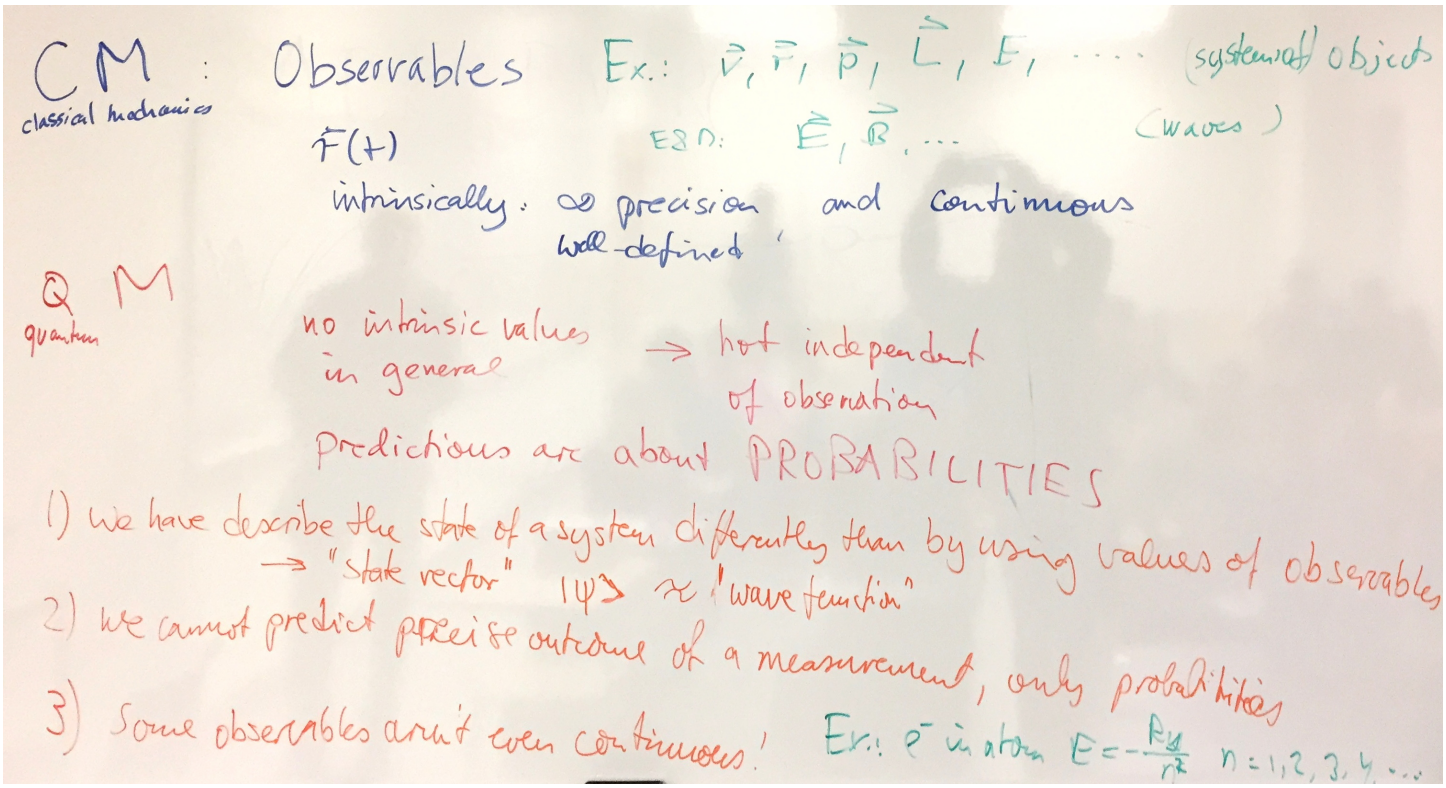
Lecture Notes PHYS323 9/20/16

Concluded (for now) discussion of Special Relativity
Short mention of General Relativity: Scheme to incorporate gravity into Special Relativity not as a force, but as a curvature of space-time.

