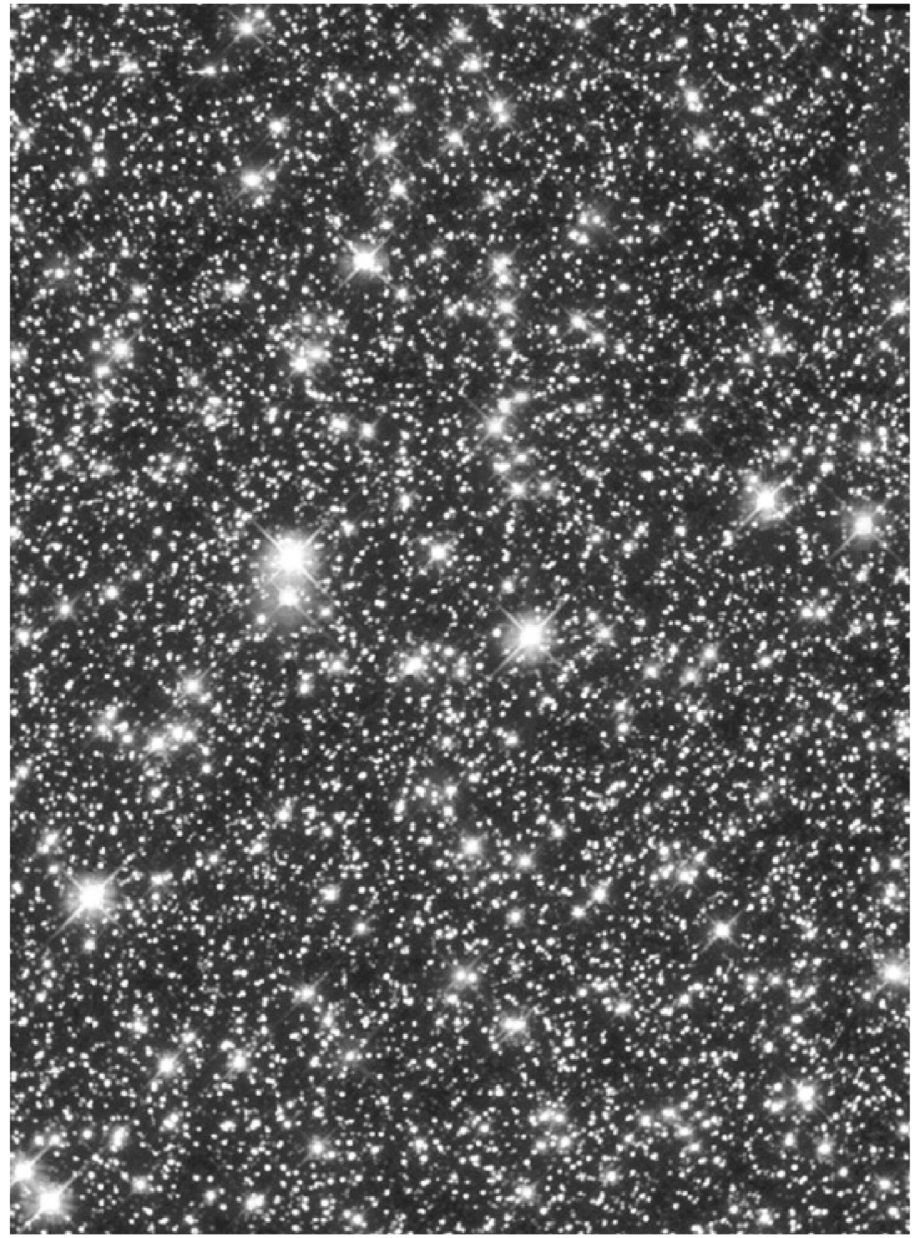
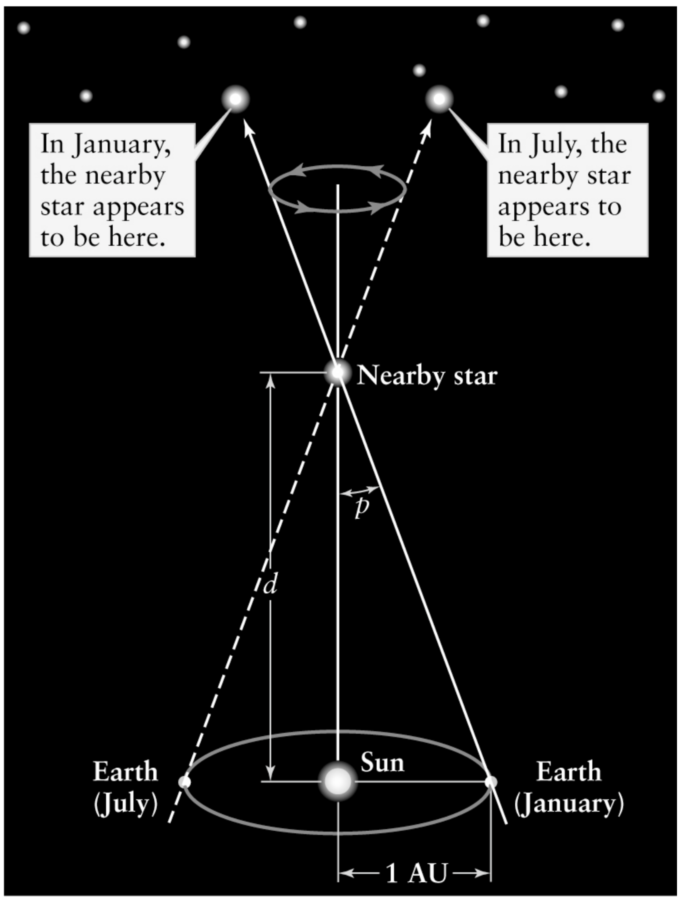
Stellar Astrophysics:

The Continuous Spectrum of Light

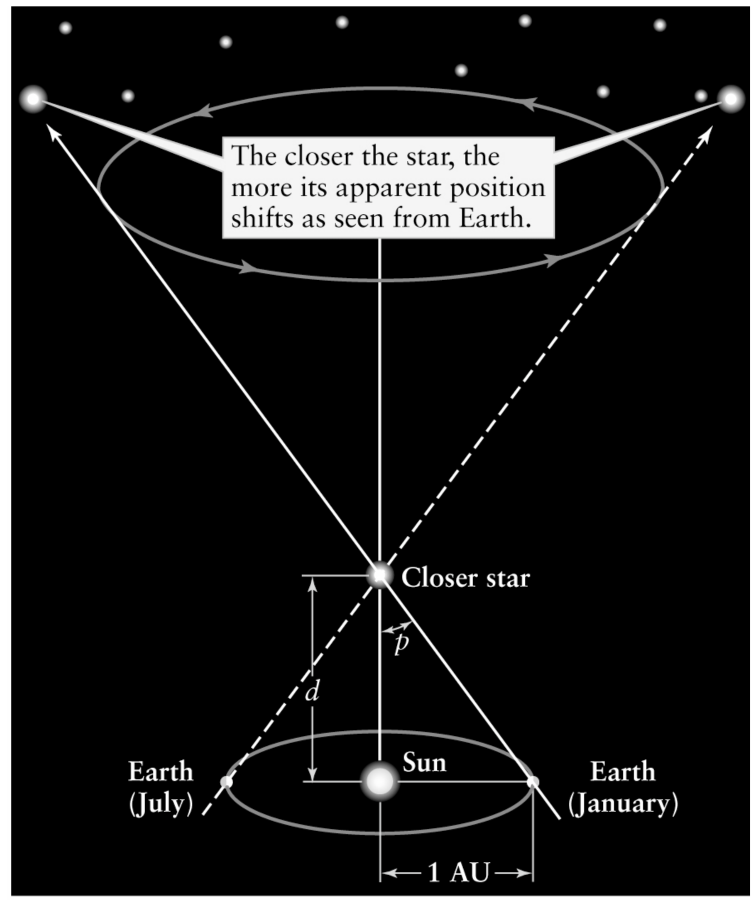


Distance Measurement of Stars

Distance Sun - Earth	1.496×10^{11} m	1 AU	1.581×10^{-5} ly
Light year	9.461×10^{15} m	6.324×10^{4} AU	1 ly
Parsec (1 pc)	3.086×10^{16} m	2.063×10^{5} AU	3.262 ly

