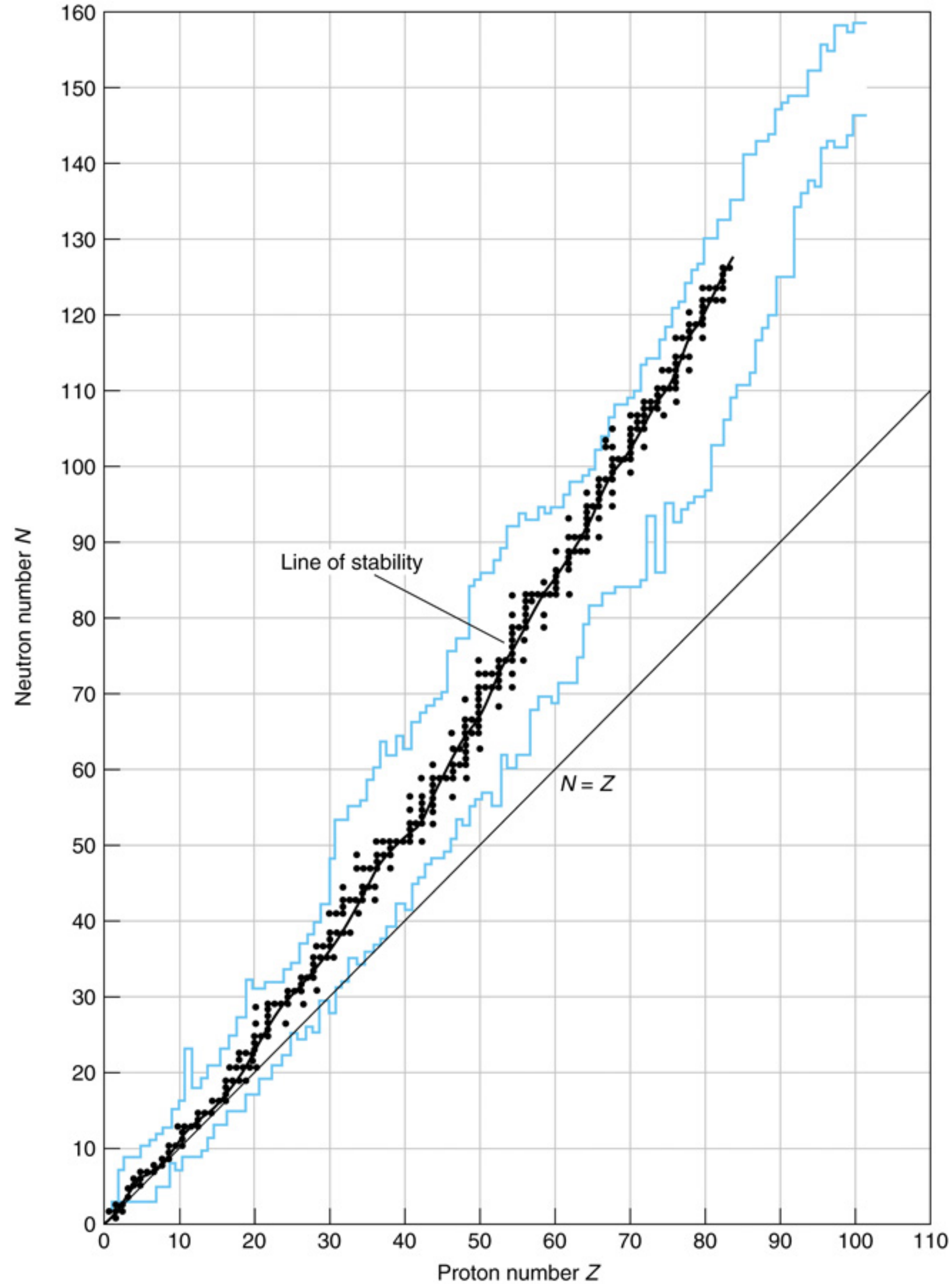
$B_{nuclear} = Zm_pc^2 + Nm_Nc^2 - M_Ac^2$ (MeV's)