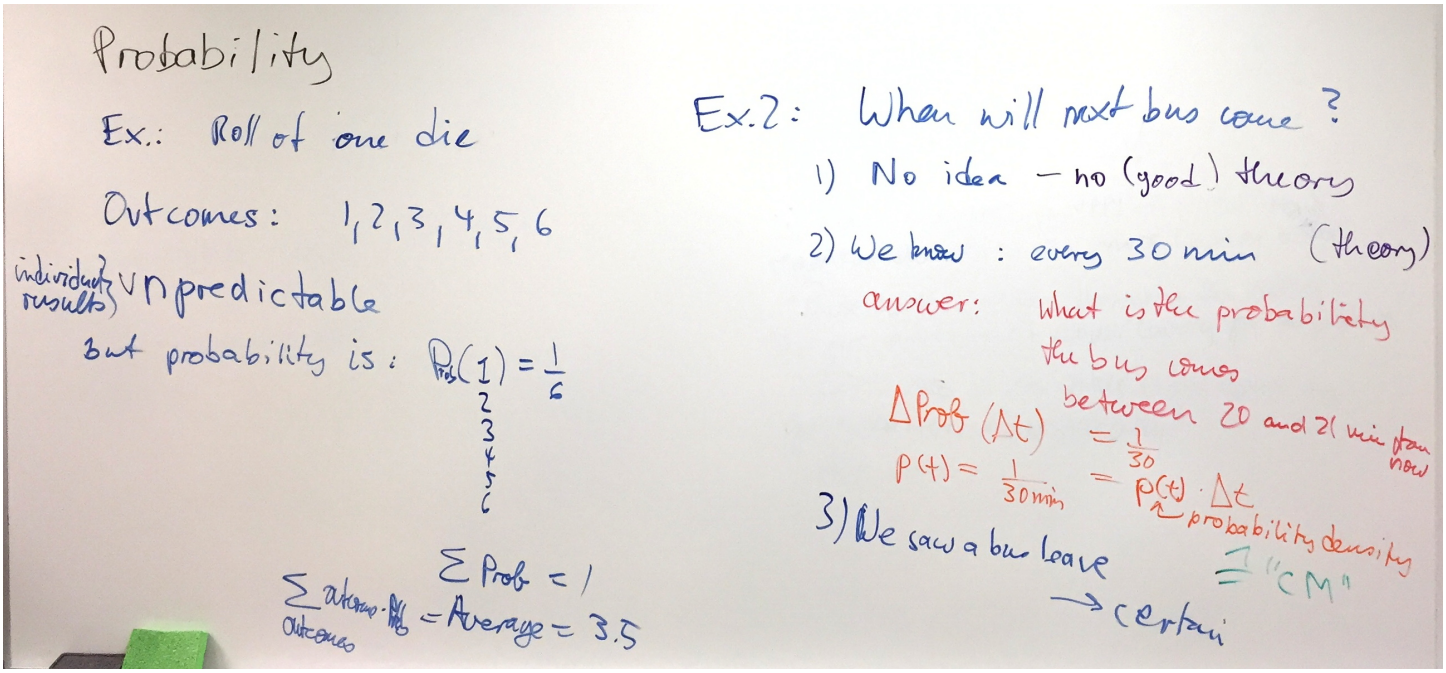
Now pivoting to Quantum Mechanics (QM). First question: What "obvious" everyday notions about reality do we have to give up on in the light of this 2nd early 20th-century revolution in Physics?



As this screen shot shows, the underlying idea of Classical Mechanics (CM) - including Special Relativity - is that observables like velocity, position, momentum, angular momentum, energy etc. are "real" properties that particles or systems of particles "have" - while we can't **measure** them with infinite precision, they do **have** precise values on a continuum (real numbers). In particular, it makes sense to talk about these quantities as well-defined functions of time - **r**(t), **p**(t) etc. Furthermore, Newtonian dynamics connect them among each other - e.g., position determines force, and force in turn determines change of momentum.
Similarly for waves, e.g. electromagnetic waves, that can be characterized by continuous, arbitrarily precise (in principle) values of electric and magnetic fields everywhere in space.

Quantum Mechanics is more akin to the idea by Plato that we cannot observe the world "as it really is", but rather we can only derive laws about our observations. Objects like