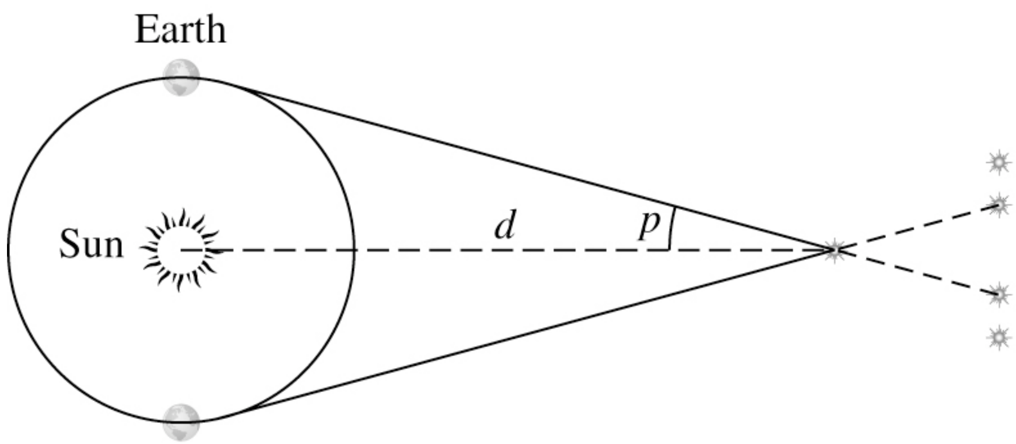


(c) Parallax of a nearby star



(d) Parallax of an even closer star

Distance Measurement of Stars

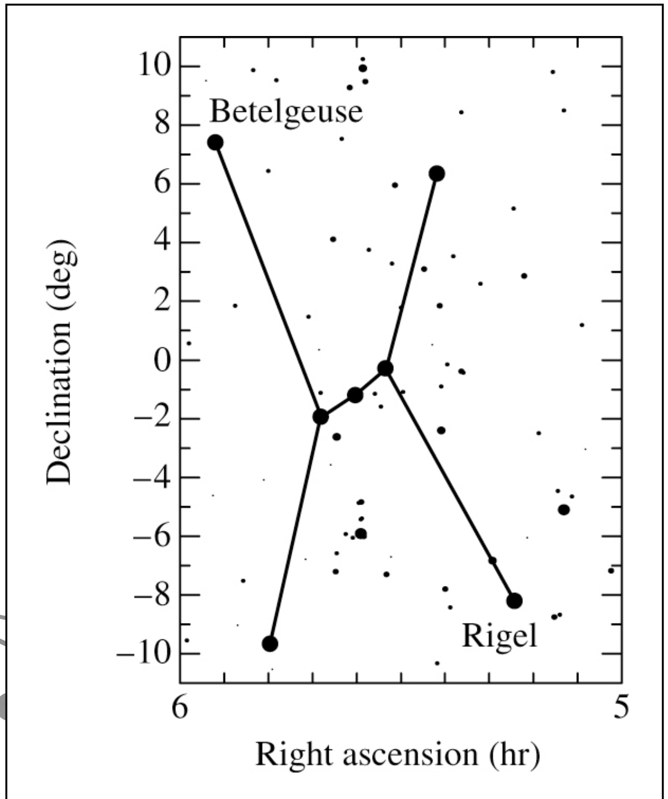


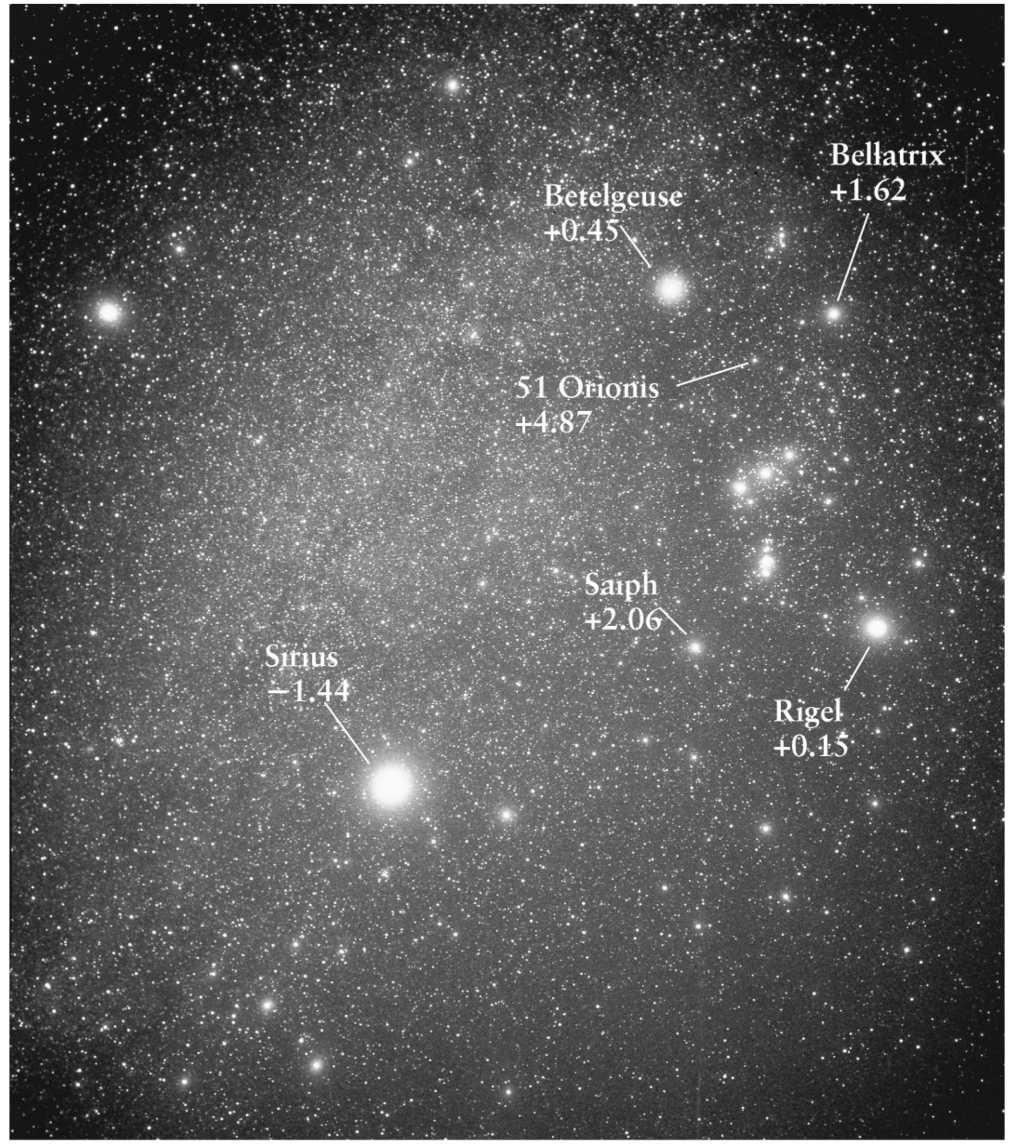
$$d = \frac{1\ \text{AU}}{\tan p} \approx \frac{1\ \text{AU}}{p} = \frac{206{,}265\ \text{AU}}{p''} = \frac{1\ \text{pc}}{p''}$$