M_A = Atomic mass

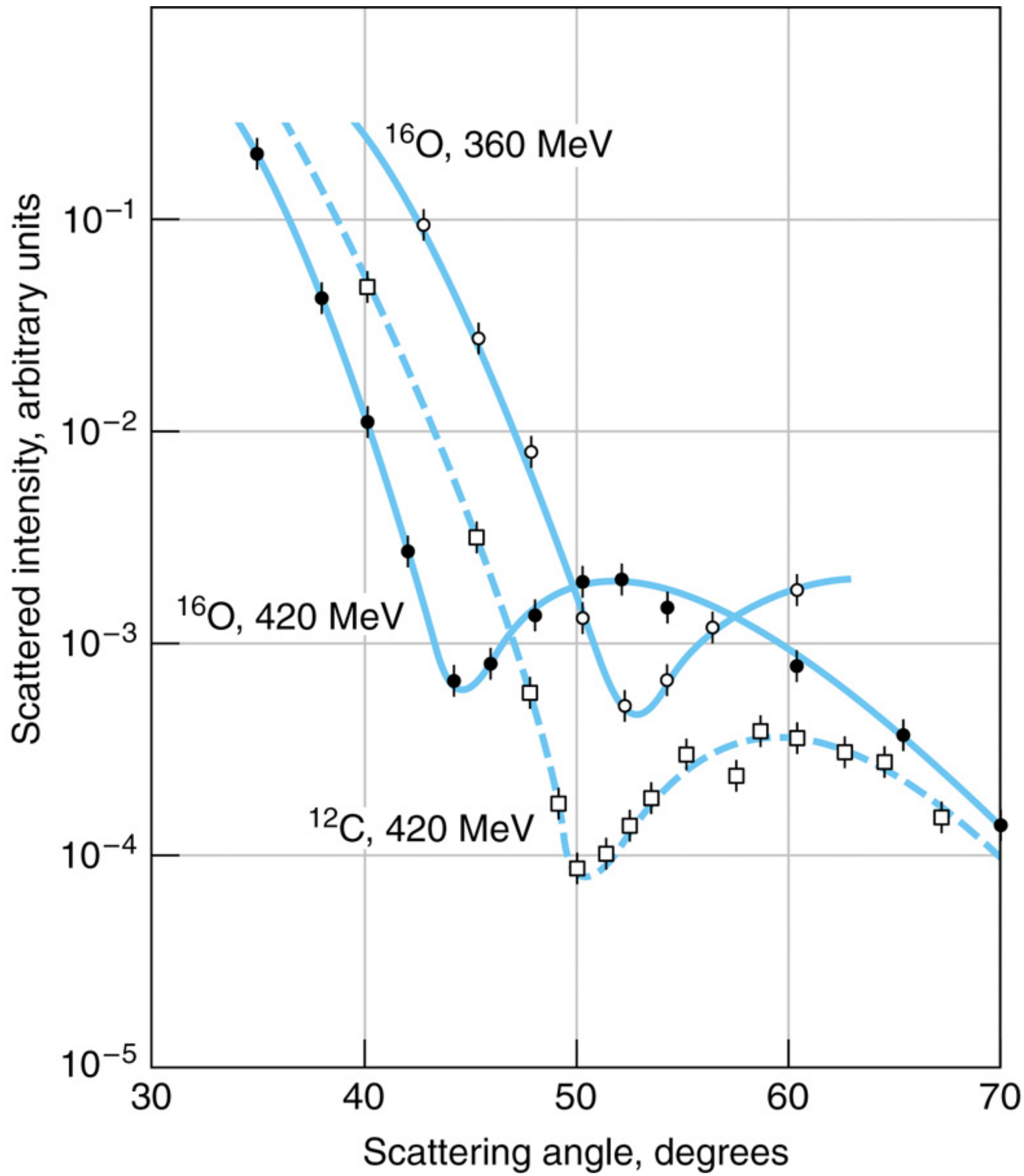
Nuclear Binding energies



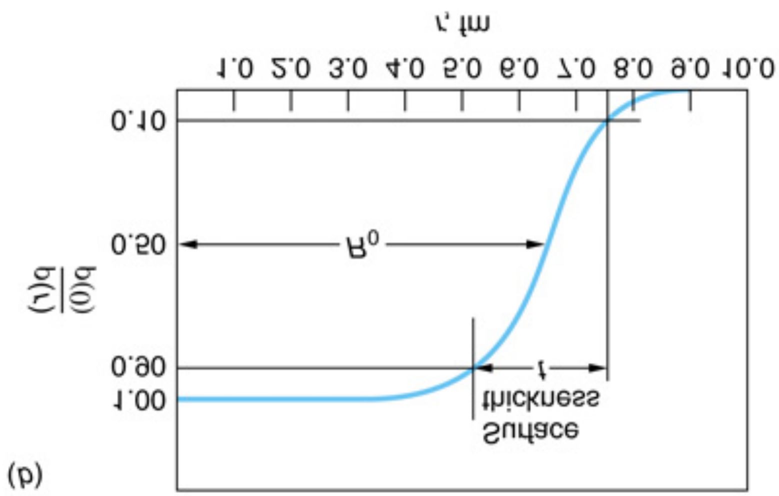
Stable nuclei

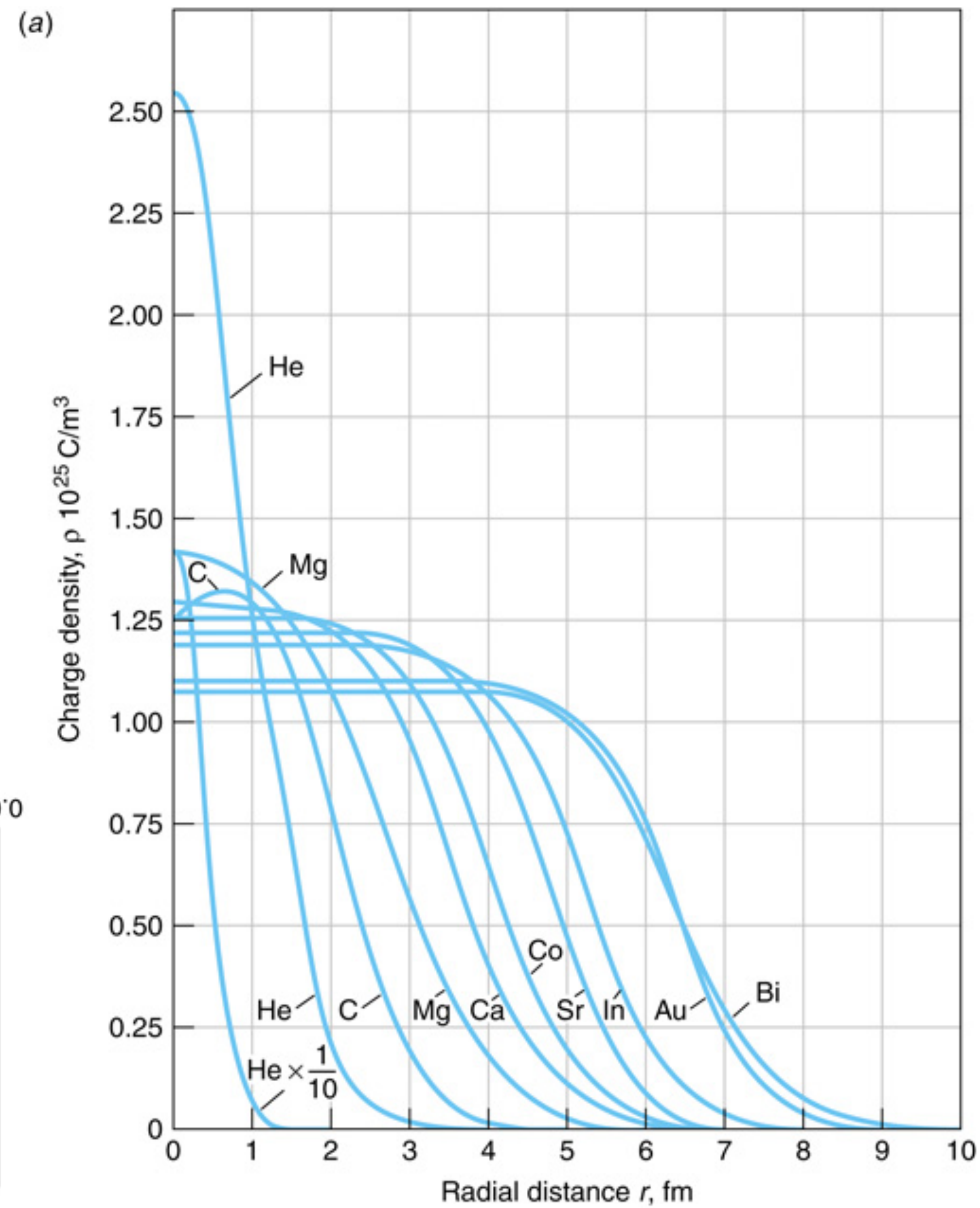


Electron Scattering

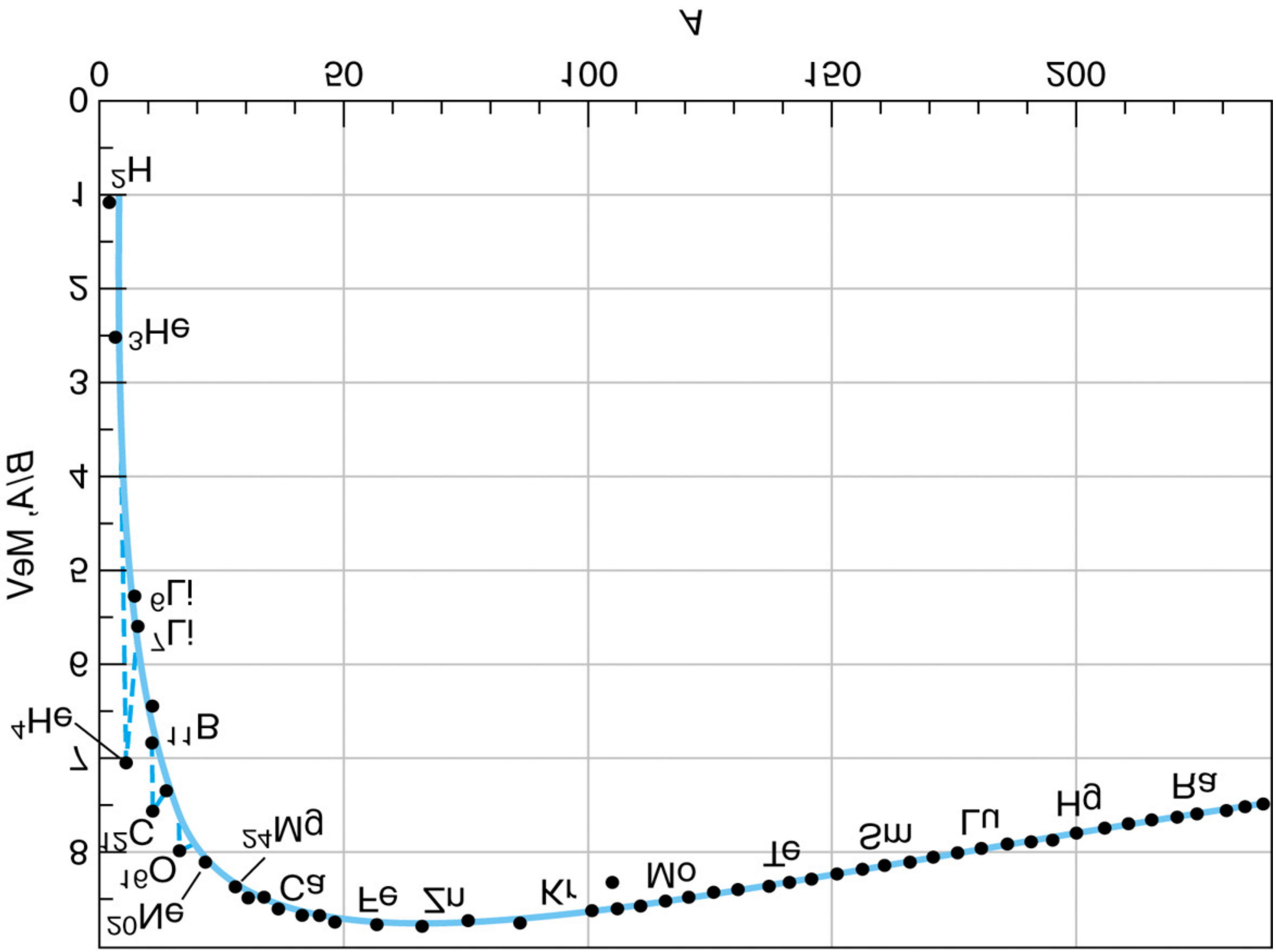


Electron Scattering -> Nuclear Density





Nuclear Binding energies

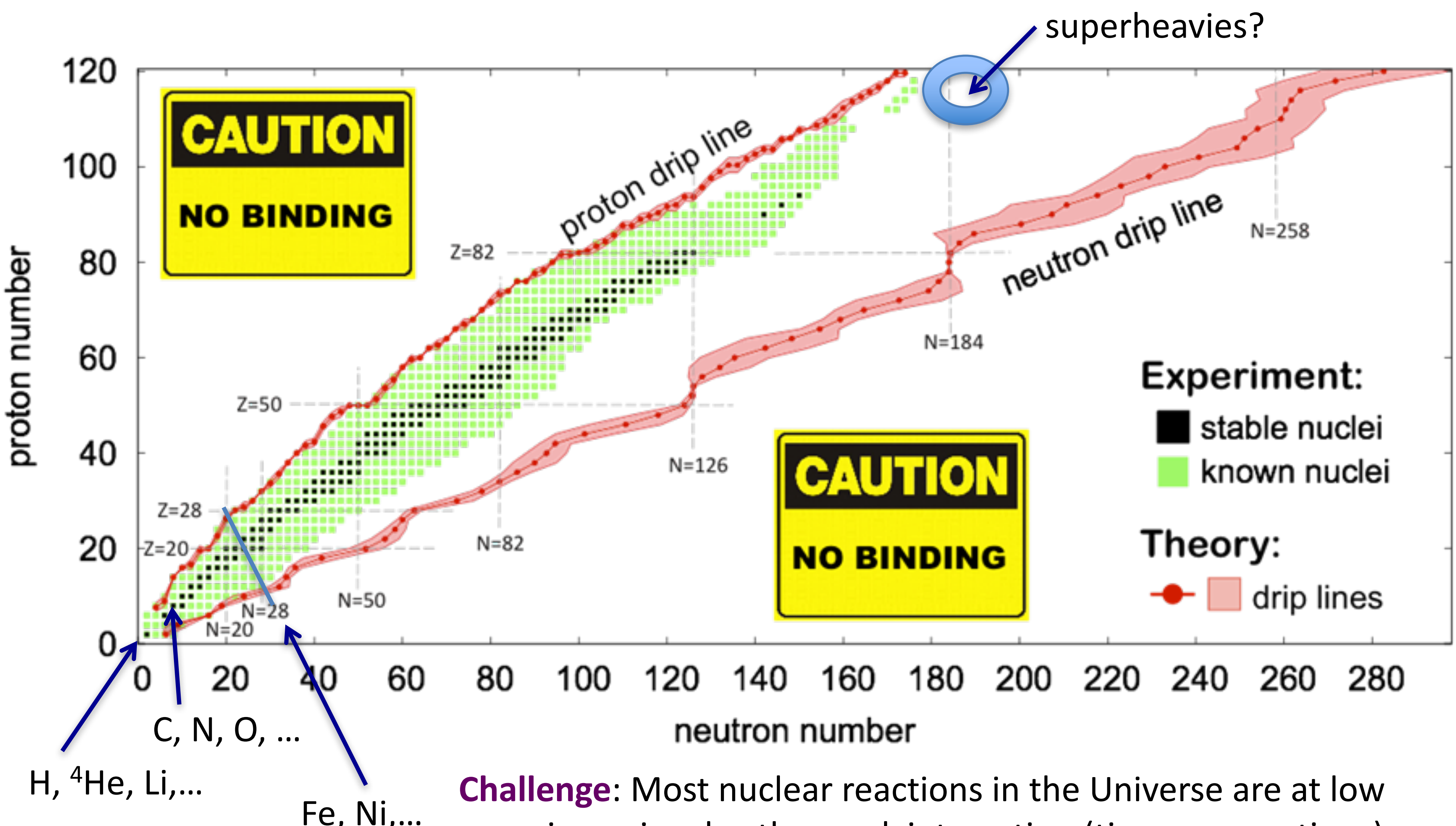


$$M(A,Z) = N\cdot M_n + Z\cdot M_p + Z\cdot m_e - |B|/c^2 \;; \quad |B| = a_V A - a_S A^{2/3} - a_C \frac{Z^2}{A^{1/3}} - a_a \frac{(N-Z)^2}{4A} \pm \frac{\delta}{A^{1/2}}$$

	Weizsäcker mass formula								
	aV	aS	aC	aA	Delta	Mn	Mp	Mel	a.m.u.
	15.67	17.23	0.714	93.15	11.2	939.56533	938.27234	0.510999	931.494
	Z	N	A	I3	Mass	Binding Energy	B.E. per nucl.	Mass Excess	Nuclear mass
	77	116	193	-19.5	0.000	-1532.825	-7.942	-35.272	179703.723
	Actual Masses								
D	1	1	2	0	1876.1238	-2.225	-1.112	13.136	1875.613
T	1	2	3	-0.5	2809.432	-8.482	-2.827	14.950	2808.921
He3	2	1	3	0.5	2809.413	-7.719	-2.573	14.931	2808.391
He4	2	2	4	0	3728.401	-28.296	-7.074	2.425	3727.379
Li6	3	3	6	0	5603.05	-31.996	-5.333	14.086	5601.517
Li7	3	4	7	-0.5	6535.366	-39.245	-5.606	14.908	6533.833
Be9	4	5	9	-0.5	8394.794	-58.166	-6.463	11.348	8392.750
B10	5	5	10	0	9326.991	-64.752	-6.475	12.051	9324.436
