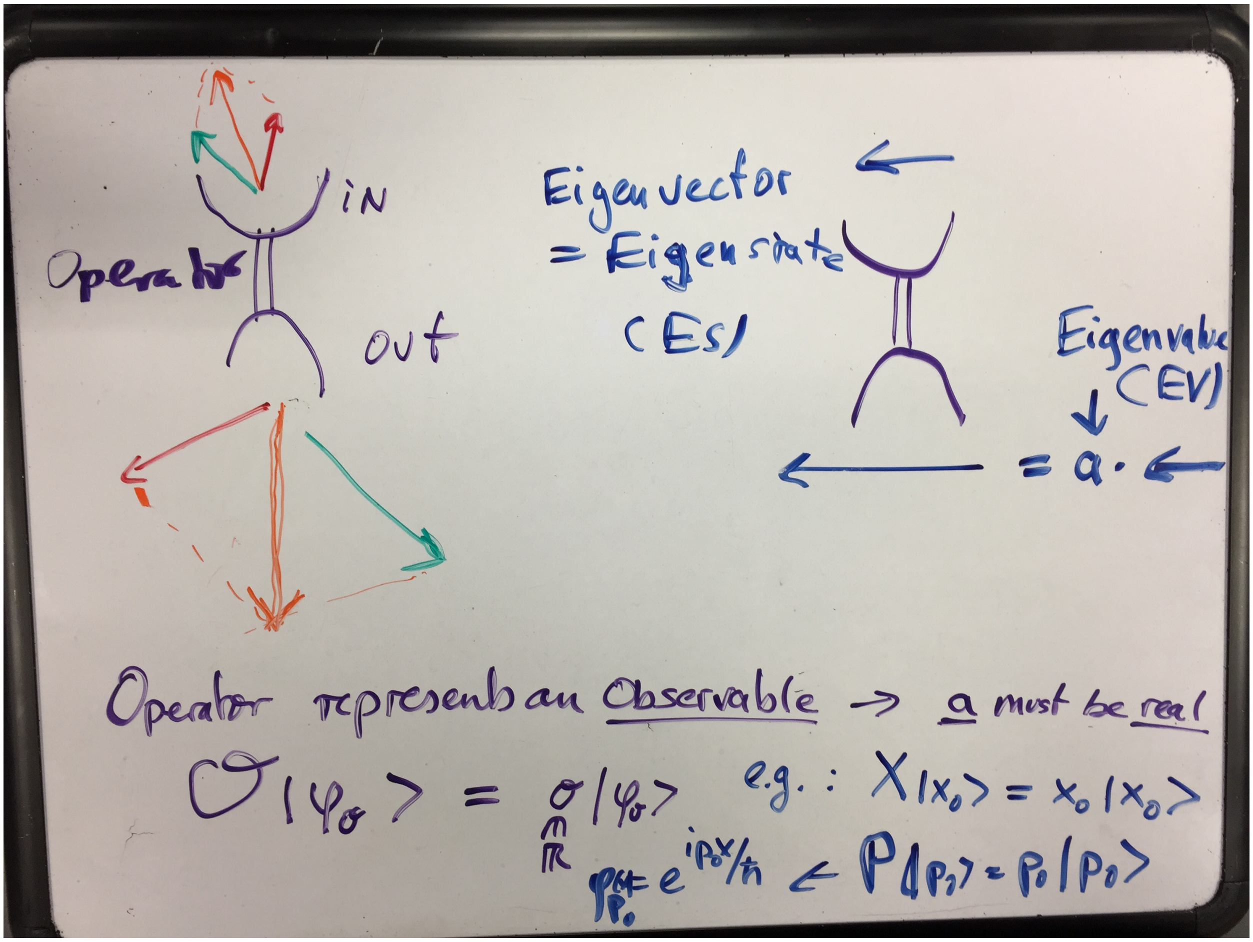
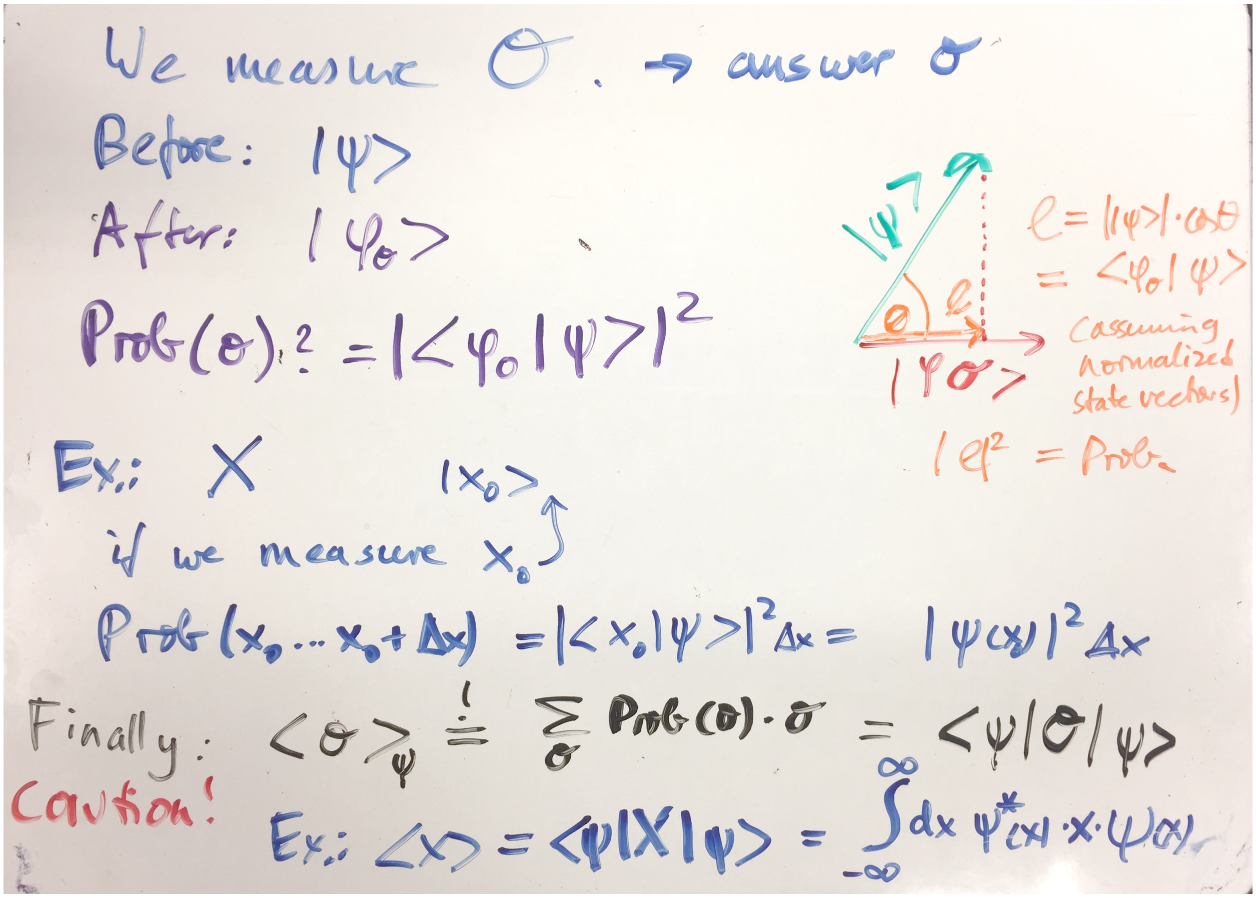