p measured in radians = 57.3º = 206,265''

p'' measured in arcseconds

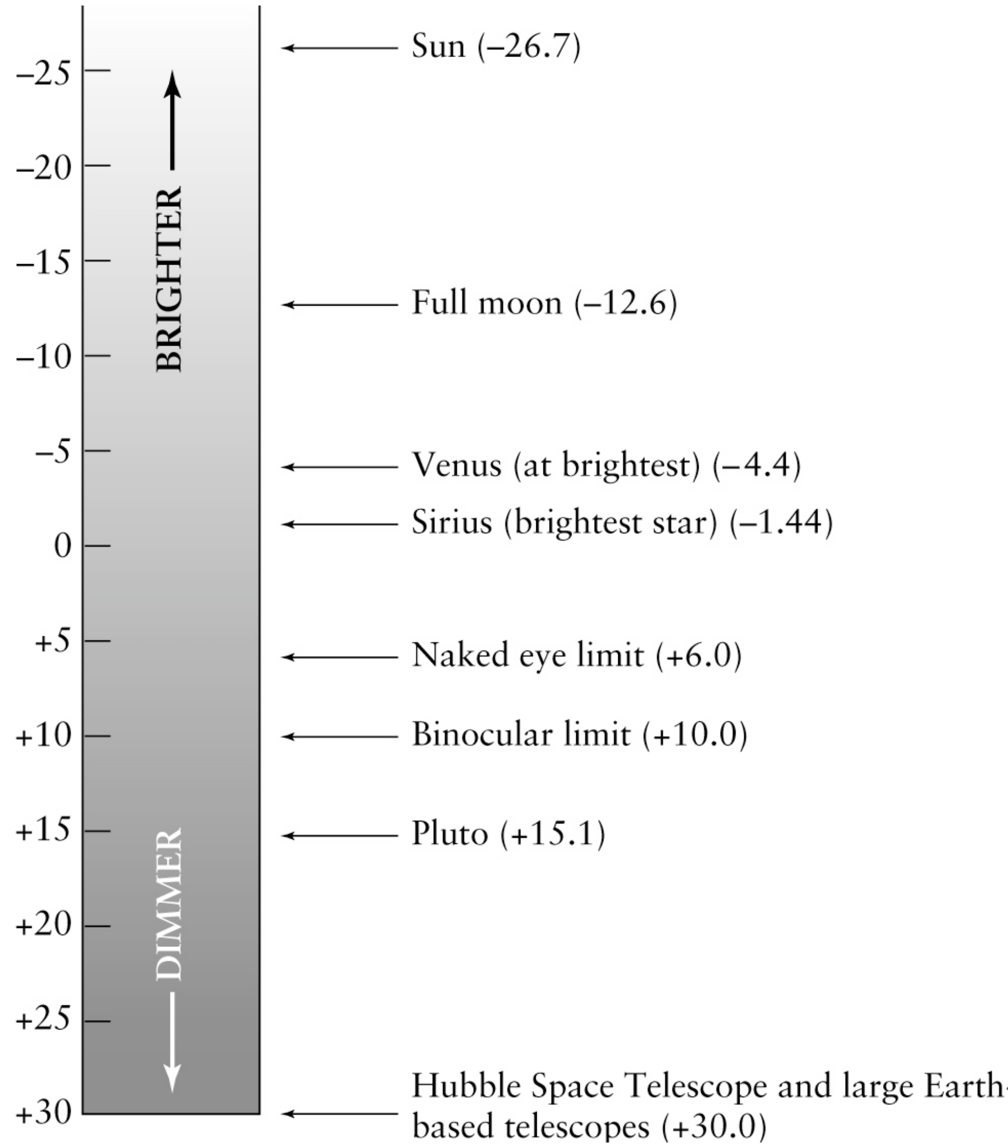
Apparent Magnitude Scale





Several stars in and around the constellation Orion labeled with their names and apparent magnitudes

Apparent Magnitude Scale



The brightness of objects in the sky is denoted by apparent magnitudes. Stars visible to the naked eye have magnitudes between m = –1.44 and about m = +6.

Apparent Magnitude Scale

Radiant Flux $F = \dfrac{L}{4 \pi r^2}$

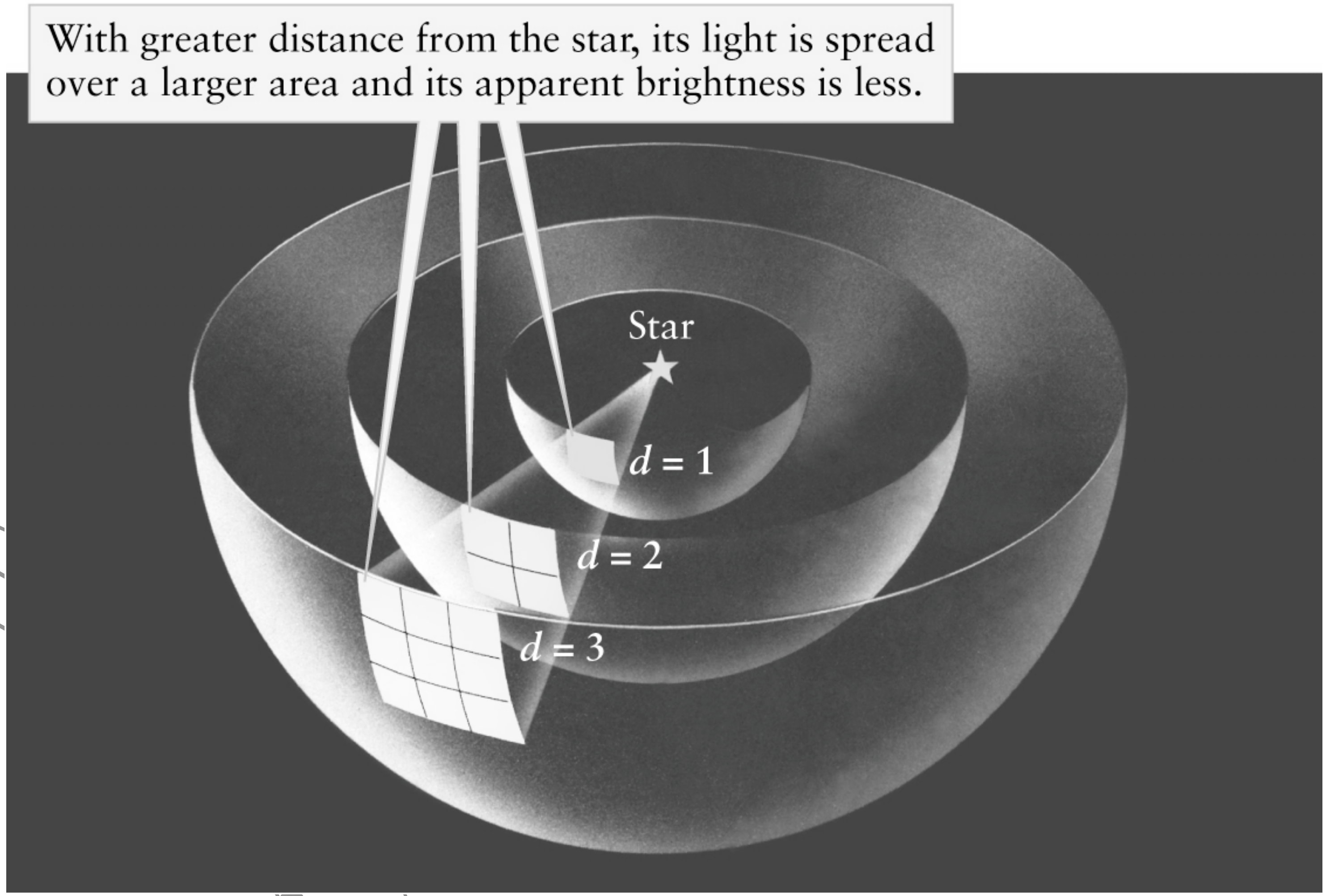
with L = Luminosity of star (energy emitted per second)

$$m_1 - m_2 = -2.5 \log_{10} (F_1 / F_2)$$

$$-\Delta m = 5 \quad \Rightarrow \quad (F_1 / F_2) = 100$$

Hipparchos (190 - 120)

