Cosm(~~et~~)ology

PHYS313

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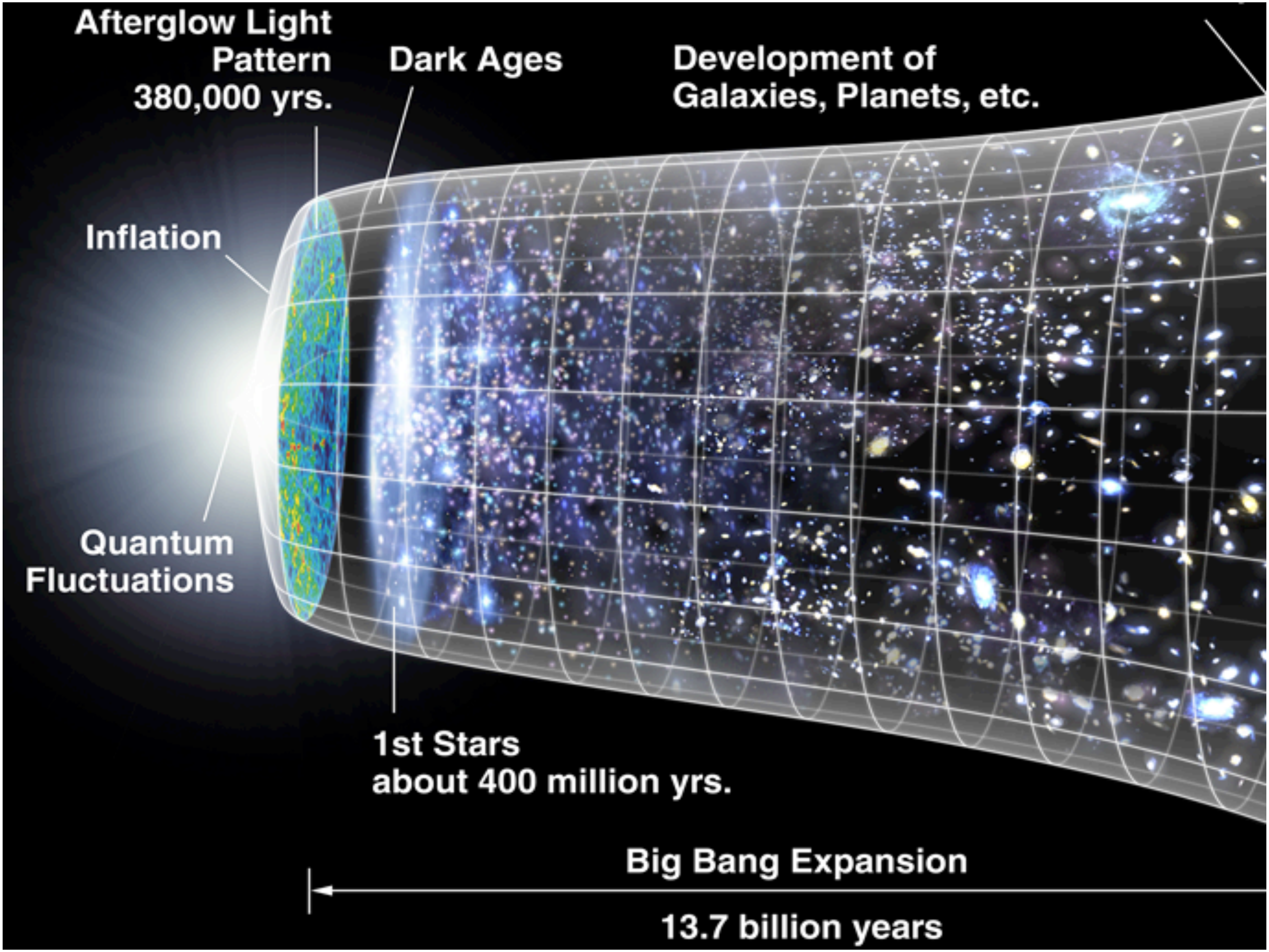
Large Scale Structure of Universe

- The Universe is expanding…
  - Hubble Constant H_0 = 70 km/s/Mpc = 1/14Gyr
- …initially it was filled with a smooth distribution of dark matter
  - and a smaller amount of nucleons + electrons
  - very small initial density fluctuations…
- …which began to clump to create the seeds of filaments, superclusters, walls, … , galaxies (central black holes?)