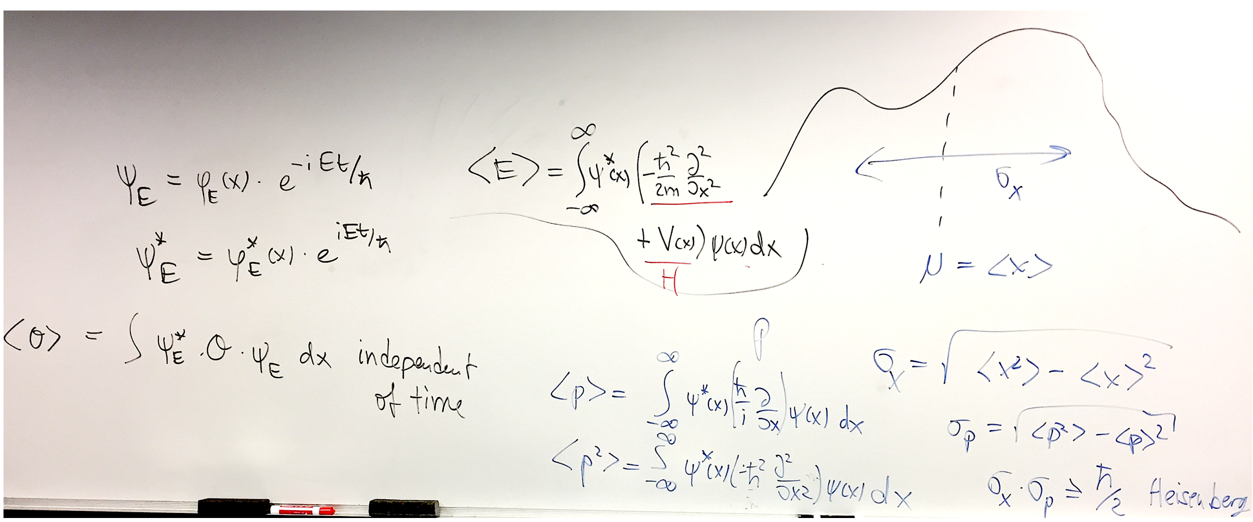
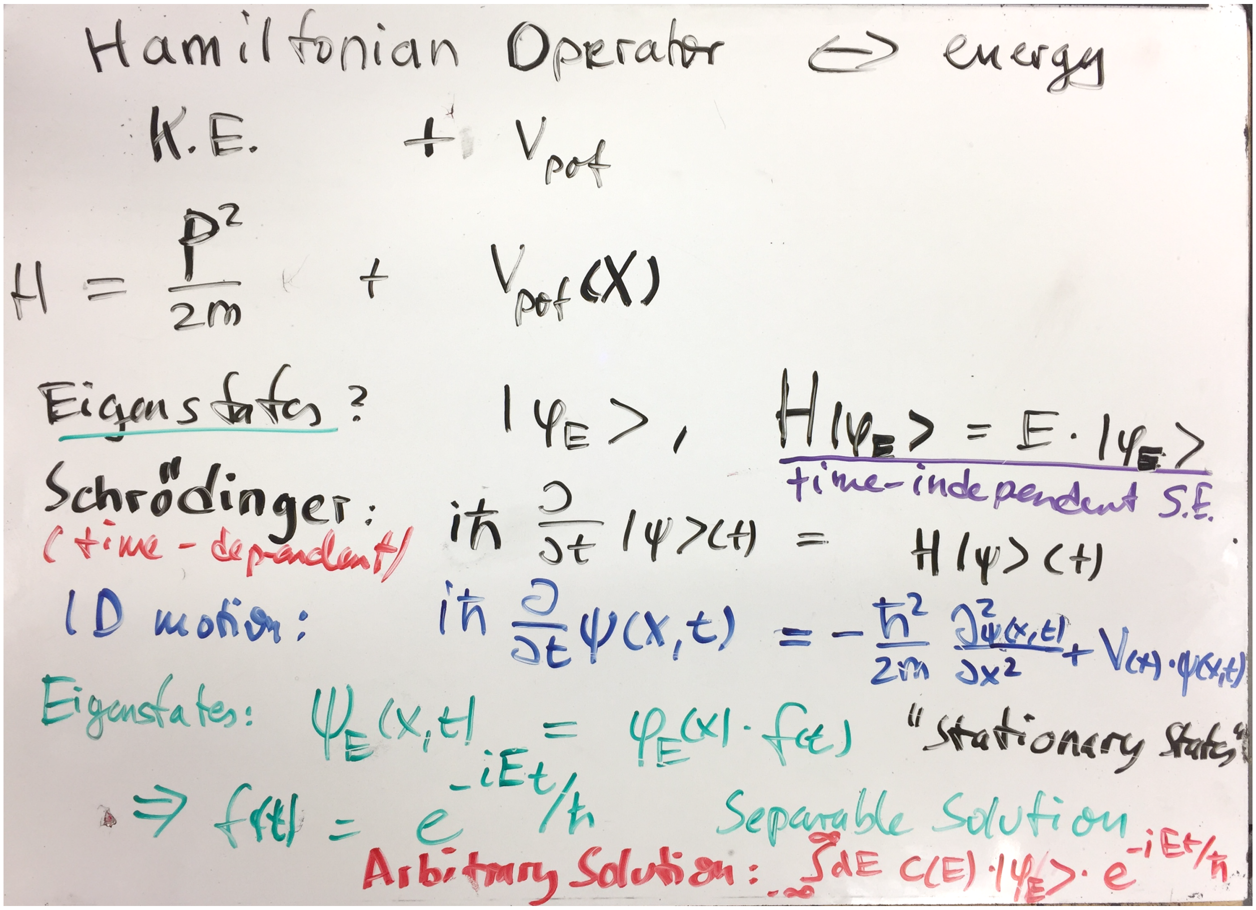