B11	5	6	11	-0.5	10255.102	-76.207	-6.928	8.668	10252.547
C12	6	6	12	0	11177.928	-92.164	-7.680	0.000	11174.862
C13	6	7	13	-0.5	12112.547	-97.110	-7.470	3.125	12109.481
N14	7	7	14	0	13043.779	-104.662	-7.476	2.863	13040.202
N15	7	8	15	-0.5	13972.511	-115.495	-7.700	0.101	13968.934
O16	8	8	16	0	14899.167	-127.622	-7.976	-4.737	14895.079
O17	8	9	17	-0.5	15834.589	-131.766	-7.751	-0.809	15830.501
O18	8	10	18	-1	16766.11	-139.810	-7.767	-0.782	16762.022
Ar40	18	22	40	-2	37224.72	-343.817	-8.595	-35.040	37215.522
K40	19	21	40	-1	37226.225	-341.530	-8.538	-33.535	37216.516
Ca40	20	20	40	0	37224.914	-342.059	-8.551	-34.846	37214.694
Ir193	77	116	193	-19.5	179743.806	-1532.089	-7.938	-34.536	179704.459
Au197	79	118	197	-19.5	183473.161	-1559.432	-7.916	-31.157	183432.792
	Weizsäcker mass formula - Comparison								
	aV	aS	aC	aA	Delta	Mn	Mp	Mel	a.m.u.
Povh et al.	15.67	17.23	0.714	93.15	11.2	939.56533	938.27234	0.510999	931.494
SSM Wong	16	17	0.6	100	25	939.56533	938.27234	0.510999	931.494
	Z	N	A	I3	Mass	Binding Energy	B.E. per nucl.	Mass Excess	Nuclear mass
Povh	77	116	193	-19.5	179743.070	-1532.825	-7.942	-35.272	179703.723
SSM Wong	77	116	193	-19.5	179560.241	-1715.654	-8.889	-218.101	179520.894