Afterglow Light
Pattern
380,000 yrs.
Dark Ages
Development of
Galaxies, Planets, etc.
Inflation
Quantum
Fluctuations
1st Stars
about 400 million yrs.
Big Bang Expansion
13.7 billion years

General Relativity - again

Reminder: Local coordinates (t,**r**) but distance defined by metric:

$$ds^2 = \begin{pmatrix} dct & d\vec{r} \end{pmatrix} \begin{pmatrix} g_{\mu\nu} \end{pmatrix} \begin{pmatrix} dct \\ d\vec{r} \end{pmatrix}$$

- Co-moving coordinate system: $\vec{r}_c = const.$ for a point (object) locally at rest relative to "Hubble flow"; true distance from origin $D = a(t) r_c$. Scale factor $a(t)$ = Radius of curvature in a curved universe (otherwise arbitrary; r_c is meant to be dimensionless). Universal time t (same everywhere; defined through Hubble parameter $H(t)$ – see later).
- Hubble law: $v_r(t) = \dfrac{dD(t)}{dt} = \dot{a}(t) r_c = \dfrac{\dot{a}(t)}{a(t)} D(t) =: H(t) D(t)$
  – At present: $H_0 = H(t_0) = \dfrac{\dot{a}(t_0)}{a(t_0)} = \dfrac{68-69 \text{ km/s}}{1 \text{ Mpc}} \approx \dfrac{1}{14 \cdot 10^9 \text{ yr}}$
  – Speed of light in co-moving coordinates: $\dfrac{dr_c}{dt} = \dfrac{c}{a(t)}$
- Redshift for light emitted at t and received at t_0: $z = \dfrac{a(t_0)}{a(t)} - 1$

Examples

- If we observe an object with redshift *z*,
  - How long ago was light emitted?

$$z+1=\frac{a(t_0)}{a(t_e)} \Rightarrow \frac{a(t_0)}{z+1}=a(t_e)=a(t_0)-\int_{t_e}^{t_0}\dot{a}(t)\,dt$$

$$\Rightarrow \int_{t_e}^{t_0}\dot{a}(t)\,dt=\frac{z}{z+1}a(t_0) \Rightarrow \text{ solve for } t_e \text{ if } \dot{a}(t) \text{ is known.}$$

  - How far is that object now?
  - How far was it when the light was emitted?

$$r_c(em.)=\int_{t_e}^{t_0}\frac{c}{a(t)}dt \Rightarrow D(em.,t_e)=a(t_e)r_c(em.);\, D(em.,t_0)=a(t_0)r_c(em.)$$

Examples

- Assume $\dot{a}(t) = \dot{a} = \text{ const.}$
  - How long ago was light emitted?

$$\int_{t_e}^{t_0} \dot{a}\,dt = \dot{a}\left(t_0 - t_e\right) = \frac{z}{z+1} a(t_0) \Rightarrow \left(t_0 - t_e\right) = \frac{z}{z+1}\frac{a(t_0)}{\dot{a}} = \frac{z}{z+1}\frac{1}{H_0}$$

  - How far is that object now?
  - How far was it when the light was emitted?

$$r_c(em.) = \int_{t_e}^{t_0} \frac{c}{a(t_0) - \dot{a}\left(t_0 - t\right)} dt = \frac{c}{\dot{a}} \ln\left(\frac{a(t_0)}{a(t_0) - \dot{a}\left(t_0 - t_e\right)}\right) = \frac{c}{\dot{a}} \ln(z+1)$$

$$\Rightarrow D(em.,t_e) = a(t_e) r_c(em.);\ D(em.,t_0) = a(t_0) r_c(em.) = \frac{c}{H_0} \ln(z+1)$$