The Inverse-Square Law



Apparent and Absolute Magnitude

- Absolute Magnitude M is defined as the apparent magnitude a star would have if located at 10 pc

$$100^{(m-M)/5} = F_{10}/F = \left(\frac{d}{10\ \text{pc}}\right)^2$$

- Solving for the Distance Modulus yields

$$m - M = 5\log_{10}\left(\frac{d}{10\ \text{pc}}\right)$$

The Speed of Light

- The speed of light in a medium is given by

$$v = \frac{c}{n} = \lambda \nu$$

- This leads to dispersion $c = n \lambda \nu = \lambda_0 \nu$

  with λ_0 the wavelength of light in the vacuum

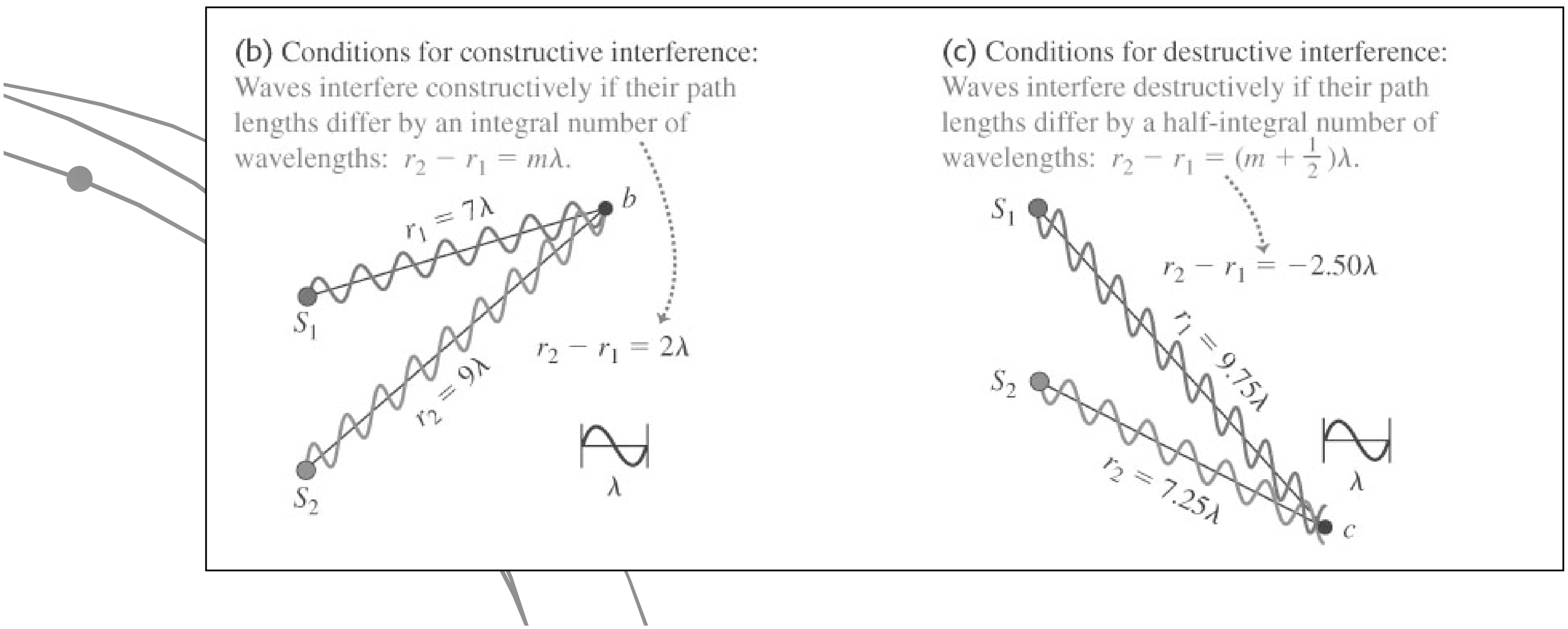
- Rømer measured in 1675 the speed of light to be $2.2 \cdot 10^{8}$ m/s by observing the difference in observed time for Jupiter moon eclipses from the calculations based on Kepler's laws

Ole Rømer (1644 - 1710)

Constructive and Destructive Interference

Path difference $r_2 - r_1 = \begin{cases} n\,\lambda & \text{constructive} \\ (n + \frac{1}{2})\,\lambda & \text{destructive} \end{cases}$

with $n = 0, \pm 1, \pm 2, \pm 3, \ldots$



Two-Source Interference of Light (Young's Experiment)

For constructive interference we find

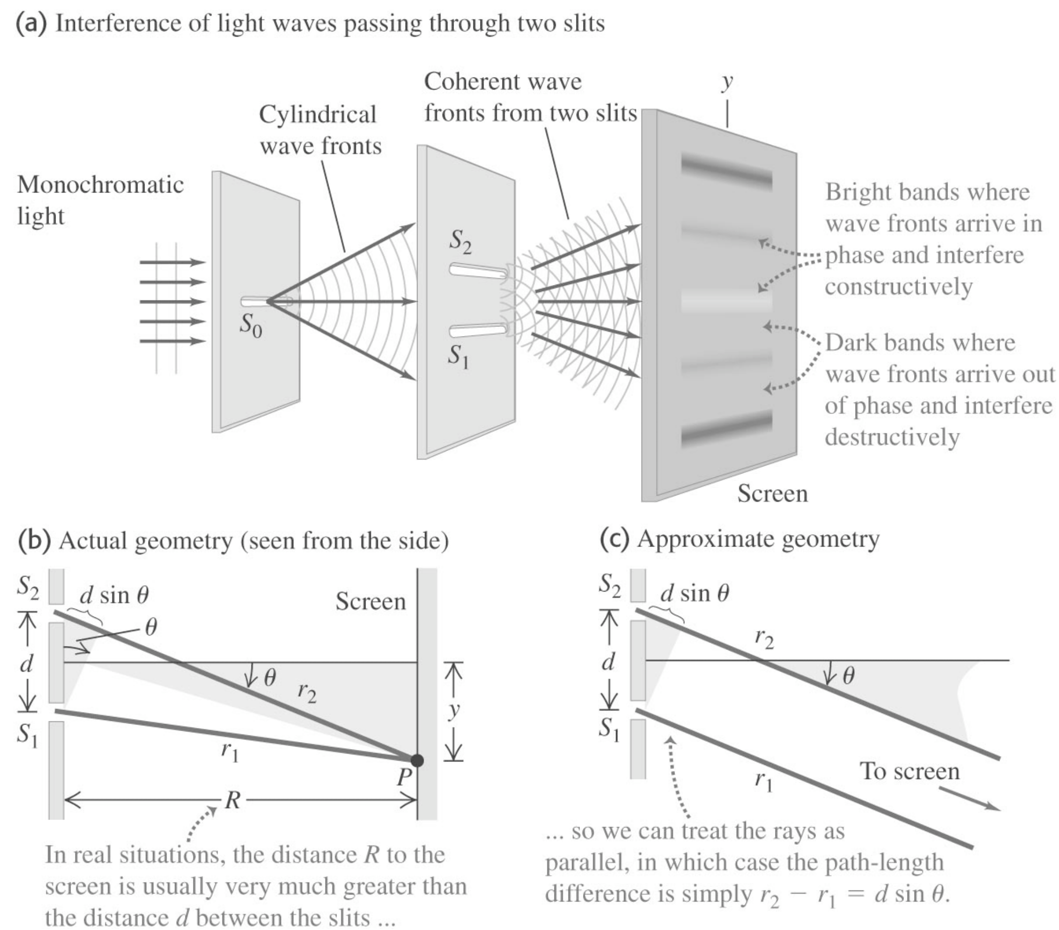
$$r_2 - r_1 = d \sin\theta = n\,\lambda$$