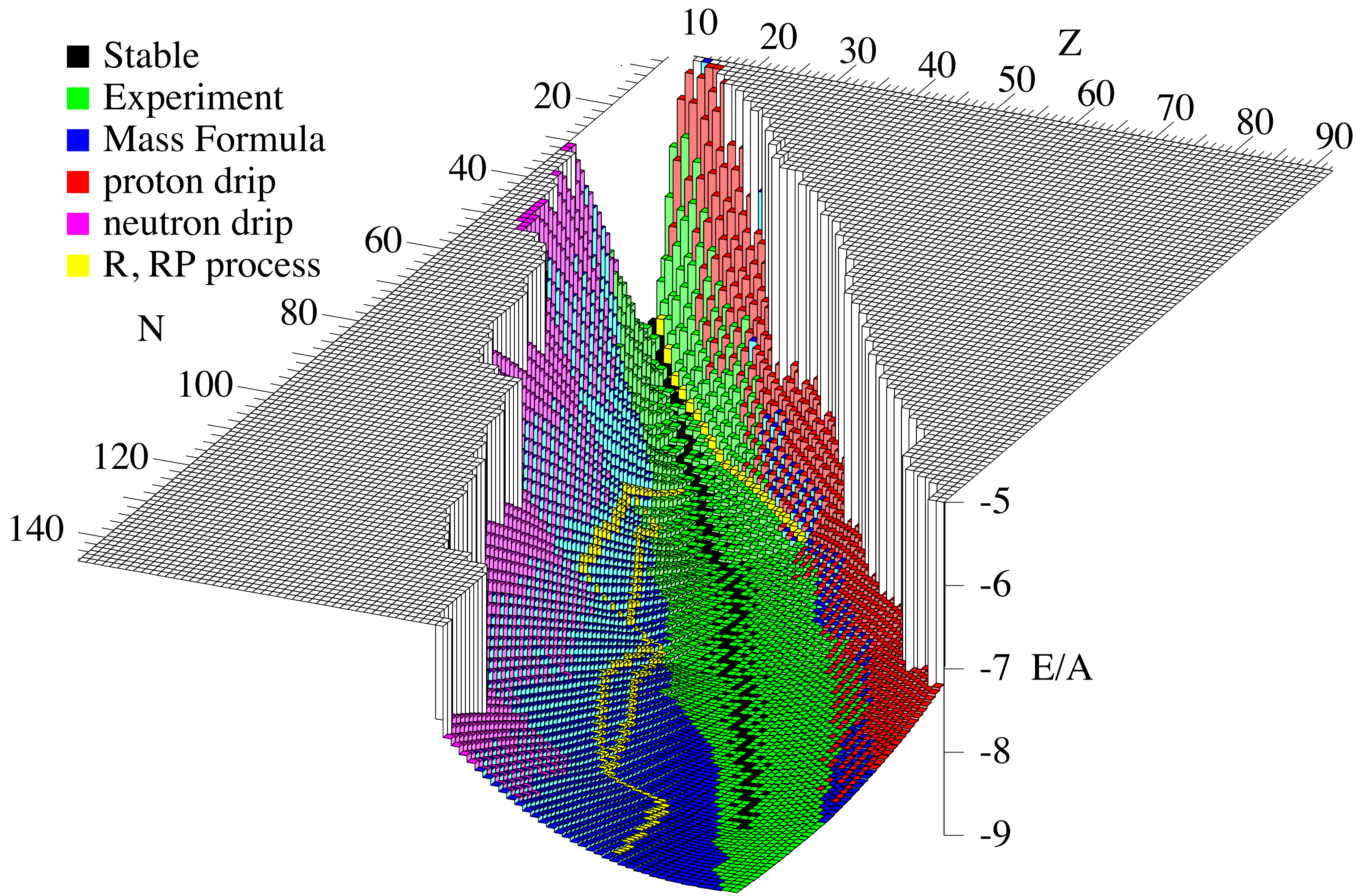
All the nuclei in the universe

n stars

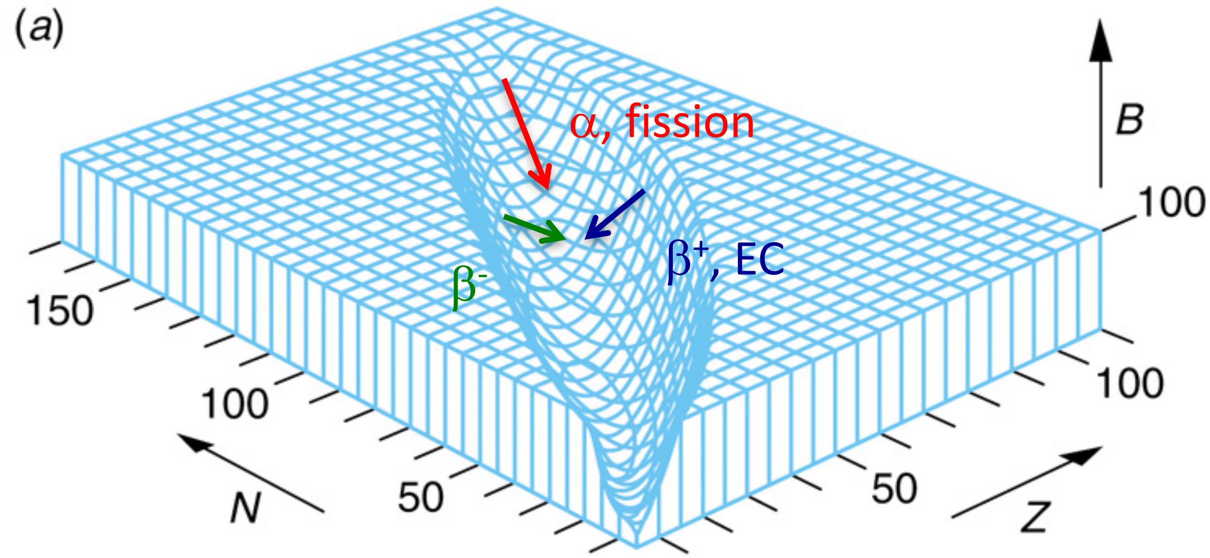


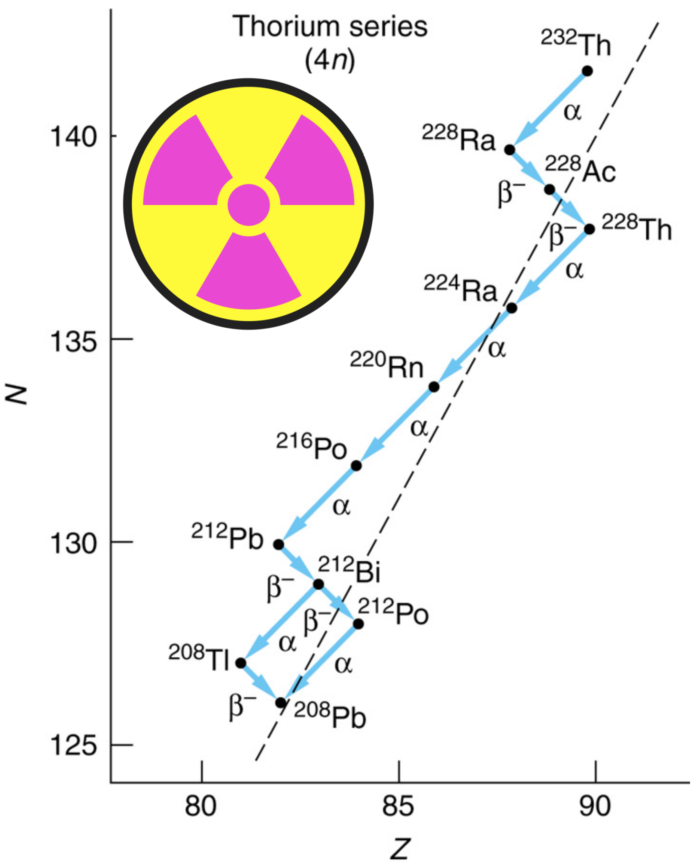
Challenge: Most nuclear reactions in the Universe are at low energies or involve the weak interaction (tiny cross sections) => Experiments and Theory are HARD! (subtle effects play big role!)

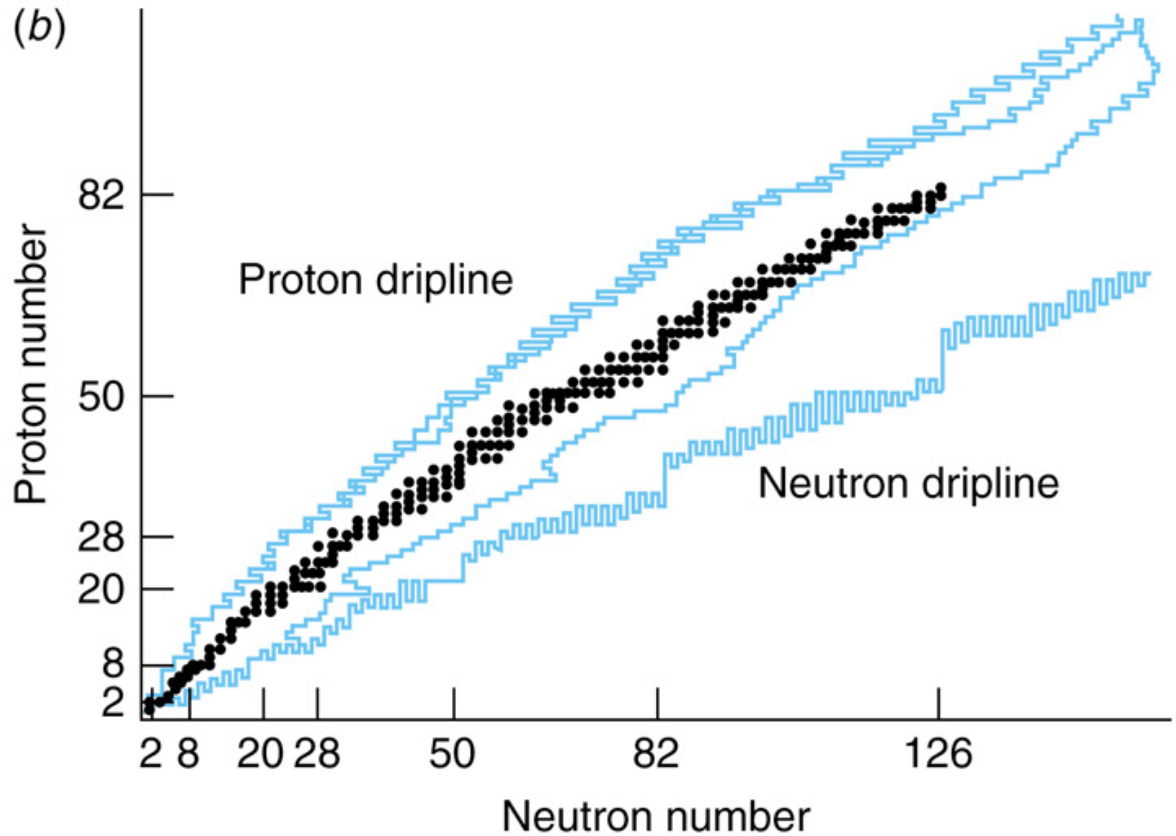
What do we know about them?