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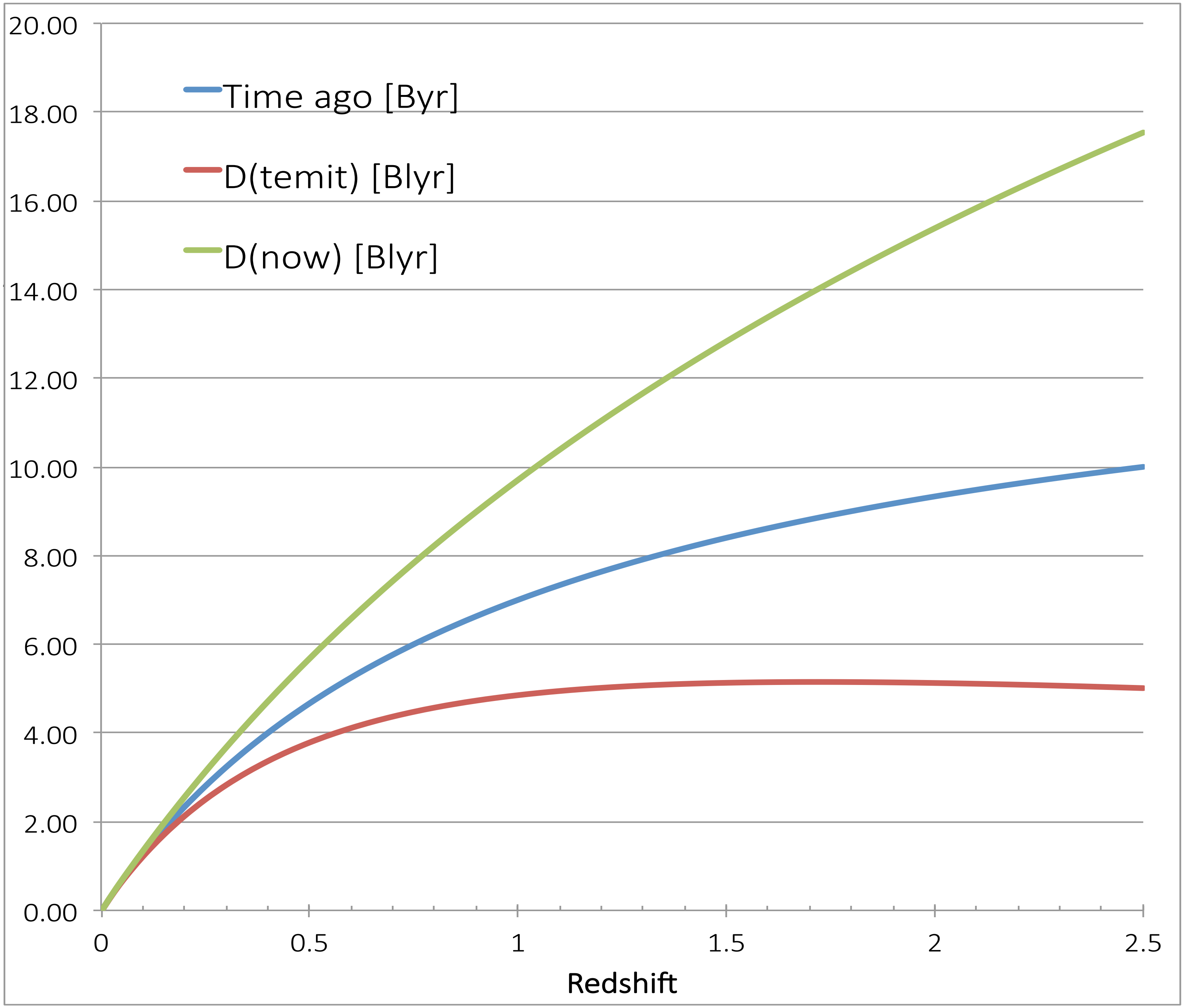

20.00
18.00
16.00
14.00
12.00
10.00
8.00
6.00
4.00
2.00
0.00
Time ago [Byr]
D(temit) [Blyr]
D(now) [Blyr]
0
0.5
1
1.5
2
2.5
Redshift

Walker-Robertson metric

$$ds^2 = dt^2 - a^2(t)\left[dr_c^2 + S_K^2(r_c)\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)\right]$$

- Closed Universe, positive curvature (K = +1):

$$S_K(r_c) = \sin(r_c)$$

- Flat Universe, no curvature (K = 0):

$$S_K(r_c) = r_c$$

- Open Universe, negative curvature (K = -1):

$$S_K(r_c) = \sinh(r_c)$$