$$n = 0, 1, 2, \dots$$

and destructive interference we find

$$r_2 - r_1 = d \sin\theta = \left(n - \tfrac{1}{2}\right)\lambda$$

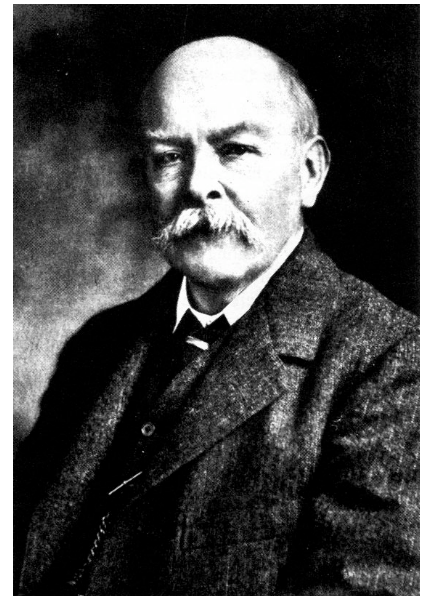
$$n = 1, 2, \dots$$



Poynting Vector

- Light is a transverse electromagnetic wave, composed of alternating electric and magnetic fields
- The *E* and *B* field vectors are perpendicular to each other and to the direction of motion of the wave

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$$

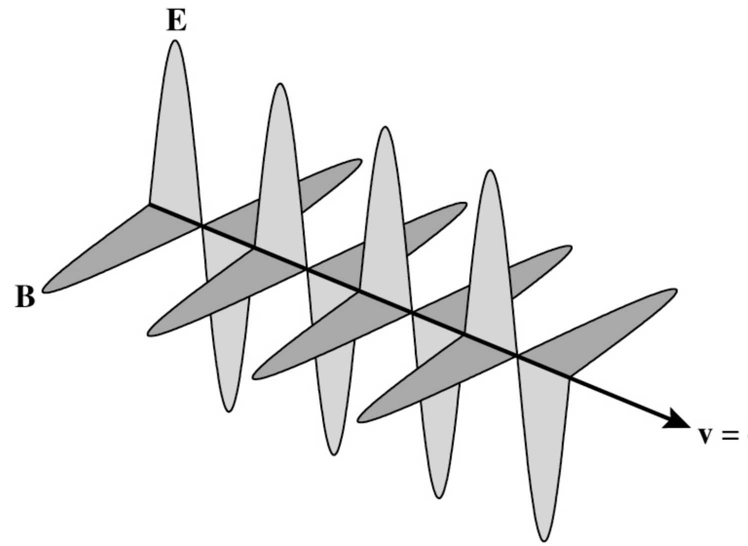


John Poynting (1852 - 1914)

- The magnitude of the time-averaged Poynting vector is given by

$$\langle S \rangle = \frac{1}{2\mu_0} E_0 B_0$$

- In vacuum $E_0 = c B_0$



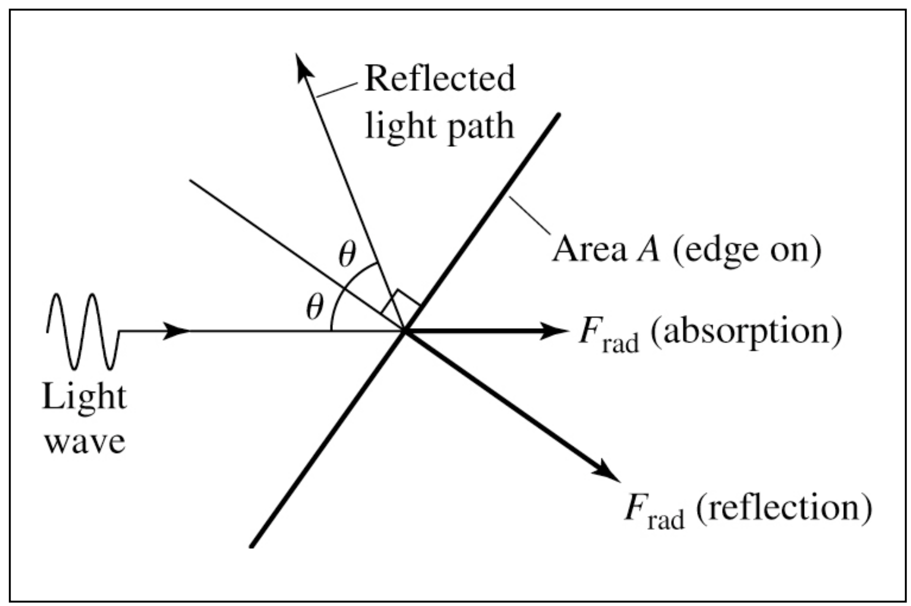
Radiation Pressure

- Electromagnetic waves carry momentum and can exert a force on a surface
- The radiation pressure depends on whether the light is reflected