Nuclear Stability and Decays







Fermi Gas Model

Interlude: Fermi Gas

- Pauli exclusion principle: No two fermions (spin 1/2 particles) can be in the same quantum state
- Heisenberg uncertainty principle: $\Delta p \cdot \Delta x \approx \hbar$ => two states are indistinguishable if they occupy the same "cell" $dV \cdot d^3p = h^3$ in "phase space" (except for factor 2 because of spin degree of freedom) => for volume V and "momentum volume" $d^3p = 4\pi p^2 dp$ we find for the Number of states between $p \ldots p+dp$:

$$dN = 2\frac{V}{h^3}4\pi p^2 dp = \frac{V}{\pi^2\hbar^3}p^2 dp \quad \Rightarrow \quad N_{tot} = \frac{V}{\pi^2\hbar^3}\frac{p_f^3}{3} \quad \Rightarrow \quad p_f = \hbar\left(3\pi^2\right)^{1/3} n^{1/3}; \quad n = \frac{N_{tot}}{V}; \quad N_{tot} = \frac{M_{star}}{0.001\text{ kg}}\frac{N_A}{2}$$

$$h = 2\pi\hbar$$

- Sirius B: p_f = 670 keV/c for electrons (semi-relativistic - m_e = 511 keV/c^2!)

Sun: $6 \cdot 10^{56}$ e$^-$

– total kinetic energy:

$$E_{tot}^{kin} = \int_0^{p_f} E(p)\frac{V}{\pi^2\hbar^3}p^2\,dp = \begin{cases} \displaystyle\int_0^{p_f}\frac{p^2}{2m}\frac{V}{\pi^2\hbar^3}p^2\,dp = \frac{1}{2m}\frac{V}{\pi^2\hbar^3}\frac{p_f^5}{5} = \frac{3}{5}N_{tot}\frac{p_f^2}{2m} = \frac{3\hbar^2}{10m}N_{tot}\left(3\pi^2\right)^{2/3}\left(\frac{N_{tot}}{V}\right)^{2/3} = \frac{3\hbar^2\left(\frac{9\pi}{4}\right)^{2/3}}{10m}\frac{N_{tot}^{5/3}}{R^2}\text{; non-rel.} \\ \displaystyle\int_0^{p_f} pc\frac{V}{\pi^2\hbar^3}p^2\,dp = \frac{Vc}{\pi^2\hbar^3}\frac{p_f^4}{4} = \frac{3}{4}N_{tot}cp_f = \frac{3}{4}\hbar c N_{tot}\left(3\pi^2\right)^{1/3}\left(\frac{N_{tot}}{V}\right)^{1/3} = \frac{3\hbar c\left(\frac{9\pi}{4}\right)^{1/3}}{4}\frac{N_{tot}^{4/3}}{R}\text{; ultra-relativistic} \end{cases}$$

White Dwarf Stability

- If R **de**creases, gravitational energy more negative:

$$\frac{dV_{pot}^{grav}}{d(-R)} = -\frac{d}{dR}\left(-\frac{3GM^2}{5R}\right) = -\frac{3GM^2}{5R^2}$$

- …while kinetic energy goes up:

$$\frac{dE_{tot}^{kin}}{d(-R)} = -\frac{d}{dR}\left(\frac{3\hbar^2}{10m}\left(\frac{9\pi}{4}\right)^{2/3}\frac{N_{tot}^{5/3}}{R^2}\right) = \frac{3\hbar^2}{5m}\left(\frac{9\pi}{4}\right)^{2/3}\frac{N_{tot}^{5/3}}{R^3}\text{;non-rel.}$$

- Compare: Equilibrium if sum of derivatives = 0

$$-\frac{3GM^2}{5R^2} + \frac{3\hbar^2}{5m}\left(\frac{9\pi}{4}\right)^{2/3}\frac{N_{tot}^{5/3}}{R^3} = 0 \Rightarrow \quad R = \frac{\hbar^2 N_{tot}^{5/3}}{m_e GM^2}\left(\frac{9\pi}{4}\right)^{2/3} \propto \frac{M^{5/3}}{M^{6/3}}$$

= 7280 (really: 5500) km / $(M/M_{sun})^{1/3}$

White Dwarf **INStability**

- If *R* **de**creases, gravitational energy more negative:

$$\frac{dV_{pot}^{grav}}{d(-R)} = -\frac{d}{dR}\left(-\frac{3GM^2}{5R}\right) = -\frac{3GM^2}{5R^2}$$

- …while kinetic energy goes up more slowly:

$$\frac{dE_{tot}^{kin}}{d(-R)} = -\frac{d}{dR}\left(\frac{3\hbar c}{4}\left(\frac{9\pi}{4}\right)^{1/3}\frac{N_{tot}^{4/3}}{R}\right) = \frac{3\hbar c}{4}\left(\frac{9\pi}{4}\right)^{1/3}\frac{N_{tot}^{4/3}}{R^2}\text{; fully rel.}$$

- Compare: Once first term is greater than 2[nd] term (depends only on *M* and *N*), no amount of shrinking can stabilize system -> collapse

Supernova remnant

- Neutron star:
  - nearly no p's, e^-'s, just neutrons
  - Remember: $R_{\text{white dwarf}} \propto 1/m_e\ M^{-1/3}$
  - $m_n = 1840\ m_e \Rightarrow R$ 1840 times smaller (really, about 500 times because only 1 e- per 2 neutrons) => of order 10 km!
  - Density: few $10^{44}/\text{m}^3 = 1/\text{fm}^3$ > nuclear density => nucleus with mass number $A = 10^{57}$
  - Chandrasekar limit: 5 solar masses (2-3 in reality?)
  - Lots depends on nuclear equation of state *), general relativity

*) Repulsive core / Nuclear superfluid / quark-gluon plasma / strange matter / pasta ?

Fermi Gas Stability revisited:

Electron Fermi Gas:

$$R = \frac{\hbar^2 N_{tot}^{5/3}}{m_e G M^2}\left(\frac{9\pi}{4}\right)^{2/3}$$

Neutron Star: replace m_e with m_n (1839 times larger), N_{tot} doubled (since N_{tot} = N_n = number of all nucleons, not = ½ N_{p+n} for electron Fermi gas.) => R is 580 times smaller (10 km)

Relativistic limit:

$$p_f = \hbar\left(3\pi^2\right)^{1/3} n^{1/3} = \hbar\left(3\pi^2\right)^{1/3}\left(\frac{N_{tot}}{V}\right)^{1/3} \propto M^{2/3}$$

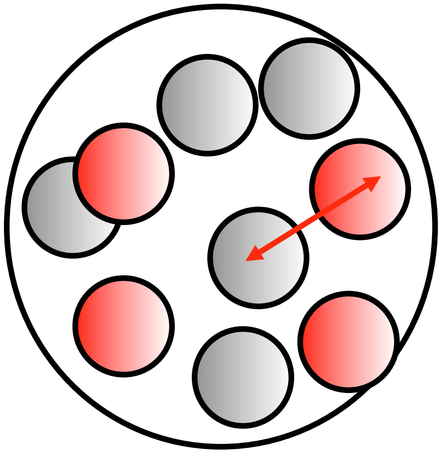
= 730 times larger for same mass M

$\Rightarrow$ p_f/m =40% of ratio for electrons for same M

$\Rightarrow$ maximum p_f could be 2.5x larger before relativity sets in => 4 times the mass (5.6 instead of 1.4 M_{sun})

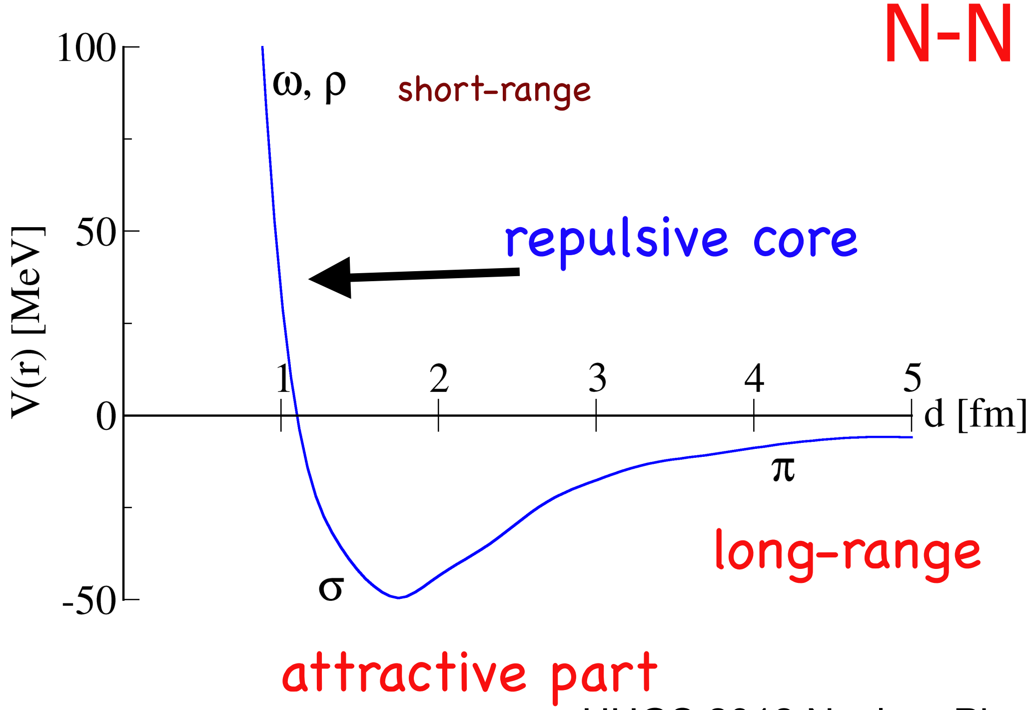
Shell Model – by Dr. Weinstein

Structure of the nucleus

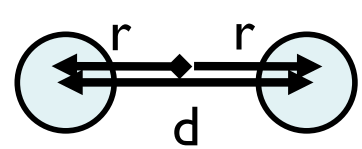


- nucleons are bound
  - energy (*E*) distribution
  - shell structure
- nucleons are not static
  - momentum (*k*) distribution

