CM (assume $V_{pot}(\vec{r})$ is known) QM

CM:

$\bullet\; m$

$\vec{r}(t), \vec{p}(t)$

$\dfrac{d\vec{r}}{dt} = \dfrac{\vec{p}}{m r} \qquad \dfrac{d\vec{p}}{dt} = -\vec{\nabla} V_{pot}$

$H = \dfrac{\vec{p}^2}{2m} + V_{pot}$

$\dfrac{dr_i}{dt} = \dfrac{\partial H}{\partial p_i} \qquad \dfrac{dp_i}{dt} = -\dfrac{\partial H}{\partial r_i}$

QM: $i\hbar \dfrac{\partial |\psi\rangle}{\partial t} = H |\psi\rangle$

Schrödinger Equation

Operators to extract Probabilities

QM:

$\vec{r}, \vec{p}$ are random variables

$\left.\begin{array}{l}\Delta \vec{r} \\ \Delta \vec{p}\end{array}\right\} =$ "uncertainty"

$\Delta \vec{r} \cdot \Delta \vec{p} \geq \dfrac{h}{4\pi} = \dfrac{\hbar}{2}$

Heisenberg!

$\Delta L_x \cdot \Delta L_z \geq \dfrac{\hbar}{2}$

Instead:

Probability Amplitude

$\left.\begin{array}{l} \mathbb{C}^3 \end{array}\right\}$ = State Vector $|\psi\rangle(t)$

= Wave function

Prob $\sim |\langle\bar{r}|\psi\rangle|^2$

Reminder: Need to understand vectors and operators

Vector   Coordinates = Basis (orthonormal)

+ Components

$(x, y, z) \quad (n=3)$ Ex.: $\hat{x}, \hat{y}, \hat{z}$

n dimensional

(even ∞ dimensions: countable $n = \mathbb{Z}_0$

continuous $n = \mathbb{R}$ Ex: $f(x)$

can be added

can multiply with scalar

$\to$ Ex.: $\mathbb{R}$

Vector Space: Vectors (basis) + Scalar field

QM: $n = 2$ (coin), ... ∞ (continuous: particle on x-axis)
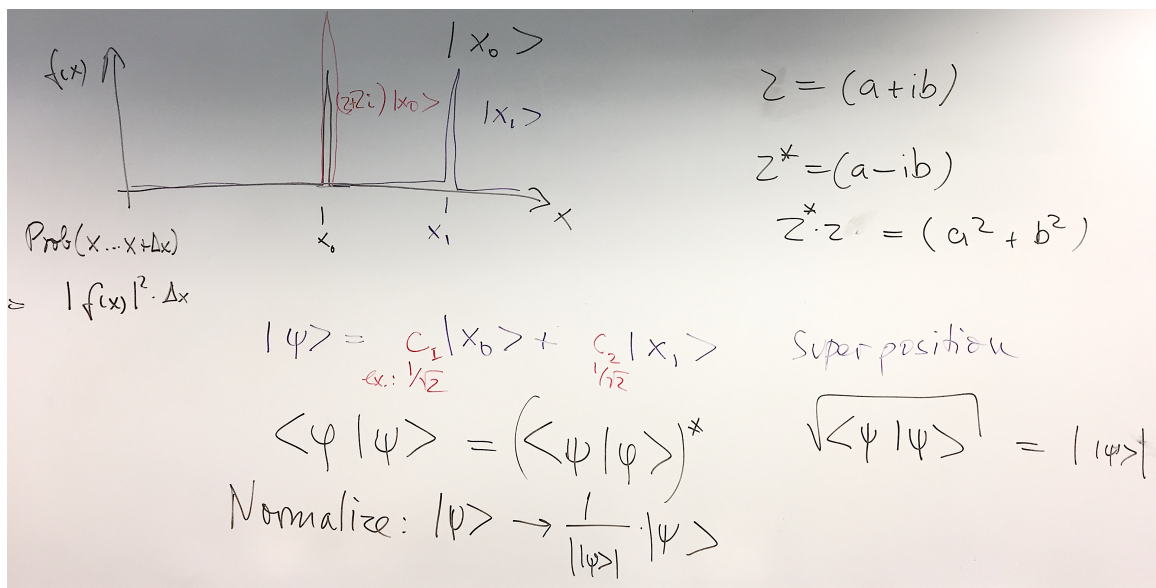
Basis: "fundamental states"

$n=2$: $|heads\rangle; |Tails\rangle$ | $n=\infty$: $|x\rangle \quad x \in \mathbb{R}$

Scalar field = $\mathbb{C}$

for any $c \in \mathbb{C}$ $|\psi\rangle$ and $c|\psi\rangle$ describe SAME physical state

Scalar Product: $\langle\psi|\varphi\rangle$
$= \psi_1^* \varphi_1 + \psi_2^* \varphi_2 + \ldots$

$|x_0\rangle$

$f(x)$

$(2\pi i)|x_0\rangle$ $|x_1\rangle$

x_0 x_1

$\text{Prob}(x \ldots x+\Delta x)$

$\approx |f(x)|^2 \cdot \Delta x$

$z = (a+ib)$

$z^* = (a-ib)$

$z^* \cdot z = (a^2 + b^2)$

$|\psi\rangle = c_1 |x_0\rangle + c_2 |x_1\rangle$ Superposition

ex.: $\frac{1}{\sqrt{2}}$ $\frac{1}{\sqrt{2}}$

$\langle \varphi | \psi \rangle = \left(\langle \psi | \varphi \rangle \right)^*$ $\sqrt{\langle \psi | \psi \rangle} = \||\psi\rangle\|$

Normalize: $|\psi\rangle \to \frac{1}{\||\psi\rangle\|} \cdot |\psi\rangle$

Example: infinite-dimensional vectors = functions f(x). Superposition, normalization,…

Operator   Hilbert space

$$O : |\psi\rangle \longrightarrow |\varphi\rangle$$

$$c_1 |\psi\rangle \longrightarrow c_1 |\varphi\rangle$$

$$|\psi_1\rangle + |\psi_2\rangle \longrightarrow |\varphi_1\rangle + |\varphi_2\rangle$$

linear transformation   (Matrix)

O: Eigen vector $|\varphi_0\rangle$ $O|\varphi_0\rangle = g|\varphi_0\rangle$

Example   Eigen value

$$X|x_0\rangle = x_0 |x_0\rangle$$