$$F_{\text{rad}} = \frac{\langle S \rangle A}{c} \cos \theta$$

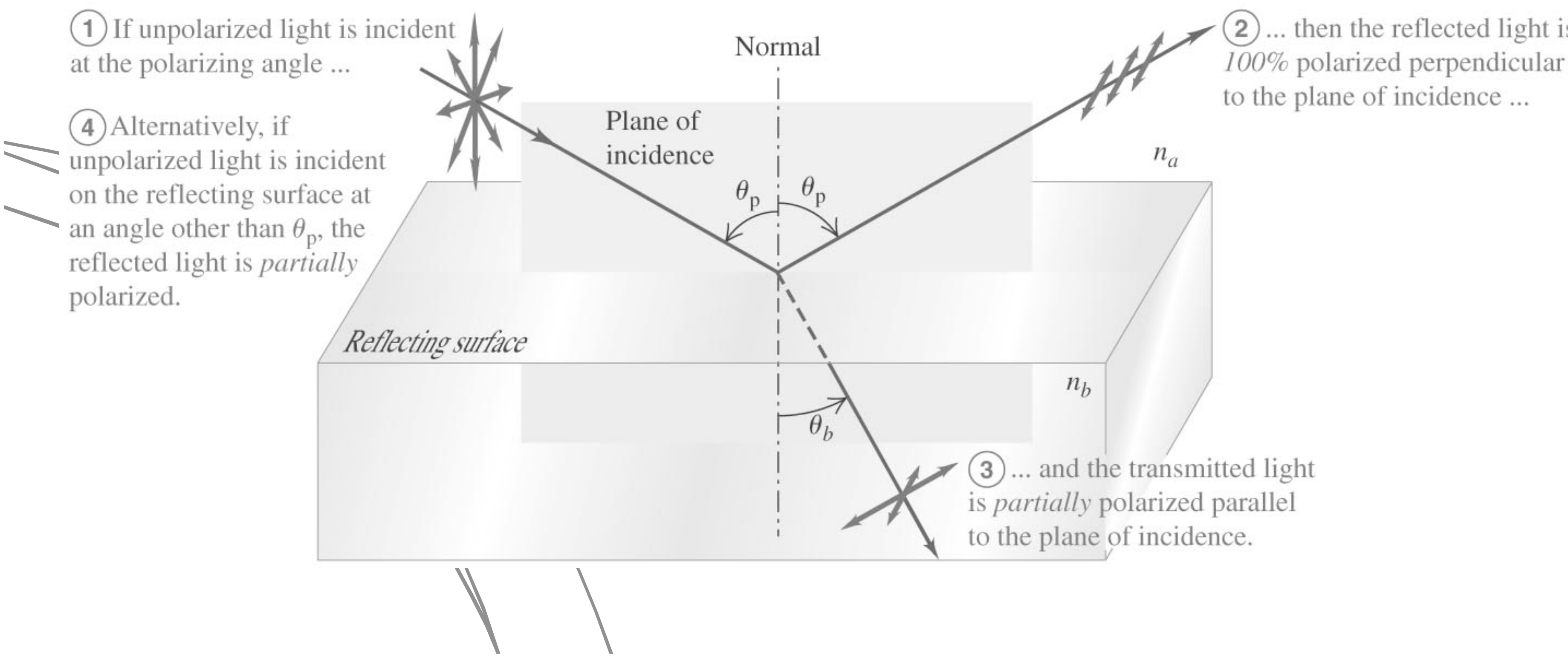
- or absorbed by the surface

$$F_{\text{rad}} = \frac{2 \langle S \rangle A}{c} \cos^2 \theta$$



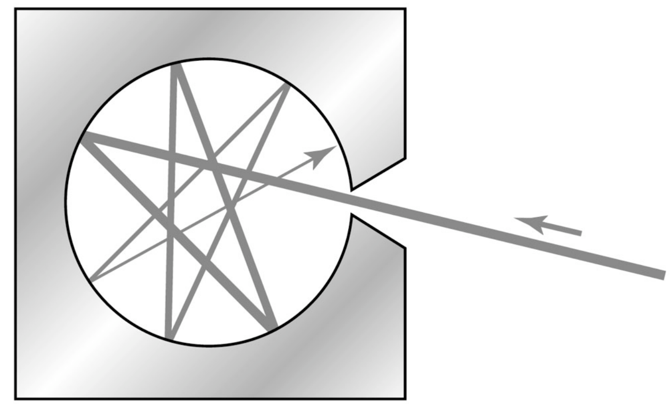
Polarization

- Electric field vectors that are perpendicular to the plane of incidence of a wave are more likely reflected than others
- This leads to a polarization of the reflected light



Blackbody Radiation

- When matter is heated, it emits radiation
- A blackbody absorbs all radiation falling on it and reflects none. It is also a perfect emitter
- An example of a blackbody is a cavity in some material. Incoming radiation is *absorbed by the cavity*



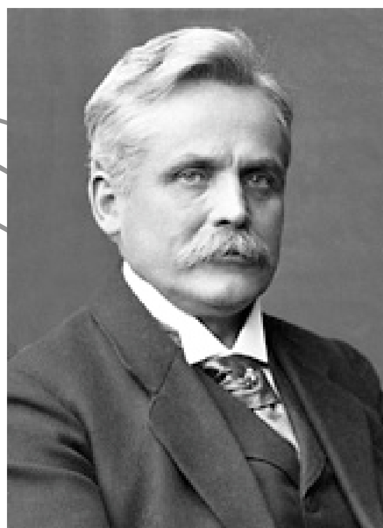
- Blackbody radiation is interesting because the radiation properties of the blackbody are independent of the particular material of the container. It therefore has a universal character. We can study, for example, the properties of intensity versus wavelength at fixed temperature, …

Wien's Displacement Law

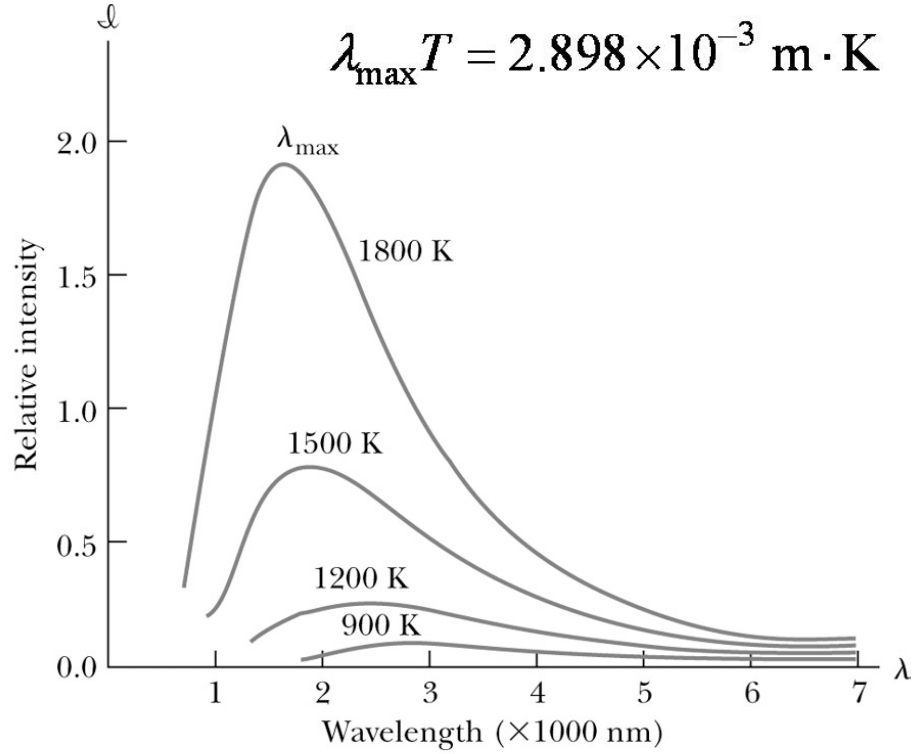
- The intensity $\mathcal{I}(\lambda, T)$ is the total power radiated per unit area per unit wavelength at a given temperature
- **Wien's Displacement law**:

  The maximum of the distribution shifts to smaller wavelengths as the temperature increases

$$\lambda_{max} T = 2.898 \times 10^{-3} \text{ m} \cdot \text{K}$$

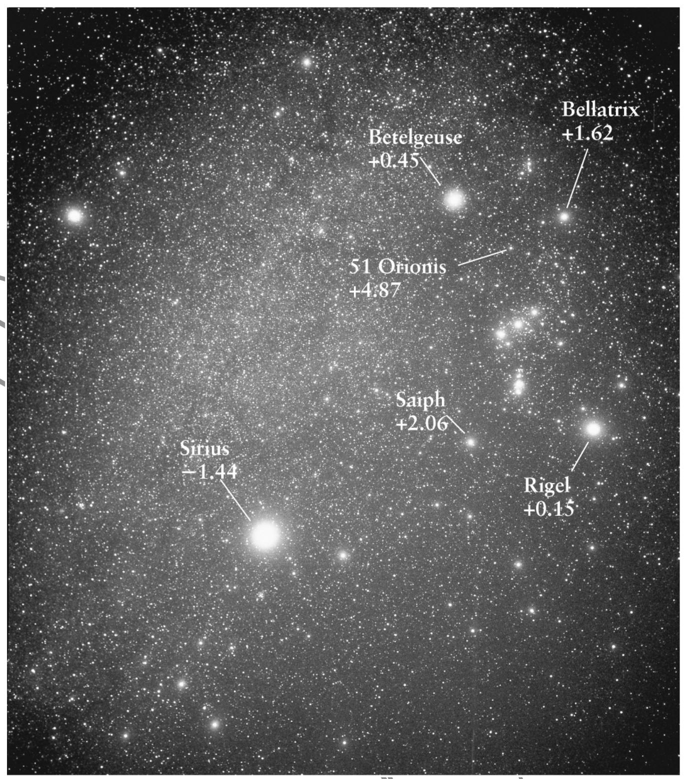


Wilhelm Wien (1864 - 1928)