determined by the
N-N potential

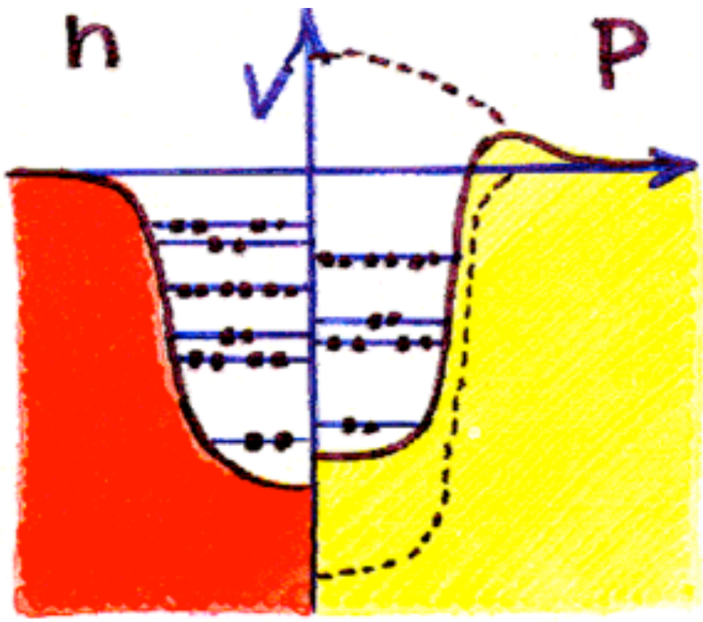


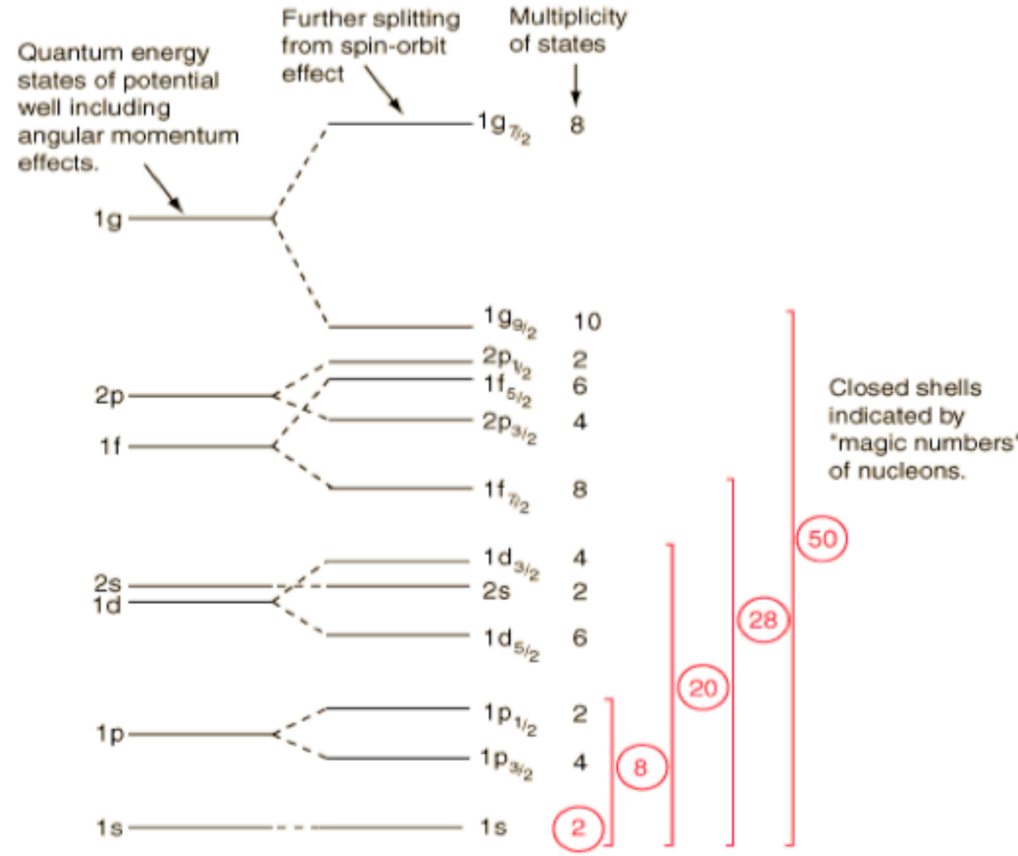
on average:
Net binding energy: ≈ 8 MeV
distance: ≈ 2 fm



Strong repulsion ⟹ NN correlations

Shell Structure (Maria Goeppert-Mayer, Jensen, 1949, Nobel Prize 1963)





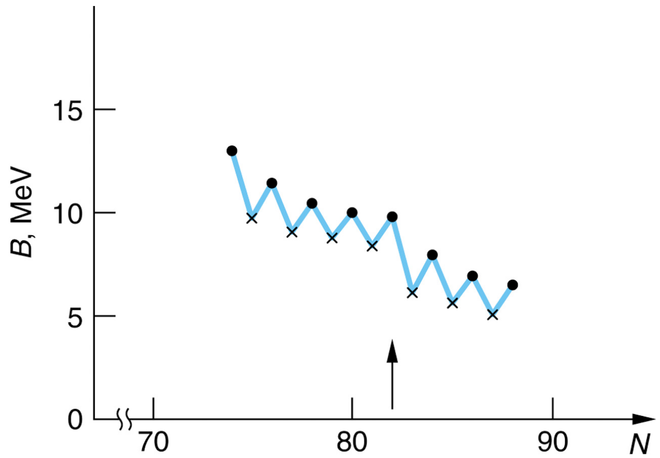
nuclear density 10^{18} kg/m^3

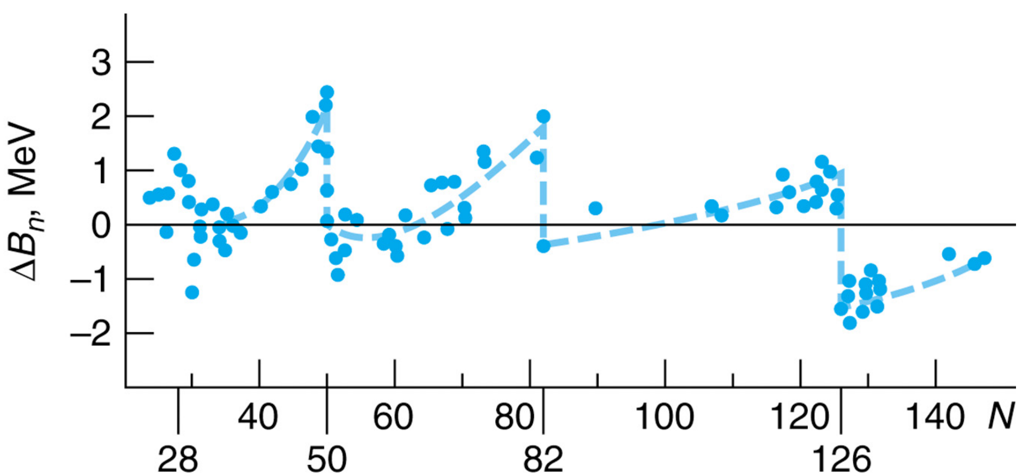
With the enormous strong force acting between them and with so many nucleons to collide with, how can nucleons possibly complete whole orbits without interacting?