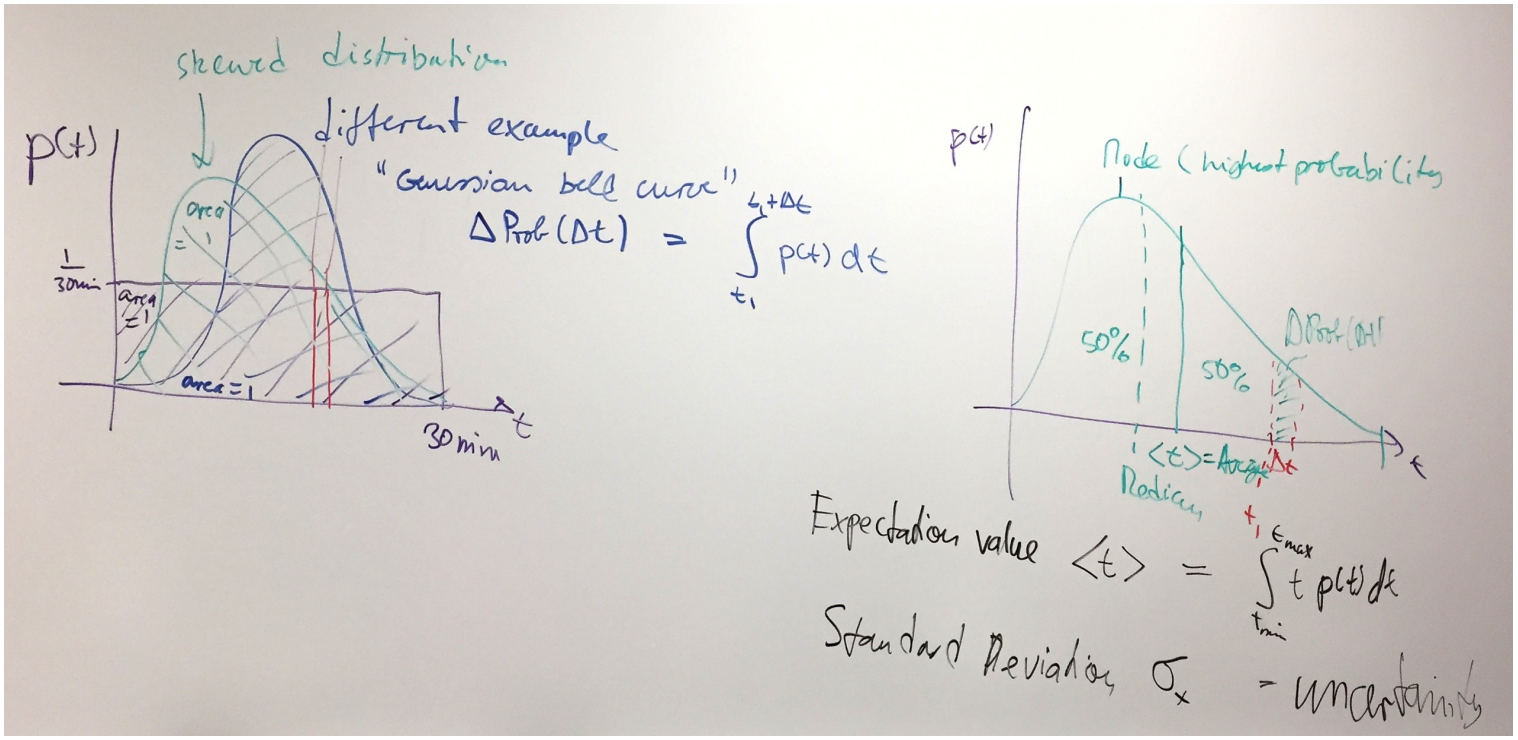
particles or systems are not characterized by precise values of observables, but rather by their "state vectors" which contain all the information we **can** have about those objects (from observations). Instead of predicting precise values for observables, these state vectors (a.k.a "wave functions") only predict **probabilities** for certain outcomes. Even more strange, some observables can only assume discrete values out of a pre-determined set (like eyes on a die) - not a continuous range of values (example: energy of an electron bound in an atom). Hence, to study QM we must study probability:



Distinguish between **discrete** outcomes (eyes on a die) with a given probability for each outcome and **continuous** range of outcomes (wait times for a bus) with a given **probability distribution** - in that case, the probability of a certain observation with a given range is proportional to the size of that range: $\Delta\text{Prob}(x \ldots x+\Delta x) = p(x) \cdot \Delta x$. In the first case, the probability for a set of outcomes (e.g., "more than 3 eyes" is the sum over the individual outcomes (here: 1/6 + 1/6 + 1/6 = ½), and the sum over all probabilities is 1. In the second case, the probability for an outcome in a larger range is the integral of $p(x)$ over that range, and the integral over the whole possible range of outcomes is 1.

Some examples and definitions (see screen shot on next page):
- flat distribution (all outcomes equally likely
- "Gaussian bell curve" - symmetric around maximum
- skewed distribution (long "tail" on one side of maximum

For each distribution, one can define the
- Mode: The value x for which the distribution (or probability) reaches its maximum; the most likely outcome
- Median: The value x for which the probability of a result below x is equally likely (50%) as for a result above x ("split the distribution in the middle")
- Average or Mean or Expectation Value: Best estimate of result (see below)



Can define the expectation value for any function of the observable, $f(x)$:

$$\langle f \rangle = \int f(x)p(x)dx$$

(the integration goes over the full range of possible outcomes)

Expectation value for the variable itself (i.e., its Mean or Average):

$$\mu \equiv \langle x \rangle = \int xp(x)dx$$

Expectation value for the mean square deviation from the mean (known as the variance):

$$\sigma^2 = \left\langle (x-\mu)^2 \right\rangle = \int (x-\mu)^2 p(x)dx = \int x^2 p(x)dx - 2\mu \int xp(x)dx + \mu^2 \int p(x)dx =$$

$$= \langle x^2 \rangle - 2\mu\langle x \rangle + \mu^2 = \langle x^2 \rangle - \langle x \rangle^2$$

The square root of the variance is known as the "standard deviation" and provides a measure on the "uncertainty" of the outcome, i.e., how far any given measurement likely will deviate from the average:

$$\Delta x \equiv \sqrt{\sigma^2} = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$$