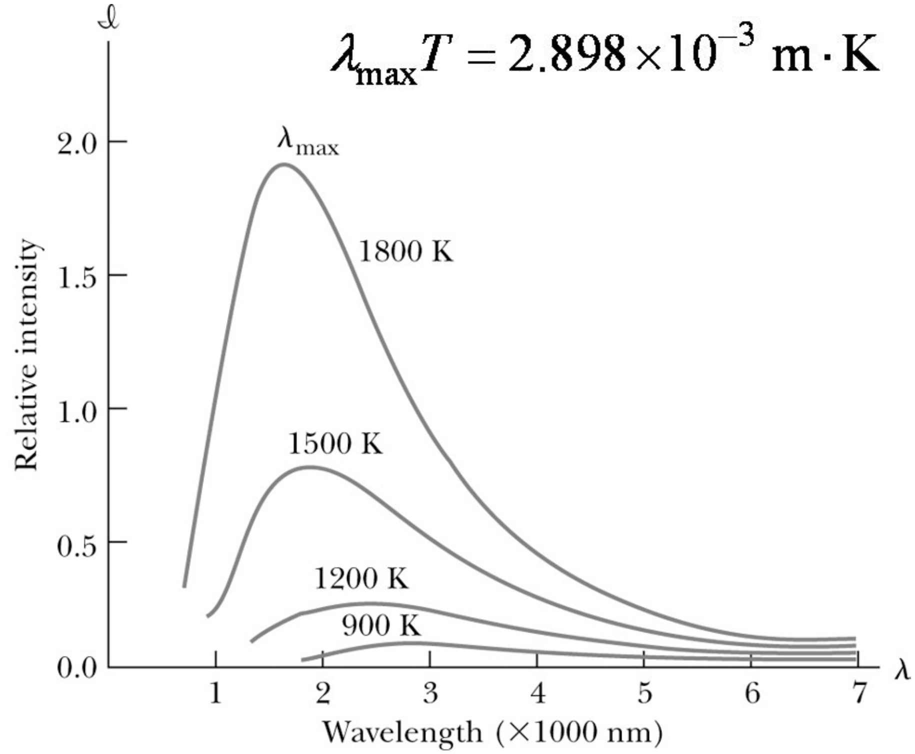


Example for Wien's Displacement Law

Betelgeuse has a surface temperature of 3600 K and Rigel of 13,000K. Treating the stars as blackbodies, we can calculate their peak wavelength of the continuous spectrum to be 805 nm and 223 nm.



$$\lambda_{max} T = 2.898 \times 10^{-3} \text{ m} \cdot \text{K}$$



Stefan-Boltzmann Law

- The luminosity of a blackbody of area A increases with the temperature

$$L = A\,\sigma\,T^4$$

- This is known as the **Stefan-Boltzmann law**, with the constant σ experimentally measured to be 5.6704 x 10^{-8} W / (m^2 · K^4)

- For a star of radius R we obtain

$$L = 4\,\pi\,R^2\,\sigma\,T_e^{\,4}$$

with T_e the effective temperature (different form blackbody)

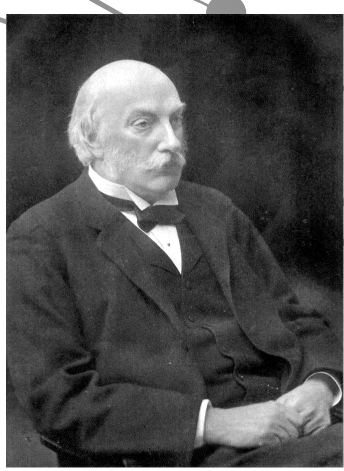
- For the surface flux of a star we get

$$F = \sigma\,T_e^{\,4}$$

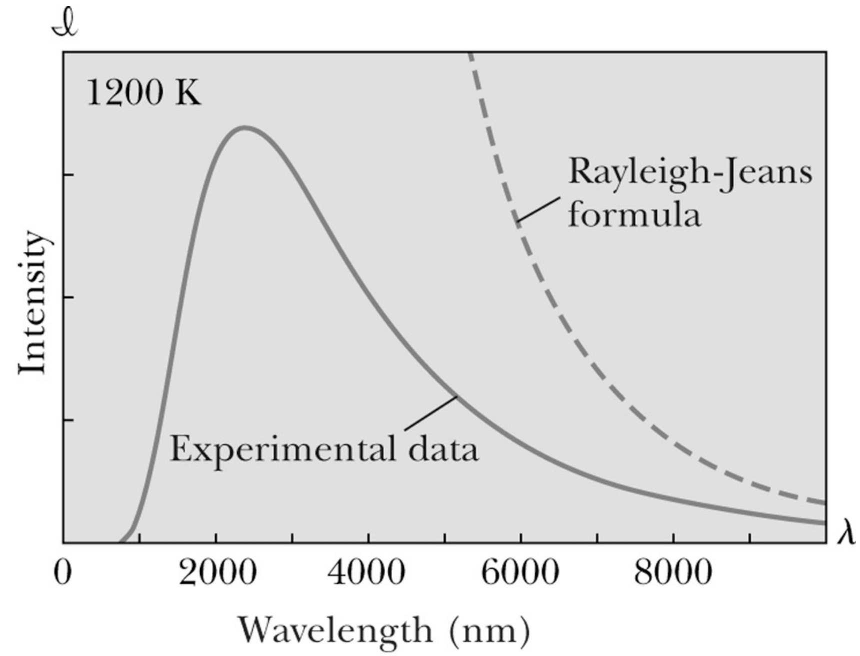
Rayleigh-Jeans Formula

- Lord Rayleigh used the classical theories of electromagnetism and thermodynamics to predict the blackbody spectral distribution
- The formula fits the data at long wavelengths, but it deviates strongly at short wavelengths
- This problem for small wavelengths became known as the *ultraviolet catastrophe*

$$B(T) = \frac{2ckT}{\lambda^4}$$



John Strutt (1842 - 1919)



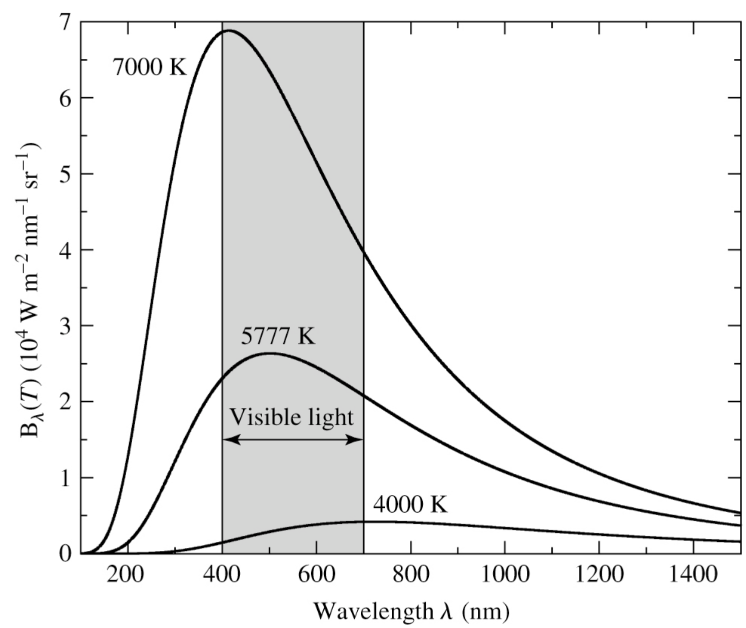
Planck's Radiation Law

- Planck assumed that the radiation in the cavity was emitted and absorbed by some sort of *oscillators* contained in the walls
- Planck used Boltzmann's statistical methods to arrive at the following formula that fit the blackbody radiation data

$$B(T) = \frac{2c}{\lambda^4} \frac{hc}{\lambda} \frac{1}{e^{hc/\lambda kT} - 1}$$

with k the Boltzmann constant

Max Planck (1858 - 1947)