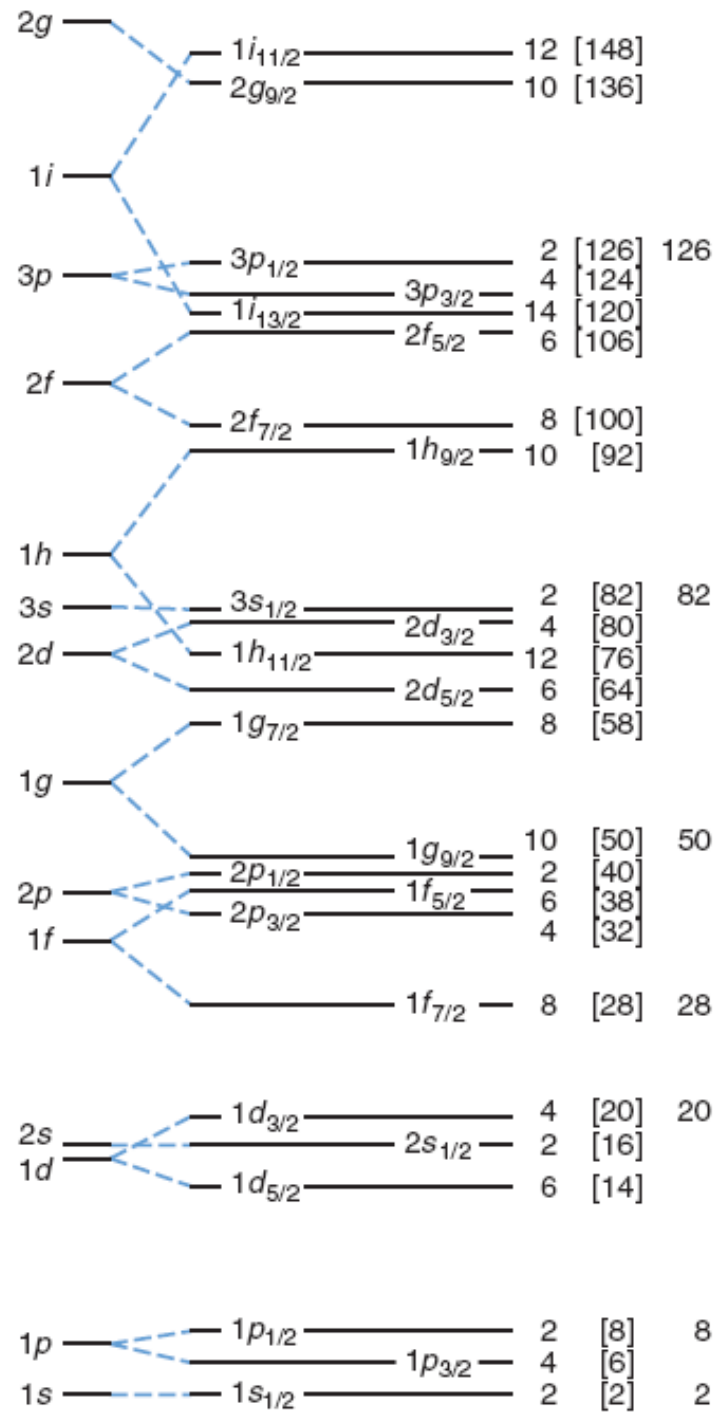
But: there is experimental evidence for shell structure

Pauli Exclusion Principle: → nucleons can not scatter into occupied levels: Suppression of collisions between nucleons

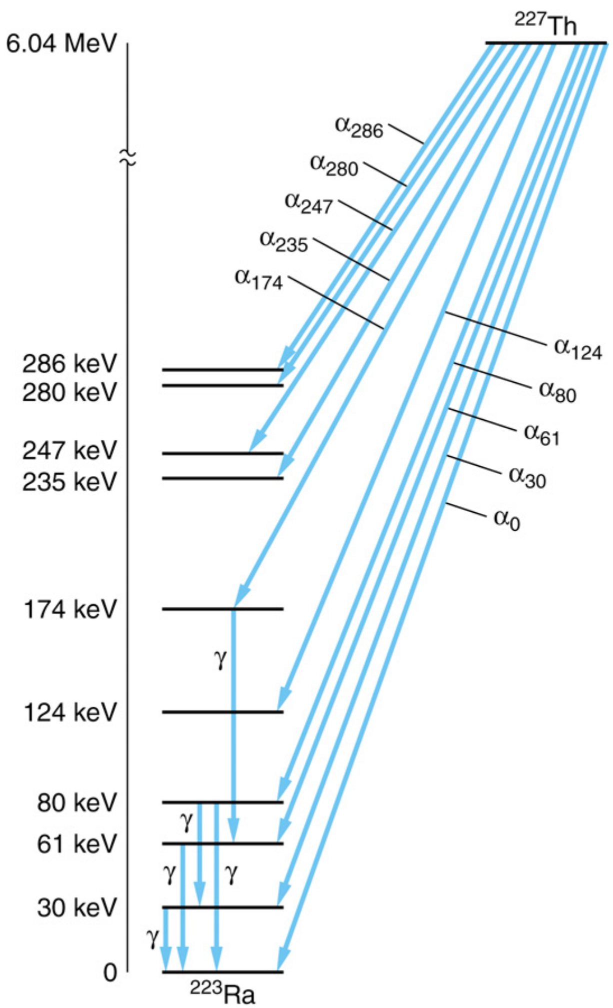
Shell structure and Nuclear Level Scheme