Pictorial representation of how a linear operator works, using 2D vectors as an example.



Observables, eigenstates, eigenvalues, measurements and pictorial representation of projection. Further examples:





Hamiltonian Operator represents the observable “Energy”
Eigenstates are vectors representing states of the system with defined, infinitely precise energy values (e.g., the energy levels of atoms)
Eigenvalues are the possible outcomes of the measurement of energy

Biggest importance: Hamiltonian describes how a state vector changes over time:
(time-dependent) SCHRÖDINGER EQUATION (SE).

Special solutions (separable) can be written as product of an eigenstate of the Hamiltonian and a purely time-dependent phase factor. These states are essentially “stationary” (meaning no observable will change its expectation value with time, unless the observable depends on time explicitly). Even better, ANY solution of the SE can be written as linear superposition of these special solutions. -> Hence most of QM is just looking for eigenstates of the Hamiltonian (i.e., solving the so-called “time-independent SE”).

Time-independent SE:

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \varphi_E(x)}{\partial x^2} + \underline{V(x)}\cdot\varphi_E(x) = E\cdot\varphi_E(x)$$

Ex.: free motion $\longrightarrow V(x) = 0$

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \varphi_E(x)}{\partial x^2} = E\,\varphi_E(x)$$

$$\varphi_E \Rightarrow \varphi_{p_e}(x) = e^{i p_e x/\hbar}$$

$$\Rightarrow +\frac{\cancel{\hbar}^2}{2m}\left(\frac{i p_e}{\cancel{\hbar}}\right)^2 e^{i p_e x/\hbar} = E\, e^{i p_e x/\hbar}$$

$$\Rightarrow \frac{p_e^2}{2m} = E \longrightarrow \varphi_E(x,t) = e^{i p_e x/\hbar}\cdot e^{-i\frac{p_e^2}{2m}t/\hbar}$$