Planck's Radiation Law

Planck made two modifications to the classical theory

- The oscillators (of electromagnetic origin) can only have certain discrete energies determined by

$$E_n = n\,h\,\nu$$

  with n is an integer,

  ν is the frequency and

  h = 6.6261 x 10^{-34} J·s is called Planck's constant

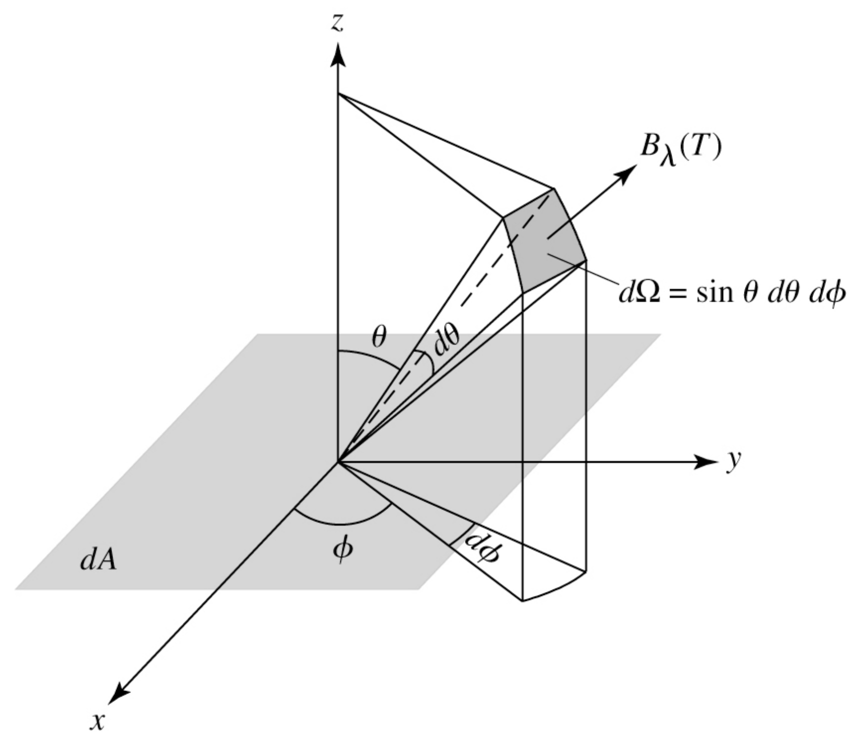
- The oscillators must absorb or emit energy in discrete multiples of the fundamental quantum of energy given by

$$\Delta E = h\,\nu$$

Blackbody Radiation

The amount of radiation energy emitted by a blackbody of T and surface area A per unit time having a wavelength between λ and $\lambda + d\lambda$ into a solid angle $d\Omega = \sin\theta \, d\theta \, d\phi$ is given by

$$B(T)\, d\lambda \, dA \cos\theta \, d\Omega$$
$$= B(T)\, d\lambda \, dA \cos\theta \, \sin\theta \, d\theta \, d\phi$$



Blackbody Radiation

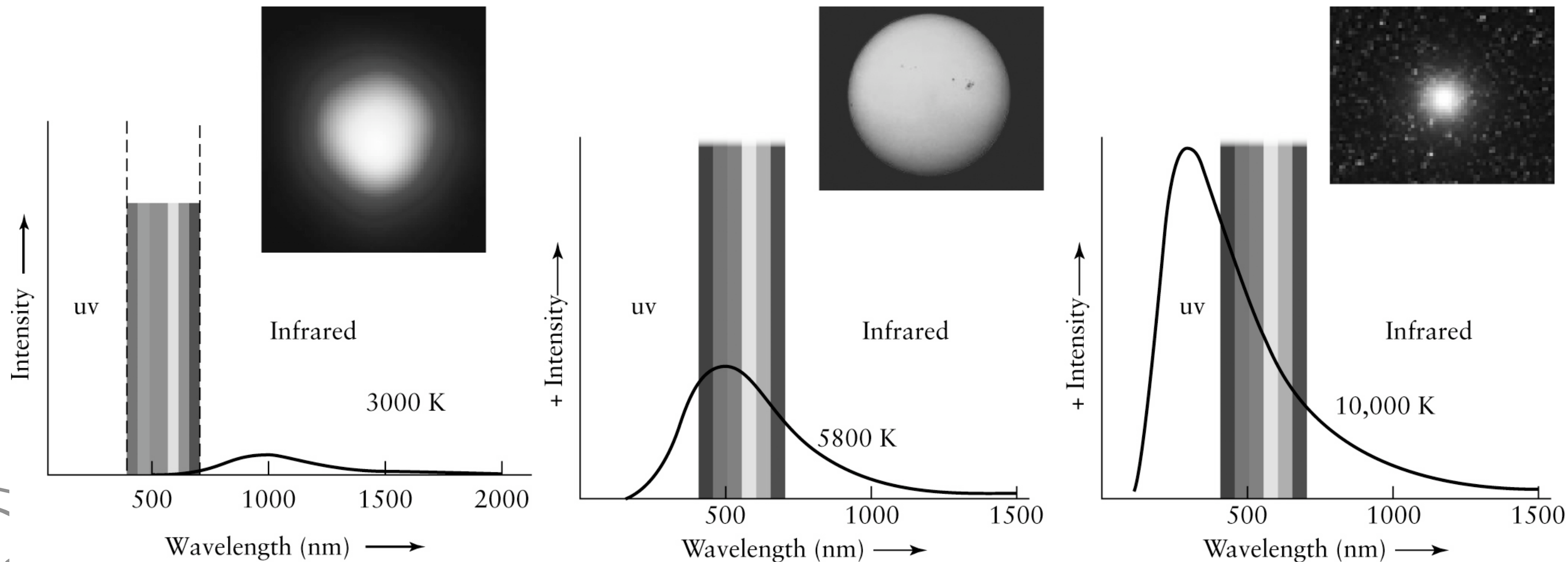
- Considering a star as a spherical blackbody of radius R and temperature T with each surface area dA emitting radiation isotropically
- The energy per second emitted by this star with wavelength between λ and $\lambda + d\lambda$ is the monochromatic luminosity

$$L \; d\lambda = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} \int_{A} B \; d\lambda \, dA \cos\theta \; \sin\theta \, d\theta \, d\phi$$

$$= 4 \pi^2 R^2 B \, d\lambda$$

$$= \frac{8 \pi^2 R^2 h c^2}{\lambda^5 (e^{hc/\lambda kT} - 1)} \; d\lambda$$

Temperature and Color



- The intensity of light emitted by three hypothetical stars is plotted against wavelength
- Where the peak of a star's intensity curve lies relative to the visible light band determines the apparent color of its visible light
- The insets show stars of about these surface temperatures

Color Indices

- The color of a star can be determined by using filters of narrow wavelength bands
- The apparent magnitude is measured through three filters in the standard *UBV* system
  - Ultraviolet *U* band 365 nm ± 34 nm
  - Blue *B* band 440 nm ± 49 nm
  - Visual *V* band 550 nm ± 45 nm