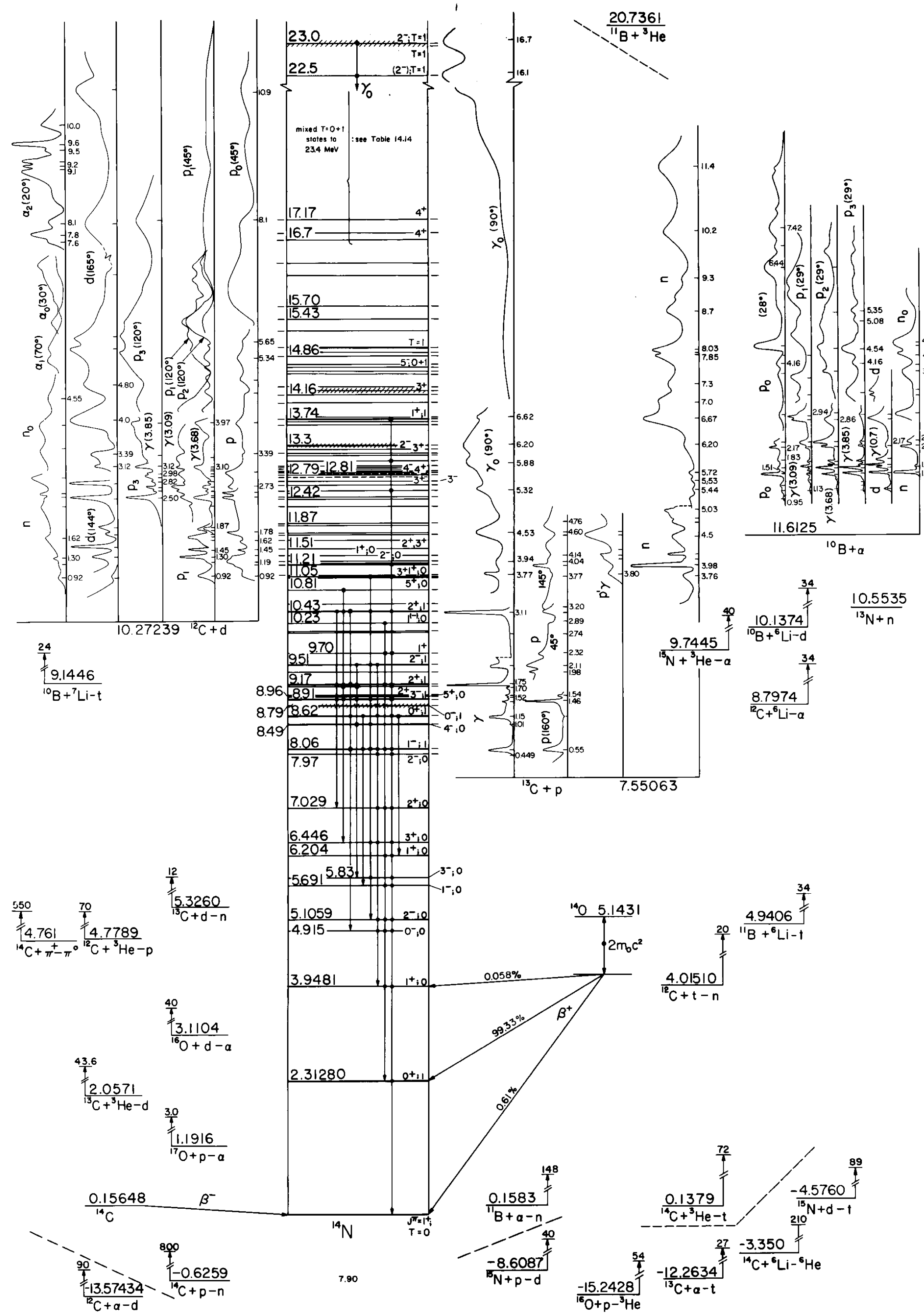




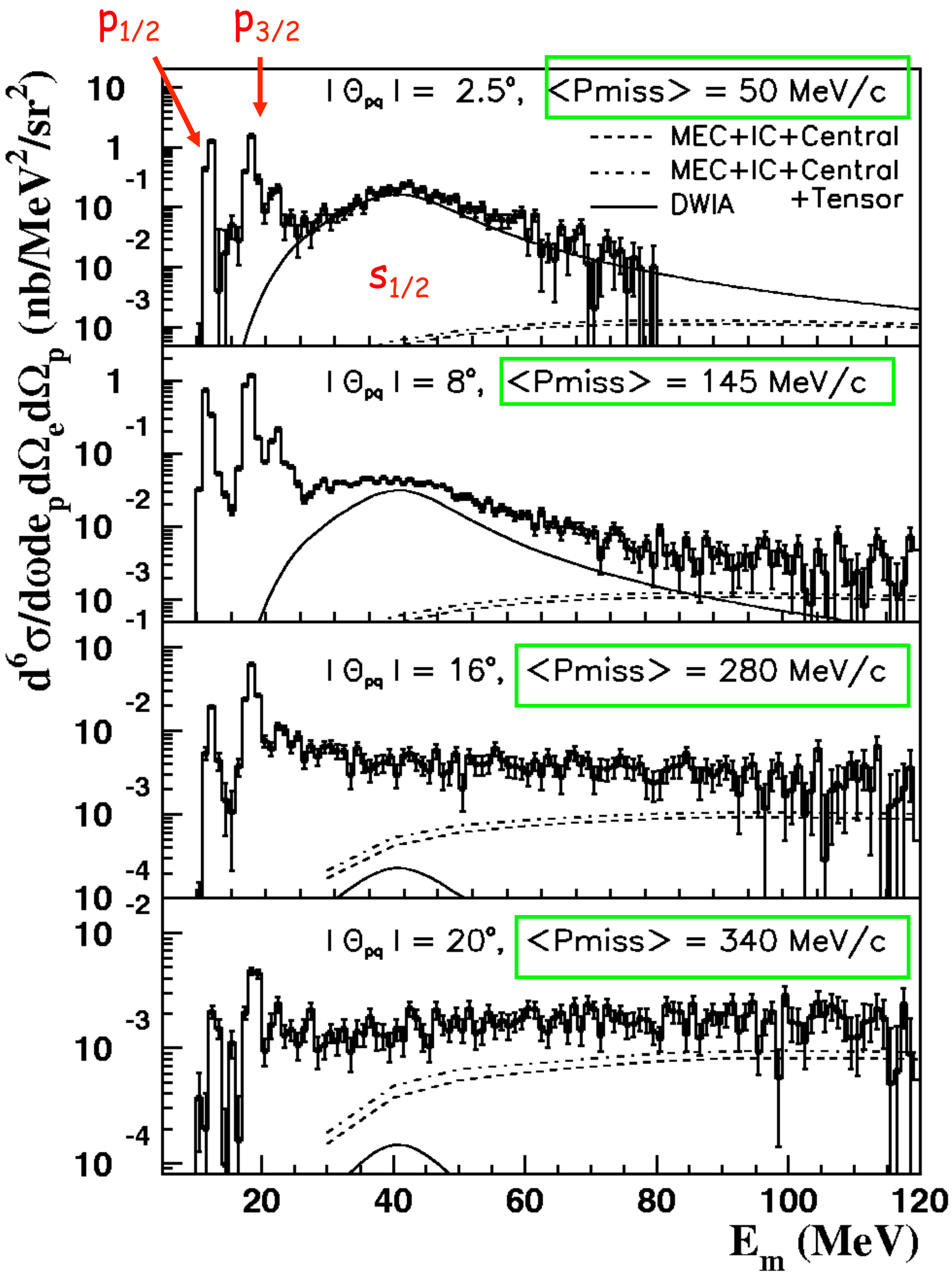


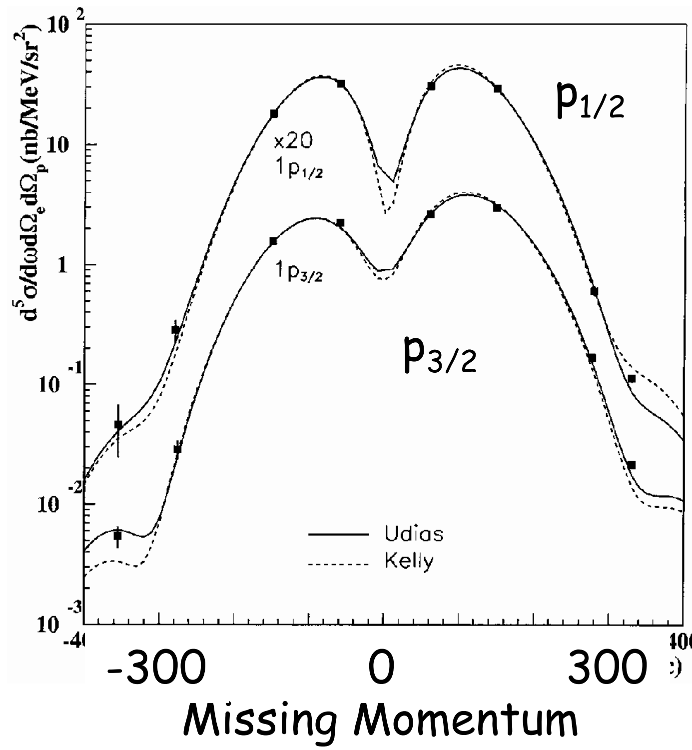
Evidence for excited states





O(e,e'p) and shell structure





$1p_{1/2}$, $1p_{3/2}$ and $1s_{1/2}$ shells visible

Momentum distribution as expected for $l = 0, 1$

Fissum et al, PRC 70, 034606 (2003)

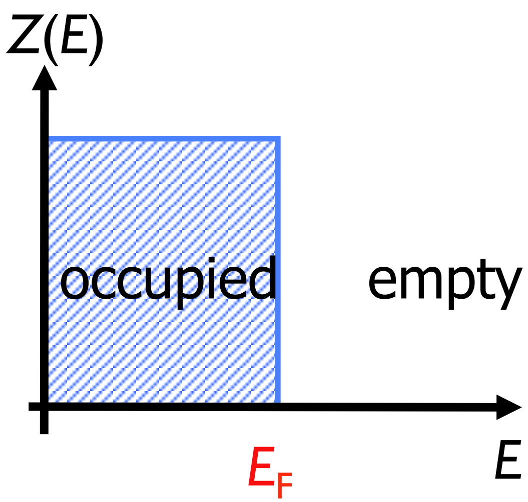
Magnetic moments

- "Natural" unit: 1 nuclear magneton $\mu_{\mathrm{N}} = \frac{e\hbar}{2m_{\mathrm{p}}}$
- Classical prediction: $\boldsymbol{\mu} = \mu_N J$
- Generally: $\boldsymbol{\mu} = \mu_N \, g_J \, J$
- Dirac/Relativity: for J = S, $g_S = 2$ (pretty good for electrons)
- For protons, $g_S = 5.58 \rightarrow$ anomalous moment $\kappa = (g-2)/2 = 1.79$; for neutrons $g_S = -3.83 \rightarrow$ $\kappa = -1.91$ (huh? n is neutral!!!)
- Orbital motion only: $g_L = 1$ (p), 0 (n)
- For nucleon w/ S,L,J: $\frac{\mu}{\mu_N} = \left(g_l \pm \frac{g_s - g_l}{2l+1} \right) j \quad \text{where} \quad j = l \pm \frac{1}{2}$

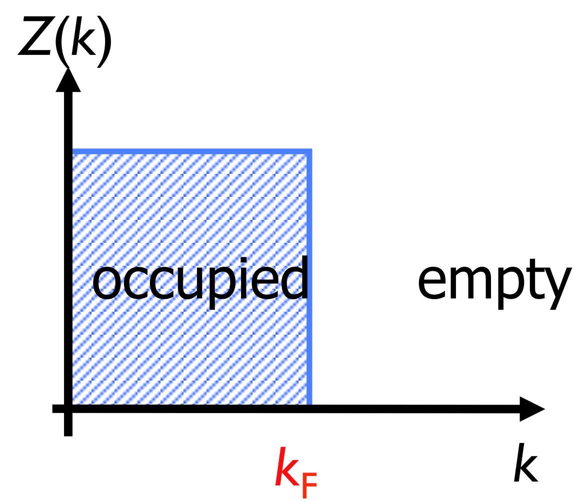
Independent Particle Shell model (IPSM)

- single particle approximation:
  nucleons move independently from each other
  in an average potential created by the other nucleons (mean field)
  - spectral function $S(E,k)$:
    probability of finding a proton with initial momentum k and energy E in the nucleus
- factorizes into energy & momentum part

nuclear matter:



$$E_F = \frac{k_F^2}{2m_p}$$



nuclei: $$S(\vec{p},E) = \sum_i |\Phi_a(p)|^2 \delta(E+\epsilon_a)$$

Not 100% accurate, but a good starting point